BEARING CAPACITY OF STRIP FOOTING ON ANISOTROPIC AND NONHOMOGENEOUS CLAYS

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ABSTRACT

Limit analysis (upper bound) has been applied for bearing capacity of strip footing on anisotropic and nonhomogeneous clays, by assuming a mechanism similar to Prandtl mechanism but with varying wedge angles. N_c values for different anisotropic coefficients and degrees of nonhomogeneity are given. The results confirm the fact that the bearing capacity of nonhomogeneous and anisotropic clays obtained by assuming a circular slip surface is on the higher side of the actual value.

Key words: anisotropy, bearing capacity, limit analysis, nonhomogeneity, upper bound
IGC: E 3

INTRODUCTION

Certain clay deposits are found to exhibit nonhomogeneity and anisotropy to a great degree (Bishop, 1966; Casagrande and Carrillo, 1944; Davis and Christian, 1971; Liveneh and Komornik, 1967; Lo, 1965). Studies of several investigators indicate that anisotropy and nonhomogeneity have considerable influence on the bearing capacity of soil (Davis and Brooker, 1973; Davis and Christian, 1971; Raymond, 1967; Reddy and Srinivasan, 1967, 1970 and 1971; Salencon, 1974 a, 1974 b and 1976; Liveneh and Greenstein, 1973). Most of these studies were concerned either for clays exhibiting nonhomogeneity or anisotropy. Reddy and Srinivasan (1967, 1971) considered the effect of both anisotropy and nonhomogeneity and their solution was based on the assumption of a circular failure surface. Reddy and Srinivasan (1970) studied the effect of anisotropy and nonhomogeneity on the bearing capacity of c - ϕ soils including $\phi=0$ condition of soils. Chen (1975) has applied the limit analysis for bearing capacity of footings on a single layer and two layered clays, exhibiting both the anisotropy and nonhomogeneity. Chen's analysis was again based on the assumption of a circular failure surface and hence agreed with the solution of Reddy and Srinivasan (1967).

However, for soils exhibiting anisotropy and nonhomogeneity, the assumption of a circular failure surface is found to considerably overestimate the bearing capacity (Davis and Brooker, 1973; Salencon, 1974 a). The assumption of a circular failure surface by many investigators was due to the fact that for complex problems like anisotropic and nonhomogeneous soils, solutions become very difficult. Limit analysis and particularly the analysis based on the upper bound theorem has been found to be a convenient tool for such problems. Herein, limit analysis has been applied for bearing capacity of strip

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Written discussions on this paper should be submitted before January 1, 1982.

footing on anisotropic and nonhomogeneous clays, by assuming a mechanism similar to Prandtl mechanism but with varying boundary wedge angles.

ANALYSIS

The problem is treated as a two dimensional problem and the analysis is based on the second theorem of Drucker and Prager (1952). The following other assumptions are made.

1. The Prandtl type of failure mechanism is valid (Fig.1).
2. The footing is a surface footing.
3. The soil is assumed to be weightless.
4. The variation of cohesion for an angle i of the inclination of major principal stress with horizontal is given by

$$c_i = c_H + (c_v - c_H) \sin^2 i \quad (1)$$

where

c_i = the strength corresponding to an inclination i of the major principal stress with the horizontal direction,

c_H = the strength corresponding to horizontal direction of major principal stress, and

c_v = the strength corresponding to vertical direction of major principal stress.

5. The ratio of c_v/c_H is constant for a given soil and is denoted by k .

6. The variation of cohesion with depth is linear and cohesion at depth h is given by

$$c_H = c_{HS} + \beta h \quad (2)$$

where

c_{HS} = cohesion in horizontal direction at $h=0$ (Fig.2).

Fig. 1. Assumed failure mechanism

Fig. 2. Variation of cohesion with depth

Fig. 3. Velocity diagram

The failure mechanism for a footing of width B shown in Fig.1 consists of a central wedge abc with wedge angle ξ , a radial shear zone bcd , and a straight line de with boundary angle γ . The angles made by the lines bc and bd with the vertical are designated as θ_1 and θ_2 , respectively. Referring to Fig.3, if V_0 is the relative velocity at c , relative velocity along bc is $V_r = V_0 \tan \xi$ and the vertical velocity $V_1 = V_0 \sec \xi$. Since the failure mechanism assumed is symmetrical, only one half needs to be considered for analysis.

