STRESS-STRAIN RELATIONSHIP OF SOILS AS ANISOTROPIC BODIES UNDER THREE DIFFERENT PRINCIPAL STRESSES

SHŌJI YOSHIDA*

ABSTRACT

The author has shown in his previous paper (Yoshida, 1978) that a set of mean normal stress and deviatoric stress (which is not identical to principal stress difference) can be adopted reasonably to describe the stress-strain relationship for soils; furthermore, the author's new idea that dilatancy is defined as the volumetric strain induced by deviatoric stress led to the conclusion that dilatancy is a phenomenon caused by anisotropy in deformation. These were verified by the results of the triaxial tests by Matsuoka (1974) under radial isotropy.

The stress-strain relationship in a more general case with three different principal stresses under orthogonal anisotropy is developed in this paper, and is verified by the empirical results obtained by Ko and Scott (1967, 1968). Herein, two types of coefficients of deformation, D and G , are introduced for 'shear' and they are assumed to be expressed by the parameters of mean normal stress and principal stress ratios. Five constants n , (a, α) and (b, β) are used to define these coefficients of deformation and were determined directly from the results of conventional triaxial compression tests.

Key words: anisotropy, compression, deformation, dilatancy, drained shear, stress-strain curve

IGC: G2

INTRODUCTION

Now that we have become capable of performing two or three dimensional stress-deformation analyses for soils through the development of computers and numerical analytical techniques, the clarification of the stress-strain relationship for soils has become increasingly important in practice.

Recognizing that the soils are essentially anisotropic deformable bodies, the stress-strain relationship under three different principal stresses, in which new coefficients of deformation were introduced, was presented in a simplified form expressed in terms of mean normal stress and deviatoric stresses. The predictions of strain behaviour under three different principal stresses from the data of conventional triaxial compression tests were then compared with the observed behaviour.

Hereafter in this paper, all stresses are implied as being the effective stresses, and both

* Professor, Research Institute for Hazards in Snowy Area, Niigata University, 2-nocho, Igarashi, Niigata. Formerly Associate Professor, Department of Agricultural Engineering, Niigata University.

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stresses and strains in compression are taken as being positive.

CHARACTERISTICS OF SOILS REGARDED AS ANISOTROPIC DEFORMABLE BODIES

The difficulty in formulating the stress-strain relationship for soils is probably due to the following characteristics of soils in the deformation behaviour : ① compressibility ; ② large deformability ; ③ nonlinear behaviour ; ④ dilatancy ; and, ⑤ anisotropy in structure or deformation. Of these characteristics, compressibility and dilatancy are not usually treated in the plasticity theory for metals but they are regarded as the fundamental features of the granular materials such as soils. It is, therefore, necessary that the stress-strain relationship for soils should include the compressibility and be able, in general, to account for dilatancy phenomenon. Dilatancy has usually been defined qualitatively as volumetric change due to 'shear'. In describing the stress-strain relationship for soils, sets of mean principal stress and any one of the following quantities : principal stress difference, principal stress ratio, or octahedral shear stress have usually been taken as general expressions for stress in the field of soil mechanics.

In the previous paper (Yoshida, 1978), stress was, on the contrary, expressed in terms of mean normal stress and deviatoric stress (which is not identical to principal stress difference), as is usually known in continuum mechanics, and dilatancy was defined as a volumetric change due to deviatoric stress.

The stress tensor is expressed as, σ , and the strain tensor as, ϵ , in terms of their isotropic parts, σ_m and ϵ_v , and deviatoric parts, σ' and ϵ' , respectively ; i.e., we decompose σ and ϵ as :

$$\sigma = \sigma_m + \sigma', \quad \epsilon = \epsilon_v + \epsilon' \quad (1)$$

where

$$\begin{aligned} T_r \sigma &= T_r \sigma_m \quad \text{and} \quad T_r \sigma' = 0 \\ T_r \epsilon &= T_r \epsilon_v \quad \text{and} \quad T_r \epsilon' = 0 \end{aligned} \quad (2)$$

in which the notation " T_r " represents a trace of a tensor and "'" represents the deviatoric part of a tensor.

