MECHANICAL PROPERTIES OF PARTICULATE MATERIAL UNDER GENERAL STRESS CONDITIONS

RYOSUKE KITAMURA*

ABSTRACT

The explanation is performed about the modified triaxial apparatus which can independently control three principal stresses. This apparatus can generate different principal stresses in the cubic specimen by means of the loading system placed in the conventional triaxial chamber. By using this apparatus, the drained shearing tests are carried out under general stress conditions including the repeated loading and the test results are shown. The used material is sandy soil called Toyoura sand. The numerical experiments are also carried out by applying the mechanical model which has been proposed by the author (Kitamura, 1981 a, b) to the mechanical behaviours of particulate material. The results of numerical experiments are compared with the stress-strain relations of Toyoura sand under general stress conditions. From the comparison of these results, it clear that the proposed model can follow the mechanical behaviours of particulate material by using several model parameters, which reflect the mechanical properties at the particle scale, when the energy transferred into or out the specimen, the initial void ratio and the distribution of contact angles are known. Thus, it may be concluded the proposed model is a unified, simple and comprehensive one that can express the various mechanical behaviours of particulate material.

Key words: deformation, granular material, laboratory test, sandy soil, static, stress-strain curve, test equipment

IGC: D 6

INTRODUCTION

In soil mechanics it has been realized that one of the most essential subjects is to establish general constitutive equations for soil with the development of deformation and stability analyses. The conventional triaxial testing apparatus has been often used to attain the above subject in the previous researches. This apparatus is a good one because the element test can easily be performed with good accuracy. But the stress condition generated in the specimen is limited to be axi-symmetric. On the other hand, the stress conditions in the real ground are generally not axi-symmetric but the magnitudes and directions of principal stresses are arbitrary. Thus, it is necessary to investigate the mechanical properties under general stress conditions and to make up the model which can follow the comprehensive mechanical behaviours of soil.

* Lecturer, Faculty of Engineering, Kagoshima University, 21-40, Kourimoto 1-chome, Kagoshima.
Written discussions on this paper should be submitted before July 1, 1982.

In this paper the newly modified triaxial testing apparatus is explained. The apparatus can generate three different principal stresses in the cubic specimen. The intermediate principal stress is transmitted to the specimen by the rigid plates. The loading system for intermediate principal stress is similar to the one that has been developed by Lade and Duncan (1973) in principle. The shearing tests are carried out with Toyoura sand under general stress conditions. The data obtained from these tests are applied to the mechanical model which has been proposed by the author in the previous papers (Kitamura, 1981 a, b) and the numerical experiments are also carried out. The stress-strain relations obtained by the numerical experiments are compared with those obtained by the shearing tests with Toyoura sand. From these comparisons it is apparent that the proposed mechanical model can versatily represent the various mechanical behaviours of particulate material such as sand under general stress conditions including the stress reverse.

TRUE TRIAXIAL APPARATUS

In this paper let's call the apparatus which can generate three different principal stresses in the specimen the true triaxial apparatus.

The true triaxial apparatus are largely divided into three groups by means of the method of generating stresses in the specimen in Fig.1. The true triaxial apparatus with hollow cylindrical specimen belong to the first group. These apparatus are called the follow cylindrical apparatus. The apparatus in this group generate three different principal stresses by applying the different pressures on the inside and outside of hollow cylindrical specimen, and further giving the axial and torsional forces on the ends of specimen. The second group includes the apparatus which have an additional loading equipment to the convetional triaxial apparatus and generate three different principal stresses in the vertical and two horizontal directions of cubic or rectangular prism specimen. The apparatus in this group are called the modified triaxial apparatus in the following description. This group is further divided into two subgroups by means of the loading system, i.e., one is equipped with the flexible rubber bags filled with liquid and the other is equipped with the rigid plates such as steel plates for the additional loading system. The apparatus in the third group are usually called the box type triaxial apparatus. In these apparatus three different principal stresses are generated in the specimen by applying the forces on planes of cubic or rectangular prism specimen. This group is also divided into two subgroups by means of the loading system equipped with the flexible rubber bags or the rigid plates which transmit the loading forces into the specimen.