Rate of external work done by the foundation load

$$= \frac{1}{2} q B V_0 \sec \xi \quad (3)$$

Rate of energy dissipation along bc

$$= (\text{Average } c_i \text{ along } bc) \times \text{length } bc \times V_r \quad (4a)$$

$$\text{At } a, \quad c_i = c_{HS} [1 + (k-1) \sin^2 i] \quad (5)$$

$$\text{at } c, \quad c_i = \left[c_{HS} + \frac{1}{2} \beta B \tan \xi \right] [1 + (k-1) \sin^2 i] \quad (6)$$

$$\text{where} \quad i = \xi + \phi \quad (7)$$

Substituting for average c_i , length bc , and V_r , Eq. (4a) reduces to

$$[1 + (k-1) \sin^2 (\xi + \phi)] \left[c_{HS} + \frac{1}{4} \beta B \tan \xi \right] \frac{BV_0}{2 \cos \xi} \quad (4b)$$

Rate of energy dissipation along cd

$$= \int_0^{\theta_1} c_i V_0 r d\theta + \int_0^{\theta_2} c_i V_0 r d\theta \quad (8a)$$

portion cf portion fd

$$\text{For the portion } cf, \quad c_i = \left[c_{HS} + \frac{\beta B}{2 \cos \xi} \cos \theta \right] [1 + (k-1) \sin^2 i] \quad (9)$$

where $i = \theta + \phi$ and

$$\theta = \text{the angle made by the radial line with the vertical.} \quad (10)$$

$$\text{For the portion } fd, \quad c_i = \left[c_{HS} + \frac{\beta B}{2 \cos \xi} \cos \theta \right] [1 + (k-1) \sin^2 i] \quad (11)$$

$$\text{where } i = \theta - \phi \quad (12)$$

Substituting for $\theta_1 = 90 - \xi$ and $\theta_2 = \eta$, Eq. (8a) reduces to

$$\begin{aligned} & \left\{ c_{HS} (\theta_1 + \eta) + \frac{\beta B}{2 \cos \xi} (\cos \xi + \sin \eta) + c_{HS} (k-1) \left[\frac{\theta_1 + \eta}{2} - \frac{1}{4} \sin 2(\theta_1 + \phi) - \frac{1}{4} \sin 2(\eta - \phi) \right] \right. \\ & + \frac{\beta B (k-1)}{2 \cos \xi} \left[\frac{1}{3} \cos^2 \phi (\cos^3 \xi + \sin^3 \eta) + \sin^2 \phi \left(\cos \xi + \sin \eta - \frac{1}{3} \cos^3 \xi - \frac{1}{3} \sin^3 \eta \right) \right. \\ & \left. \left. + \sin 2\phi \left(-\frac{1}{3} \sin^3 \xi + \frac{1}{3} \cos^3 \eta \right) \right] \right\} \frac{BV_0}{2 \cos \xi} \quad (8b) \end{aligned}$$

Rate of energy dissipation in the radial shear zone

$$= \int_0^{\theta_1} c_{iav} r d\theta V_0 + \int_0^{\theta_2} c_{iav} r d\theta V_0 \quad (13a)$$

portion bcf portion bfd

where c_{iav} = the average c_i along a radial line.

In portion bcf , for a radial line

$$i = 90 - \theta + \phi \quad (14)$$

In portion bfd , for a radial line

$$i = 90 - \theta - \phi \quad (15)$$

In Eq. (13a), c_{iav} is calculated as the average of c_i given by Eq. (5) and Eq. (9), but with i by Eqs. (14) and (15). Using these expressions, Eq. (13a) reduces to

$$\begin{aligned} & \left\{ c_{HS} (\eta + 90 - \xi) + \frac{\beta B}{4 \cos \xi} (\sin \eta + \cos \xi) + c_{HS} (k-1) \left[\frac{\eta + 90 - \xi}{2} + \frac{1}{4} \sin 2(\xi + \phi) \right. \right. \\ & \left. \left. + \frac{1}{4} \sin 2(\eta + \phi) \right] + \frac{\beta B (k-1)}{4 \cos \xi} \left[\cos^2 \phi \left(\cos \xi - \frac{1}{3} \cos^3 \xi + \sin \eta - \frac{1}{3} \sin^3 \eta \right) + \sin^2 \phi \right. \right. \\ & \left. \left. \times \left(\frac{1}{3} \cos^3 \xi + \frac{1}{3} \sin^3 \eta \right) + \sin 2\phi \left(-\frac{1}{3} \sin^3 \xi + \frac{1}{3} \cos^3 \eta \right) \right] \right\} \frac{BV_0}{2 \cos \xi} \quad (13b) \end{aligned}$$

Rate of energy dissipation along de

$$= (\text{Average } c_i \text{ along } de) \times \text{length } de \times V_0 \quad (16a)$$

$$\text{Length } de = \frac{B}{2\cos\xi} \cot\eta \quad (17)$$

Along de , $i = \eta - \phi$ and c_i at e is obtained by Eq. (5).