When the stress-strain relationship for soils is assumed to be expressed as:

$$\epsilon_v = C_1 \sigma_m + C_2 \sigma' \quad (3)$$

$$\epsilon' = C_3 \sigma_m + C_4 \sigma' \quad (4)$$

the second term on the right-hand side of Eq. (3) must not be zero in order to satisfy the above new definition ; that is, dilatancy is the volumetric change caused by the deviatoric stress ; and, consequently the first term on the right-hand side of Eq. (4) also must not be zero. Both the isotropic part of σ' and the deviatoric of σ_m , however, always vanishes, $T_r \sigma' = 0$ and $(\sigma_m)' = 0$, and hence both C_2 and C_3 must not be scalars but tensors in order to satisfy the conditions below,

$$T_r(C_2 \sigma') \neq 0 \quad \text{and} \quad (C_2 \sigma')' = 0 \quad \text{for} \quad C_2 \sigma'$$

$$(C_3 \sigma_m)' \neq 0 \quad \text{and} \quad T_r(C_3 \sigma_m) = 0 \quad \text{for} \quad C_3 \sigma_m$$

respectively. When C_i is tensors C_i , and Eqs. (3) and (4) are combined, the following equation ensues,

$$\epsilon = K \cdot \sigma_m + D \cdot \sigma' \quad (5)$$

where

$$K = C_1 + C_3 \quad \text{and} \quad D = C_2 + C_4$$

From Eq. (5), the following equation can be written,

$$\epsilon_v = T_r(K \cdot \sigma_m) + T_r(D \cdot \sigma') \quad (6)$$

where $T_r(\mathbf{K} \cdot \boldsymbol{\sigma}_m)$ is assumed to be equal to $T_r(\mathbf{C}_1 \cdot \boldsymbol{\sigma}_m) (\neq 0)$ and $T_r(\mathbf{D} \cdot \boldsymbol{\sigma}')$ to be equal to $T_r(\mathbf{C}_2 \cdot \boldsymbol{\sigma}') (\neq 0)$.

The second term on the right-hand side of Eq. (6) represents the dilatancy defined above as the volumetric change due to deviatoric stress. As shown in Eq. (5), the coefficients of deformation, $\mathbf{K}$ and $\mathbf{D}$, must be tensors to represent dilatancy; i.e., the coefficients of deformation are dependent on direction within the soils. This means that the deformation behaviours of soils are essentially anisotropic. Consequently, it is concluded that the dilatancy defined as a volumetric change due to deviatoric stress can be taken into account only by assuming the anisotropy of deformation behaviours as described above. Conversely, the isotropy of deformation behaviours does not lead to dilatancy and, hence, soils that have dilatancy characteristics attributively, must be treated as essentially anisotropic deformable bodies. Furthermore, the first term on the right-hand side of Eq. (5) is noted as representing the anisotropic deformation due to mean normal stress or isotropic pressure.

There has been a number of investigations treating the anisotropy in stress-strain relationship for soils*. However, this anisotropy has not been described as in the forms of Eq. (5) and, therefore, even an isotropic soil was thought to have dilatancy characteristics as a fundamental property peculiar to soils.

From the above point of view, the stress-strain relationship for soils for triaxial compression test condition ($\sigma_1 \geq \sigma_2 = \sigma_3$) was thought to be expressible simply as follows (Yoshida, 1978):

$$\left. \begin{aligned} \varepsilon_1 &= \frac{K_1}{1+e_0} \log \frac{\sigma_m}{p_0} + D_1 \sigma_1' \\ \varepsilon_2 = \varepsilon_3 &= \frac{K_3}{1+e_0} \log \frac{\sigma_m}{p_0} + D_3 \sigma_3' \end{aligned} \right\} \quad (7)$$

where e_0 is an initial void ratio at $\sigma_m = p_0$, p_0 is an isotropic pressure, σ_i are principal stresses, and σ_i' are deviatoric stresses with $i=1, 2, 3$, and

$$\begin{aligned} \sigma_m &= \frac{1}{3}(\sigma_1 + 2\sigma_3) \\ \sigma_1' &= \sigma_1 - \sigma_m = \frac{2}{3}(\sigma_1 - \sigma_3) \\ \sigma_2' &= \sigma_3' = \sigma_3 - \sigma_m = -\frac{1}{3}(\sigma_1 - \sigma_3) \end{aligned}$$