The specimen in the hollow cylindrical testings can experience larger strains than the conventional triaxial apparatus. Thus, the apparatus in the first group are beneficial to the experimental researches for the residual strength and the critical void ratio. Furthermore, as these apparatus can be easily given the alternating torsional forces on the ends of specimen, they are recently used in dynamic soil tests. However, comparing them with the conventional triaxial apparatus, the stresses and strains induced in the specimen tend to be non-uniform. In the second or third group the specimens being in contact with the flexible rubber bags are subjected to the uniform forces because the pressure in liquid is

isotropic and uniform. However, as the rubber bag is flexible, the deformation occuring in the specimen tends to be non-uniform and the expansion of the rubber bag under high pressure often causes some troubles at the corners of the specimen. On the other

Fig. 1. Classification of true triaxial apparatus

hand, as contrasted with the flexible rubber bags, the rigid plates generate the uniform deformation in the specimen. But, because of difficulty to accurately measure the contact area between the loading rigid plates and the specimen, some errors take place in the calculation of stresses.

MODIFIED TRIAXIAL APPARATUS

The modified triaxial apparatus is designed and constructed by Adachi and Kitamura (1979) for the purpose of studying the mechanical behaviours of particulate material under general three dimensional stress conditions. This apparatus belongs to the second group in Fig.1 and has rigid plates for additional loading. The schematic diagram of this modified triaxial chamber is illustrated in Fig.2 and its whole view is given in Photo.1. The loadings in three orthogonal directions of a cubic specimen are applied by controlling of air pressures. The vertical loading (X_1 -axis) is applied to the specimen through the bellofram cylinder mounted on the loading frame. The loading in one of the horizontal directions (X_2 -axis) is transmitted through the bellofram cylinder set up in the triaxial chamber and the loading in another horizontal direction (X_3 -axis) is applied by chamber water pressure. The central axis of triaxial chamber does not coincide with the one of specimen as shown in Fig.2(a) so that the horizontal loading system shown in Photo.2 might be set in the triaxial chamber. The chamber (see (2) in Fig.2) is made of acrylic resins whose dimension is 26 cm and 30 cm in inner and outer diameters respectively and 25 cm in height.

The horizontal loading system consists of two inter-connected rigid plates (see (4) in Fig. 2) which sit opposite each other. One of the rigid plates is provided with a bellofram cylinder (see (12) in Fig.2) which applies the horizontal load. Each plate is composed of two sliding sections connected by springs (see (16) in Fig.2). Each plate possesses four little rollers (see (6) and (11) in Fig.2) to guarantee for smooth horizontal movement of

Legend of Fig. 2

(1) piston, (2) acrylic resins, (3) top loading cap, (4) lateral loading plate, (5) porous stone, (6) roller, (7) roller guide, (8) pedestal, (9) coupler, (10) bellofram, (11) roller, (12) bellofram cylinder, (13) slide callipers, (14) rigid bar, (15) specimen, (16) spring

Fig. 2. (a) Front view of modified triaxial apparatus

Fig. 2. (b) Side view of modified triaxial apparatus

Photo. 1. Whole view of modified triaxial apparatus

Photo. 2. Horizontal loading system

exchanging the cap and pedestal, the conventional axi-symmetrical triaxial tests can be performed with the cylindrical specimens, whose dimensions are 5 cm or 10 cm in diameter and 10 cm in height.

the rigid plates under the condition of contact between the plates and the cap or the pedestal. This loading system is similar to the one that has been developed by Lade and Duncan (1973) in principle.