$$\text{At } d, \quad c_i = \left[c_{HS} + \frac{\beta B}{2\cos\xi} \cos\eta \right] [1 + (k-1)\sin^2 i] \quad (18)$$

Substituting for average c_i and length de , Eq. (16a) reduces to

$$[1 + (k-1)\sin^2(\eta - \phi)] \left[c_{HS} + \frac{\beta B}{4\cos\xi} \cos\eta \right] \frac{BV_0}{2\cos\xi} \cot\eta \quad (16b)$$

Equating the rate of external work done (Eq. (3)) to the rate of energy dissipation (Eqs. (4), (8), (13) and (16)) and introducing the nondimensional parameters, $\nu = \beta B/c_{HS}$, $q' = q/c_{vs}$, where c_{vs} = the cohesion in the vertical direction at the surface, the expression for q' is obtained as

$$q' = N_c = f(\nu, k, \phi, \xi, \eta) \quad (19)$$

N_c is the bearing capacity factor.

For the minimum values of q' , ξ and η should satisfy the Eq.

$$\frac{\partial q'}{\partial \xi} = 0 \quad \text{and} \quad \frac{\partial q'}{\partial \eta} = 0 \quad (20)$$

RESULTS AND DISCUSSION

The angle ϕ between the major principal stress and the failure plane is found to be nearly constant and equal to 35 degrees (Lo, 1965) and the same has been assumed in this analysis also. N_c values have been obtained for $k=0.8$ to 2.0 and $\nu=0$ to 1.2.

Fig. 4. Comparison of N_c values

Table 1. Comparison of N_c values

Type of analysis	$k=0.8$			$k=1.0$			$k=2.0$		
	ν			ν			ν		
	0.0	0.8	1.2	0.0	0.8	1.2	0.0	0.8	1.2
Present analysis	6.07	7.78	8.52	5.14	6.56	7.21	3.23	4.24	4.56
Conventional method	6.28	8.23	9.09	5.52	7.19	7.93	3.98	5.08	5.58
Reddy and Srinivasan (1970)	6.0	8.00	8.50	5.14	6.59	7.27	3.50	4.90	5.80
Liveneh and Greenstein (1973)	—	—	—	5.14	6.74	7.61	—	—	—
Salencon (1974a)	—	—	—	5.14	6.79	7.54	—	—	—

Fig. 4 shows the comparison of N_c values obtained by the present analysis and the conventional method. Table 1 compares the values of N_c obtained by the present analysis with those of others. As could be seen from the table, for the case of isotropic and

nonhomogeneous cases, the values of N_c obtained by the present analysis compare well with those of Liveneh and Greenstein (1973), and Salencon (1974a). For clays with $k=2.0$, $\nu=0$; and $k=2.0$, $\nu=1.2$; the conventional method overestimates the value of N_c by about 23.2 per cent and 22.6 per cent respectively. For the case of clays with $k=0.8$, $\nu=0$; and $k=0.8$, $\nu=1.2$, the conventional method overestimates the value of N_c by

about 3.3 per cent and 6.7 per cent, respectively. This result is to be expected since in the present analysis energy dissipation in the radial shear zone is taken into account in addition to energy dissipation along the failure plane, whereas with the circular slip surface, energy dissipation along the failure surface only is considered.

CONCLUSION

The limit analysis (upper bound) applied herein, for bearing capacity of anisotropic and nonhomogeneous clays, confirms that the analysis based on circular slip surface overestimates the bearing capacity for nonhomogeneous and anisotropic clays. The values obtained in the present analysis are therefore more nearer to the actual values.

NOTATION

- B =width of foundation
 N_c =bearing capacity factor
 V_1, V_0, V_r =vertical and relative velocities (Fig. 3)
 c_H, c_v =cohesion corresponding to horizontal and vertical directions respectively
 c_i =cohesion at an inclination i of major principal stress with horizontal direction
 c_{av} =average cohesion along a line
 c_{HS} =cohesion corresponding to horizontal direction at surface
 h =depth of a point below the surface
 k =coefficient of anisotropy
 q =ultimate bearing capacity
 $q'=q/c_{vS}$ =ultimate bearing capacity expressed as a nondimensional quantity
 β =variation of cohesion with depth
 ξ, η =angles which define the failure mechanism (Fig. 1)
 θ =angle between the radial line and vertical line
 θ_1, θ_2 =angles subtended by the radial lines bc and bd (Fig. 1)
 $\nu=\beta B/c_{HS}$ =nondimensional parameter expressing variation of cohesion with depth
 ϕ =angle between major principal stress and failure plane

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(Received January 16, 1980)