In Eq. (7) the forms of the first terms on the right-hand side were assumed considering the experimental evidence, as is well known, with respect to consolidation, and by referring to the forms adopted in the investigations by Matsuoka (1974 B) and others. Since the first term on the right-hand side of Eq. (7) is not expressed exactly in the form of $\mathbf{C}_1 \cdot \boldsymbol{\sigma}_m$ as in Eq. (5), Eq. (7) may be lacking in mathematical unity. Furthermore, when D_i are regarded as being a function of stress in order to consider nonlinear behaviour, D_i can probably no longer construct a tensor. It is, however, held in Eq. (7) that (a) independent variables are mean principal stress and deviatoric stresses, and (b) anisotropy of deformation is expressed by $K_1 \neq K_3$ and $D_1 \neq D_3$. The adopting deviatoric stresses σ_i' and not principal stress difference, $\sigma_1 - \sigma_3$, therefore, makes possible the expression of the stress-strain relationship in such simple forms as Eq. (7) and consideration, as a matter of course, of the factors $2/3$ and $-1/3$ for principal stress difference $\sigma_1 - \sigma_3$.

Considering, furthermore, the experimental facts that compressible bodies like soils become hard with the increase of mean normal stress and induced anisotropy in structure

* The problem on dilatancy and anisotropy was investigated by Tatsuoka (1976) from another point of view.

or deformation varies with principal stress ratio (Holubec, 1968 ; Moroto, 1972 ; Oda, 1972), the two coefficients of deformation, D_1 and D_3 , in Eq. (7) were assumed to be expressed in the forms with parameters of mean principal stress, σ_m , and principal stress ratio, σ_1/σ_3 , as follows:

$$D_1 = \frac{a}{\sigma_m^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\alpha, \quad D_3 = \frac{b}{\sigma_m^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\beta \quad (8)$$

where n , (a, α) and (b, β) are constants. The values of constants (a, α) and (b, β) were determined directly from a triaxial compression test ($\sigma_1 \geq \sigma_2 = \sigma_3$) with constant mean principal stress whose value is unit, for example, $\sigma_m = 1.0 \text{ kgf/cm}^2$. The values of a and b thus determined are considered to reflect the initial anisotropic state at $\sigma_m = 1$. When the values of a and b are determined from the results of the compression test with $\sigma_m = \text{const.}$, whose value is not unit ($\sigma_m = p_0 \neq 1$), Eq. (8) must be written in the following manner:

$$D_1 = \frac{a}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\alpha, \quad D_3 = \frac{b}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\beta \quad (9)$$

When $p_0 = 1$ in Eq. (9), Eq. (9) coincides with Eq. (8). It may, therefore, be reasonable to treat σ_m in Eq. (8) as a nondimensional quantity: $\sigma_m \equiv \sigma_m/1$. n would be an exponent to be determined from a series of triaxial compression tests with $\sigma_m = \text{const.}$ whose values are different in each test.

K_1 and K_3 in Eq. (7) may be determined from the isotropic compression tests or the isotropic consolidation test. When soil behaviours are dealt with approximately, instead of K_1 and K_3 ,

$$C/3 \doteq K_1 \doteq K_3$$

can be used, where C corresponds to the compression index C_c in the cases where the mean principal stress increases while it corresponds to the swelling index C_s in the cases where the mean principal stress decreases.