The vertical deformation is measured with the dial gauge which can be read to 1/100 mm. The horizontal deformation is measured with the slide callipers (see (13) in Fig.2) which is mounted on the inter-connected rigid bar (see (14) in Fig.2) and can be read to 2/100 mm. The volumetric change in the specimen is measured with the burette which can be read to 0.05 ml. The pedestal (see (8) in Fig.2) is provided with the porous stone (see (5) in Fig.2) which is set in the center of pedestal and has 1 cm in diameter, and the drainage line which is connected to the burette. By

MATERIAL AND TESTING PROCEDURES

All the tests to be shown in this paper have been carried out on Toyoura sand whose mean diameter of particles is 0.2 mm, uniformity coefficient is 1.6 and specific gravity is 2.65. The material is boiled more than one hour in order to deair. Specimens are prepared by pouring saturated sand into mould. The size of cubic specimen is $10 \times 10 \times 10$ cm and in the case of cylindrical specimen it is 10 cm in diameter and 10 cm in height. The initial void ratio is controlled by the number of tamping with a rod, i.e., the dense specimens are prepared by dividing the specimen into three layers and tamping each layer about 15 times by the rod whose diameter is 1 cm. On the other hand, the loose specimens are prepared by the following way, i.e., a specimen is divided into two layers and sand is poured into the mould filled with water and lightly tamped 4 times on each layer. After sealing the specimen by O-rings, the suction with 0.5 kN/m^2 is imposed on the specimen in order to stand by itself when the mould is dissolved. Then, the size of specimen is measured by the slide callipers. The horizontal loading system is set on the rails (see (7) in Fig.2) for the tests under three different principal stress states. The preparation of triaxial tests is finished by putting on the acrylic resins cell and filling the triaxial chamber with water. After specimens are consolidated under the isotropic pressures, the shearing tests are conducted under the drained condition. The initial void ratios of specimens after the isotropic consolidation in the experiments are listed up in Table 1 with stress conditions of tests.

For the tests under the constant confining pressures the strain-controlled method is adopted with the vertical strain rate of $0.05\%/min$. In these tests the dimensions of specimen are 10 cm in diameter and 10 cm in height. The stress-controlled method is adopted for the other tests with various stress paths including the stress reverse under the constant mean effective principal stress. In these cases the cubic specimens are used. The stress-controlled tests are performed such that the difference between the maximum and the minimum principal stresses is increased or decreased by 10 kN/m^2 in every loading or unloading stage and the next stage is commenced after the vertical strain rate decreases less than $0.005\%/min$ at the present stage. The monotonous loading tests under the constant mean effective principal stress are performed along the stress paths as shown in Fig.3 which illustrates

Fig. 3. Radial stress paths on II-plane

Fig. 4. (a) Loading and unloading processes of one-way repeated loading test

Fig. 4. (b) Loading and unloading processes of two-way repeated loading test

the radial stress paths on the octahedral plane (II-plane) in three effective principal stress space. The radial stress paths with $\theta=0^\circ$ and $\theta=180^\circ$ in Fig.3 correspond to the conventional triaxial compression and extension tests respectively.

In the following description, let's denote the vertical principal stress and strain by σ_1 and ϵ_1 , one of horizontal principal stress and strain in the direction loaded by the bellofram cylinder by σ_2 and ϵ_2 , and the other horizontal principal stress and strain by σ_3 and ϵ_3 in order that the subscripts are coincided with the reference axes in the previous papers (Kitamura, 1981 a, b). Thus, the conventional triaxial extension test σ_2 and σ_3 are the maximum principal stress and σ_1 is the minimum principal stress. The repeated loading tests are carried out under the constant confining pressure and the constant mean effective principal stress. For later convenience the loading and the unloading processes in these repeated tests are divided and named as shown in Fig.4.

In order to examine the accuracy of the horizontal loading system, an axi-symmetric triaxial compression test is carried out under the constant mean effective principal stress, $\sigma_m=2 \times 10^2 \text{ kN/m}^2$, by setting the horizontal loading plates in the chamber. The vertical strain ϵ_1 , the horizontal strain ϵ_2 and the volumetric strain ϵ_v are measured. The stress-strain relation between the strains ϵ_1 and ϵ_2 , and the stress ratio σ_1/σ_3 is shown in Fig.5. When infinitesimal strain conditions can be assumed, the following relation is satisfied.