When the two coefficients of deformation, D_1 and D_3 , obtained in this way from the triaxial compression tests on Toyoura sand (Matsuoka, 1974 A) and Fujinomori clay (Matsuoka, 1974 B), were used in Eq. (7), the results of the two types of triaxial tests were well accounted for. The two types of triaxial tests were the extension test with $\sigma_1 = \sigma_2 > \sigma_3$, $\sigma_m = \text{const.}$ on Toyoura sand (Matsuoka and Nakai, 1974 C), and the compression test with $\sigma_3 = \text{const.}$, in which the mean principal stress σ_m varied continuously, on Fujinomori clay (Matsuoka, 1974 B).

From Eqs. (7) and (9) for triaxial compression, the following equations ensue,

$$\begin{aligned} v = \epsilon_1 + 2\epsilon_3 &= \frac{C_0}{1+e_0} \log \frac{\sigma_m}{p_0} + 2(D_1 - D_3) \cdot \frac{\sigma_1 - \sigma_3}{3} \\ &= \frac{C_0}{1+e_0} \log \frac{\sigma_m}{p_0} + \frac{2(aR^\alpha - bR^\beta)}{(\sigma_m/p_0)^n} \cdot \frac{\sigma_1 - \sigma_3}{3} \end{aligned}$$

For sands, the volumetric change due to the increase of mean principal stress, σ_m , is generally a contraction, but volumetric change may become an expansion at higher stress ratios than at a certain value of R (El-Sohby, 1969). This can be explained by the above equation as follows. Under the condition $a > b$ and $\alpha < \beta$, one gets $D_1 < D_3$ at higher stress ratios than at a certain value of R , and, further, when the absolute of the second term (=an expansion due to dilatancy) on the right-hand side of the above equation becomes superior to the first term (=a contraction due to mean principal stress σ_m) on the right-hand side of the above equation with the increase of principal stress ratio R , an expansion occurs while the mean principal stress, σ_m , increases. Herein, the condition $n < 1.0$ is necessary for such cases to be realized.

STRESS-STRAIN RELATIONSHIP UNDER THREE DIFFERENT PRINCIPAL STRESSES

The next discussion is on the soil behaviour in the more general case with three different principal stresses on the basis of the discussion mentioned above. It is, herein, assumed that the axes of orthogonal anisotropy in soil structure coincide with the directions of principal stresses and that three principal stresses change and keep the order of their magnitudes constant. During the process of deformation, therefore, the principal axes of stress and strain may coincide with one another and may not rotate.

As there exist two independent principal stress ratios in the case of three different principal stresses, the method of expression of the coefficients of deformation in terms of two or three principal stress ratios comes into question. Then, noting first that deviatoric stress components, σ_i' ($i=1,2,3$), are expressed identically as follows:

$$\sigma_i' = \sigma_i - \sigma_m = \frac{1}{3} \{2\sigma_i - (\sigma_r + \sigma_s)\} \equiv \frac{1}{3} \{(\sigma_i - \sigma_r) + (\sigma_i - \sigma_s)\} = q_{ir} + q_{is} \quad (10)$$

where

$$q_{ik} = \frac{1}{3}(\sigma_i - \sigma_k), \quad i, r, s \quad \text{and} \quad k=1,2,3; i \neq r, s$$

deviatoric stresses σ_i are understood as consisting of two components, q_{ir} and q_{is} , expressed in terms of principal stress differences. Thus the normal strain ϵ_i^d in i -direction due to deviatoric stress σ_i' is assumed to be produced by composing two strain components ϵ_{ir}^d and ϵ_{is}^d due to two components q_{ir} and q_{is} of σ_i' respectively;

$$\epsilon_i^d = \epsilon_{ir}^d + \epsilon_{is}^d \quad (11)$$

Next, referring to Eqs. (7) and (8) in triaxial condition, when a component q_{ir} of σ_i' is positive or a compressive stress, a compressive strain $\epsilon_{ir}^d (>0)$ is assumed to occur in i -direction and, conversely, when a component q_{is} of σ_i' is negative or an extensive stress, an extensive strain $\epsilon_{is}^d (<0)$ is assumed to occur in i -direction. The abovementioned relationships are shown in the schematic diagram in Fig. 1. In noticing that one type D_1 of coefficients of deformation is considered to be governing the positive stress and strain, and the other type D_3 of coefficients of deformation is considered to be governing the negative stress and strain, the two types of coefficients of deformation, D_{ir} and G_{is} , are defined in the more general forms of D_1 and D_3 respectively as follows:

Fig. 1. Strains due to deviatoric stresses

$$\left. \begin{aligned} D_{ir} &= \frac{a}{(\sigma_m/p_0)^n} \left(\frac{\sigma_i}{\sigma_r} \right)^\alpha & \text{for compression and } \sigma_i > \sigma_r \\ G_{is} &= \frac{b}{(\sigma_m/p_0)^n} \left(\frac{\sigma_s}{\sigma_i} \right)^\beta & \text{for extension and } \sigma_i < \sigma_s \end{aligned} \right\} \quad (12)$$

Hence, in the case of $q_{ir} > 0$ and $q_{is} < 0$, ϵ_i^d in Eq. (11) is expressed as follows:

$$\epsilon_i^d = D_{ir}q_{ir} + G_{is}q_{is} \quad (13)$$

where

$$\begin{aligned} \epsilon_{ir}^d &= D_{ir}q_{ir} = \frac{a}{(\sigma_m/p_0)^n} \left(\frac{\sigma_i}{\sigma_r} \right)^\alpha \frac{\sigma_i - \sigma_r}{3} & \text{for } \sigma_i > \sigma_r \\ \epsilon_{is}^d &= G_{is}q_{is} = \frac{b}{(\sigma_m/p_0)^n} \left(\frac{\sigma_s}{\sigma_i} \right)^\beta \frac{\sigma_i - \sigma_s}{3} & \text{for } \sigma_i < \sigma_s \end{aligned}$$

Should both q_{ir} and q_{is} , two components of σ_i' , be positive, Eq. (11) becomes

$$\epsilon_i^d = D_{ir}q_{ir} + D_{is}q_{is} \quad (14)$$

Should both q_{ir} and q_{is} be negative, Eq. (11) becomes

$$\epsilon_i^d = G_{ir}q_{ir} + G_{is}q_{is} \quad (15)$$

For example, in the case of an ordinary triaxial compression condition when $\sigma_1 > \sigma_2 = \sigma_3$; $q_{12} = q_{13} > 0$, from Eq. (14)

$$\epsilon_1^d = 2D_{13}q_{13} = \frac{2}{3} \frac{a}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\alpha (\sigma_1 - \sigma_3) \quad (16)$$

is obtained, and $q_{31} = -q_{13}$ and $q_{32} = 0$, from Eq. (15)

$$\epsilon_3^d = G_{31}q_{31} = -\frac{1}{3} \frac{b}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\beta (\sigma_1 - \sigma_3) \quad (17)$$

is obtained. Similarly, for a triaxial extension condition when $\sigma_1 = \sigma_2 > \sigma_3$; $q_{12} = 0$ and $q_{31} = q_{32} = -(1/3)(\sigma_1 - \sigma_3)$, the following equations are obtained:

$$\left\{ \begin{aligned} \epsilon_1^d &= D_{13}q_{13} = \frac{1}{3} \frac{a}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\alpha (\sigma_1 - \sigma_3) \\ \epsilon_3^d &= G_{32}q_{32} + G_{31}q_{31} = 2G_{31}q_{31} = -\frac{2}{3} \frac{b}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\beta (\sigma_1 - \sigma_3) \end{aligned} \right.$$