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = \epsilon_v \quad (1)$$

The strain ϵ_3 can be calculated by using Eq. (1) and the measured strains ϵ_1 , ϵ_2 and ϵ_v . The calculated strain ϵ_3 is also given in Fig.5. As the fabric of sand specimen is supposed to be isotropic in the horizontal directions, the strains in the horizontal directions, ϵ_2 and ϵ_3 , should be theoretically same under the axi-symmetrical stress state. From Fig.5 it is found that the measured strain ϵ_2 and the calculated strain ϵ_3 are almost same and then the accuracy of the horizontal loading system is satisfactory.

Fig. 5. Measured and calculated strains under axi-symmetric condition

EXPERIMENTAL RESULTS

As shown in Table 1, one monotonous and one repeated loading conventional triaxial compression tests are carried out under the constant confining pressures. Six monotonous and one repeated loading tests with

Table 1. Initial void ratios and stress conditions

stress condition	initial void ratio	remarks
$\sigma_r = 3 \times 10^2 \text{ kN/m}^2$	0.720	monotonous
$\sigma_r = 3 \times 10^2 \text{ kN/m}^2$	0.645	one-way repeated
$\theta = 0^\circ$	0.656	monotonous
$\theta = 15^\circ$	0.664	
$\theta = 30^\circ$	0.653	
$\theta = 45^\circ$	0.701	
$\theta = 60^\circ$	0.705	
$\theta = 180^\circ$	0.692	loose
$\theta = 0^\circ \rightarrow 180^\circ \rightarrow 0^\circ$	0.689	
		two-way repeated

Then, the strains ϵ_1 and ϵ_2 must be equal, if the fabric of sand is isotropic on the plane including X_1 - and X_2 - axes. It is found, however, the difference between the strains ϵ_1 and ϵ_2 from Fig.6(g). This fact means the anisotropy in the fabric of sand specimen on the plane including X_1 - axis. Fig.6(i) shows the two-way repeated loading test result. The stresses, τ_{oct} and σ_m , and the shear strain γ_{oct} are defined as the following equations.

$$\tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (2)$$

$$\sigma_m = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \quad (3)$$

Fig. 6.(a) Stress-strain relation for monotounous loading test under constant confining pressure, $\sigma_r = 3 \times 10^2 \text{ kN/m}^2$ **Fig. 6.(b) Stress-strain relation for one-way repeated loading test under constant confining pressure, $\sigma_r = 3 \times 10^2 \text{ kN/m}^2$** **Fig. 6.(c) Stress-strain relation for $\theta = 0^\circ$** **Fig. 6.(d) Stress-strain relation for $\theta = 15^\circ$**

medium dense specimens, and one monotonous loading test with loose specimen are also performed under the constant mean effective principal stress, $\sigma_m = 2 \times 10^2 \text{ kN/m}^2$. The stress-strain relations obtained by these shearing tests are shown by $\bullet$ marks in Fig.6. It is apparent from these figures that the stress paths and the initial void ratio influence the stress-strain relations. From Figs.6(d) and (e) the stress path for plain strain condition is supposed to exist in the range between $\theta = 15^\circ$ and 30° . The shearing test for $\theta = 60^\circ$ corresponds to the extension test for which both the stresses σ_1 and σ_2 in the direction of X_1 - and X_2 - axes are the maximum principal stress.

Fig. 6.(e) Stress-strain relation for $\theta=30^\circ$ Fig. 6.(f) Stress-strain relation for $\theta=45^\circ$ Fig. 6.(g) Stress-strain relation for $\theta=60^\circ$ Fig. 6.(h) Stress-strain relation for $\theta=180^\circ$

Fig. 6.(i) Stress-strain relation for two-way repeated loading test

Fig. 6.(j) Stress-strain relation for loose specimen, ($\theta=0^\circ$)

$$\tau_{oct} = \frac{2}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2} \quad (4)$$

The specimen for this test is subjected to the cyclic shearing stress in both triaxial compression and extension sides. The positive sign τ_{oct} represents the compression side.