In the case of three different principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$), strains due to deviatoric stress are expressed as follows:

$$\left\{ \begin{aligned} \epsilon_1^d &= D_{12}q_{12} + D_{13}q_{13} = \frac{a}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_2} \right)^\alpha \frac{(\sigma_1 - \sigma_2)}{3} + \frac{a}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\alpha \frac{(\sigma_1 - \sigma_3)}{3} \\ \epsilon_2^d &= G_{21}q_{21} + D_{23}q_{23} = -\frac{b}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_2} \right)^\beta \frac{(\sigma_1 - \sigma_2)}{3} + \frac{a}{(\sigma_m/p_0)^n} \left(\frac{\sigma_2}{\sigma_3} \right)^\alpha \frac{(\sigma_2 - \sigma_3)}{3} \\ \epsilon_3^d &= G_{32}q_{32} + G_{31}q_{31} = -\frac{b}{(\sigma_m/p_0)^n} \left(\frac{\sigma_2}{\sigma_3} \right)^\beta \frac{(\sigma_2 - \sigma_3)}{3} - \frac{b}{(\sigma_m/p_0)^n} \left(\frac{\sigma_1}{\sigma_3} \right)^\beta \frac{(\sigma_1 - \sigma_3)}{3} \end{aligned} \right\} \quad (18)$$

It is interesting to note that the forms of Eq. (18), especially with respect to principal stress ratios, σ_i/σ_j , seem to be very similar to Matsuoka's expressions (Matsuoka, 1974 A, B) which were derived on the basis of the concept of three compound mobilised planes. It should be noted, however, that there is a difference between the two is so far as the principal stress ratios are adopted as independent variables in Matsuoka's expressions while the principal stress ratios are not treated as independent variables in Eq. (18), but as parameters of the coefficients of deformation.

Next, strains ϵ_i^c due to mean principal stress σ_m are assumed to be expressed as follows:

$$\epsilon_i^c = \frac{K_i}{1+e_0} \log \frac{\sigma_m}{p_0}, \quad (i=1, 2, 3) \quad (19)$$

where K_i : coefficients of deformation

e_0, p_0 : initial void ratio and mean principal stress or isotropic pressure, respectively. In Eq. (19), $K_1 \neq K_2 \neq K_3$ corresponds to the assumption that anisotropic deformation occurs even by isotropic pressure. Such facts have been confirmed by a few experiments, (for example, Murayama et al. (1975) ; Yoshida (1978, not published)). The data available seems, however, to be too scanty for the determination of both K_i and (D_{ij}, G_{ij}) simultaneously. Therefore, in the future the coefficients of deformation K_i need to be obtained from experiments.

Thus, from Eqs. (18) and (19), the stress-strain relationship with three different principal stresses may be expressed as follows:

$$\left. \begin{aligned} \epsilon_1 &= \frac{K_1}{1+e_0} \log \frac{\sigma_m}{p_0} + (D_{12}q_{12} + D_{13}q_{13}) \\ \epsilon_2 &= \frac{K_2}{1+e_0} \log \frac{\sigma_m}{p_0} + (G_{21}q_{21} + D_{23}q_{23}) \\ \epsilon_3 &= \frac{K_3}{1+e_0} \log \frac{\sigma_m}{p_0} + (G_{32}q_{32} + G_{31}q_{31}) \end{aligned} \right\} \quad (20)$$

where

$$q_{ij} = \frac{1}{3}(\sigma_i - \sigma_j), \text{ and } \sigma_i > \sigma_j \text{ for } i < j; i, j = 1, 2, 3$$

From Eq. (20), an expression for volumetric strain is obtained.

$$\begin{aligned} v &= \epsilon_1 + \epsilon_2 + \epsilon_3 \\ &= \frac{C}{1+e_0} \log \frac{\sigma_m}{p_0} + \{(D_{12} - G_{21})q_{12} + (D_{23} - G_{32})q_{23} + (D_{13} - G_{31})q_{13}\} \end{aligned} \quad (21)$$

where

$$C = K_1 + K_2 + K_3$$

Here, the volumetric strain v is related to a component ϵ_v of isotropic tensor ϵ_v by $v = 3\epsilon_v$. By decomposing the volumetric strain v into that due to 'consolidation' or compression by σ_m , v^c , and that due to dilatancy, v^d , from Eq. (21), the following equations are obtained,

$$v = v^c + v^d, \quad \begin{cases} v^c = \frac{C}{1+e_0} \log \frac{\sigma_m}{p_0} \\ v^d = (D_{12} - G_{21})q_{12} + (D_{23} - G_{32})q_{23} + (D_{13} - G_{31})q_{13} \end{cases} \quad (22)$$

$$v^d = (D_{12} - G_{21})q_{12} + (D_{23} - G_{32})q_{23} + (D_{13} - G_{31})q_{13} \quad (23)$$