NUMERICAL EXPERIMENTS

In the previous papers (Kitamura, 1981a, b) a mechanical model for particulate material has been theoretically derived by regarding the motions of particles as a Markov process. In this model the new concepts are introduced, i.e., the concepts of the potential barrier and the potential slip plane, and the ratios of the disappearance and the appearance. Fig. 7 shows the flow chart how to obtain the stress-strain relations by using the proposed model. The flow chart for the determination of the coefficients A_i and B_{ii} in the basic equation is shown in Fig. 8.

In this section the numerical experiments are carried out in accordance with the flow charts in Figs. 7 and 8 in order to verify the validity of the proposed model. In the numerical experiments the basic equation is solved by using the finite difference method. In the following subsections the values of model parameters are determined based on the data obtained by the experiments with Toyoura sand given in the previous section.

Model Parameters used in the Proposed Model

Energy transferred into specimen; In the deformation process of particulate material the energy is transferred into or out the specimen due to the change in external actions. The amount of transferred mechanical energy is calculated by the following equation.

$$\Delta W = \sigma_1 \cdot d\epsilon_1 + \sigma_2 \cdot d\epsilon_2 + \sigma_3 \cdot d\epsilon_3 \quad (5)$$

where ΔW ; increment of mechanical energy transferred due to external actions per unit volume,

σ_1, σ_2 and σ_3 ; principal stresses,

$d\epsilon_1, d\epsilon_2$ and $d\epsilon_3$; principal strain increments.

The total energy W during the deformation process is obtained by summing up each energy increment calculated by Eq. (5).

The relations between the total energy for dense or loose specimens, W , and the stress ratio, τ/σ_N , defined by Matsuoka and Nakai (1974) are schematically shown in Fig. 9 for the monotonous loading process of shearing tests. If these curves are assumed to be hyperbolic, $1/a$ and $1/b$ are the initial slope and the asymptote respectively. $W/(\tau/\sigma_N) \sim W$ relations obtained from the shearing test of Toyoura sand are shown in Fig. 10. Figs. 10 (a) and (b) show the relations obtained by the monotonous loading test results with medium

Fig. 7. Flow chart of proposed mechanical model

Fig. 8. Flow chart for determination of A_i and B_{ii}

Fig. 9. Schematic relations between W and τ/σ_N for dense and loose specimens

Fig. 10.(a) $W/(\tau/\sigma_N) \sim W$ relation for monotonous loading test under constant confining pressure

Fig. 10.(b) $W/(\tau/\sigma_N) \sim W$ relations for monotonous loading tests under constant mean effective principal stress

Fig. 10.(c) $W/(\tau/\sigma_N) \sim W$ relation for one-way repeated loading test

dense specimens under the constant confining pressure and under the constant mean effective principal stress respectively. Fig.10(c) shows the results obtained by the first and the third loading processes for the one-way repeated loading test under the constant confining pressure. Fig.10(d) shows the relation obtained by the first, the second and the third loading processes for the two-way repeated loading test under the constant mean effective principal stress. Fig.10(e) is the results of compression tests for the loose and the dense specimens. It is found that the parameters a and b for the loose specimen are larger than those for the dense specimen which corresponds to the difference of the relations for the dense and the loose specimens shown in Fig.9. In Figs.10(a)~(e) it is found that $W/(\tau/\sigma_N)$ is linearly related to W , i.e., the following hyperbolic equation is empirically derived between the total energy W and the stress ratio τ/σ_N for such tests.

$$\tau/\sigma_N = \frac{W}{a + b \cdot W} \quad (6)$$

Fig. 10. (d) $W/(\tau/\sigma_N) \sim W$ relation for two-way repeated loading test

Fig. 10. (e) $W/(\tau/\sigma_N) \sim W$ relations for dense and loose specimens

where a ; reciprocal number of slope of initial tangent for $\tau/\sigma_N \sim W$ relation as shown in Fig. 9 and intercept value of the vertical axis for $W/(\tau/\sigma_N) \sim W$ relation in Fig. 10.

b ; reciprocal number of asymptotic value for $\tau/\sigma_N \sim W$ relation as shown in Fig. 9 and slope for $W/(\tau/\sigma_N) \sim W$ relation in Fig. 10.