Eq. (23) shows concretely that the volumetric strain v^d due to dilatancy occurs only through anisotropy or $D_{ij} \neq G_{ji}$; conversely, in the isotropic case or $D_{ij} = G_{ji}$, $v^d = 0$; that is, no dilatancy occurs.

COMPARISON OF PREDICTED AND OBSERVED STRESS-STRAIN RELATIONS

Eq. (18) is verified by the experimental results obtained from tests (Ko and Scott, 1967, 1968) on Ottawa sand having three different principal stresses, where the mean principal stress σ_m was kept constant. Since the experiments after Ko and Scott were all carried out with $\sigma_m = 20 \text{ psi} = 1.4 \text{ kgf/cm}^2$, the value of n in Eq. (18) cannot be obtained from their data. However, in order to confirm the stress-strain relationship expressed by Eq. (18) from these experiments with $\sigma_m = \text{const.}$, it was sufficient to obtain only the values of $a' = a/(\sigma_m/p_0)^n$ and $b' = b/(\sigma_m/p_0)^n$ because they became constant. Thus, the values of the constants, (a', α) and (b', β) were obtained from their triaxial compression tests ($\sigma_1 \geq \sigma_2 = \sigma_3$). These were

$$a' = 0.071, \quad \alpha = 2.05, \quad b' = 0.045, \quad \beta = 2.3 \text{ on loose sand } (e_0 = 0.670)$$

$a'=0.054$, $\alpha=1.13$, $b'=0.026$, $\beta=1.65$ on medium sand ($e_0=0.510$)

These values were obtained for strains expressed in percentages and stresses in kgf/cm^2 . By using these constants, the stress-strain relations for the case with three different principal stresses should be predictable. In their experiments, however, the initial void ratios in each experiment with various stress paths differed slightly even on the samples having

Fig. 2. Observed and predicted results of a RS 75° test on a medium loose Ottawa sand, ($e_0=0.606$, $\lambda=0.64$) (Test data from Ko and Scott (1968))

Fig. 4. Observed and predicted results of a RS 45° test on a medium loose Ottawa sand, ($e_0=0.610$, $\lambda=0.40$) (Test data from Ko and Scott (1968))

Fig. 6. Observed and predicted results of a RS 75° test on a medium dense Ottawa sand, ($e_0=0.532$, $\lambda=0.85$) (Test data from Ko and Scott (1968))

Fig. 3. Observed and predicted results of a RS 60° test on a medium loose Ottawa sand, ($e_0=0.614$, $\lambda=0.90$) (Test data from Ko and Scott (1968)) (All the figures 2 through 9 have the same legend shown in Fig. 2.)

Fig. 5. Observed and predicted results of a triaxial extension test (RS 30°) on a loose Ottawa sand, ($e_0=0.670$, $\lambda=1.2$) (Test data from Ko and Scott (1968))

Fig. 7. Observed and predicted results of a RS 60° test on a medium dense Ottawa sand, ($e_0=0.518$, $\lambda=1.45$) (Test data from Ko and Scott (1968))

Fig. 8. Observed and predicted results of a RS 45° test on a medium dense Ottawa sand, ($e_0=0.526$, $\lambda=1.33$) (Test data from Ko and Scott (1968))

Fig. 9. Observed and predicted results of a triaxial extension test (RS 30°) on a dense Ottawa sand, ($e_0=0.502$, $\lambda=0.90$) (Test data from Ko and Scott (1968))

the same range of densities, i.e., medium loose or medium dense. These differences could have effect on the constants, a' and b' . Consequently, the stress-strain curves predicted by the use of the constants obtained above may not necessarily agree well with the observed curves. When the factor λ multiplied commonly with a' and b' for each test having various stress paths are selected precisely, the predicted curves would then agree well with the experimental results as shown in Figs. 2~9. We were not able to recognize a definite relationships between the factor λ and initial void ratio e_0 . Further experimentation is needed to learn how the constants a' and b' depend on void ratio.

CONCLUSIONS

In formulating the stress-strain relationship for soils, the following ways were adopted:

- (1) As is well known in continuum mechanics, stress is expressed in terms of mean normal stress and deviatoric stress (which is not identical to principal stress difference).
- (2) When the dilatancy of soils is defined to be a volumetric strain due to deviatoric stress, the dilatant phenomenon of soils is postulated as occurring only on the basis of anisotropy of deformation, and, therefore, soils must be treated as essentially anisotropic deformable bodies.
- (3) When considering compressibility as varying with mean normal stress and the induced anisotropy varying with principal stress ratios, the coefficients of deformation, D_{tr} and G_{ts} , with respect to 'shear' are assumed to be expressible by Eq. (12). The five soil constants, n , (a, α) and (b, β) contained in D_{tr} and G_{ts} may be determined from the conventional triaxial compression tests.

With Eq. (18), accounting for all the matters mentioned above, we can predict the stress-strain behaviour in the case of three different principal stresses with constant mean principal stress in good agreement with experimental results.

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NOTATION

- a = constant (in D_1, D_{ij})
 $a' = a/(\sigma_m/p_0)^n$ when $\sigma_m = \text{const.}$
 b = constant (in D_3, G_{ij})
 $b' = b/(\sigma_m/p_0)^n$ when $\sigma_m = \text{const.}$
 $C = K_1 + K_2 + K_3$
 C_c = compression index
 C_s = swelling index
 $C_i (i=1\sim 4)$ = coefficients of deformation (scalars)
 $\mathbf{C}_i (i=1\sim 4)$ = coefficients of deformation (tensors)
 D_1, D_{ij} = coefficients of deformation (associated with positive components of deviatoric stress)
 e_0 = initial void ratio
 D_3, G_{ij} = coefficients of deformation (associated with negative components of deviatoric stress)
 $i, j (=1, 2, 3)$ = principal directions
 $K_i; K_1, K_2, K_3$ = coefficients of deformation (associated with mean normal stress)
 n = exponent (in $D_1, D_{ij}; D_3, G_{ij}$)
 p_0 = initial isotropic pressure
 $q_{ik} = (q_i - q_k)/3$ = components of σ_i' ($\sigma_i' = q_{ir} + q_{is}$)
 $r, s, k (=1, 2, 3)$ = principal directions
 T_r = indicating trace of tensor
 $'$ = indicating deviatoric part of tensor
 v = volumetric strain
 v^c = volumetric strain due to 'consolidation' or σ_m
 v^d = volumetric strain due to dilatancy or σ_i'
 α = constant (in D_1, D_{ij})
 β = constant (in D_3, G_{ij})
 ϵ = strain tensor
 ϵ_v = isotropic part of ϵ
 ϵ' = deviatoric part of ϵ
 ϵ_v = component of ϵ_v
 $\epsilon_i; \epsilon_1, \epsilon_2, \epsilon_3$ = principal strains
 $\epsilon_i^d; \epsilon_1^d, \epsilon_2^d, \epsilon_3^d$ = principal strains due to dilatancy or σ_i'
 λ = fitting factor
 σ = stress tensor (effective)
 σ_m = isotropic part of σ (effective)
 σ' = deviatoric part of σ (effective)
 $\sigma_i; \sigma_1, \sigma_2, \sigma_3$ = principal stresses (effective)
 $\sigma_i'; \sigma_1', \sigma_2', \sigma_3'$ = principal deviatoric stresses (effective)
 $\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$ = mean principal stress or component of σ_m (effective)

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