Furthermore, it may be concluded from these figures that the parameters a and b in Eq. (6) generally depend on the initial void ratio, confining pressure, fabric of particulate material and so on, but are independent of the shearing stress paths as far as the loading process is monotonous under the constant mean effective principal stress.

The relations between τ/σ_N and W during the unloading and reloading process of repeated loading test do not satisfy the hyperbolic equation, Eq. (6), because the recoverable particle motion is predominant. Thus, in this paper these relations are examined by the logarithmic

Fig. 11. (a) $W \sim \tau/\sigma_N$ relations for first unloading and second loading processes of one-way repeated loading test

Fig. 11. (b) $W \sim \tau/\sigma_N$ relations for first and second unloading processes of two-way repeated loading test

Table 2. Model parameters used in numerical experiments

stress condition	E_1	E_2	E_3	a	b	m	r	λ_1	λ_2	κ_1	κ_2	κ_3	remarks
$\sigma_r = 3 \times 10^2 \text{ kN/m}^2$	133.0	357.5	357.5	1.550	1.340	—	—	15.0	6.36	—	0.5	0.5	monotonous
$\sigma_r = 3 \times 10^2 \text{ kN/m}^2$	94.0	209.4	209.4	3.214	1.014	—	—	20.0	6.36	—	0.5	0.5	1st loading
				—	—	9.810	2.850	0.5	12.73	—	2.5	2.5	1st unloading
				—	—	11.460	2.630	0.5	12.73	—	2.5	2.5	2nd loading
				3.214	1.014	—	—	20.0	8.49	—	0.5	0.5	3rd loading
$\theta = 0^\circ$	635.0	2 530.0	2 530.0	0.358	1.153	—	—	15.0	10.60	—	0.5	0.5	monotonous
$\theta = 15^\circ$	400.0	4 000.0	1 360.0	0.330	1.100	—	—	6.0	10.65	—	0.2	1.2	
$\theta = 30^\circ$	420.0	1 600.0	410.0	0.375	0.935	—	—	15.0	1.22	—	-0.2	1.3	
$\theta = 45^\circ$	730.0	1 400.0	700.0	0.215	0.895	—	—	10.0	4.73	—	-1.0	1.3	
$\theta = 60^\circ$	420.0	560.0	300.0	0.775	0.725	—	—	20.0	3.18	—	-2.0	2.0	
$\theta = 180^\circ$	310.0	310.0	590.0	0.787	0.863	—	—	30.0	8.49	2.0	—	—	
$\theta = 0^\circ$	197.7	482.4	482.4	0.600	1.310	—	—	15.0	3.18	—	0.35	0.35	loose
$\theta = 0^\circ \rightarrow 180^\circ \rightarrow 0^\circ$	325.0	635.0	635.0	0.343	1.088	—	—	16.0	6.36	—	0.5	0.5	1st loading
				—	—	-0.196	0.064	15.0	2.12	—	4.0	4.0	1st unloading
				3.295	1.025	—	—	30.0	1.59	1.75	—	—	2nd loading
				—	—	-0.716	0.460	15.0	2.12	4.0	—	—	2nd unloading
				2.948	0.932	—	—	20.0	6.36	—	0.7	0.7	3rd loading

where, a, m : [$\times 10^2 \text{ kN/m}^2$], $\lambda_1, \lambda_2, \kappa_1, \kappa_2, \kappa_3$: [$\times 10^{-2} \text{ m}^2/\text{kN}$]

scale. Fig.11(a) shows the relation between τ/σ_N and W in the first unloading and the second loading processes for the one-way repeated loading test under the constant confining pressure. Fig.11(b) shows the relation in the first and the second unloading processes for the two-way repeated loading test under the constant mean effective principal stress. From Fig.11 the relation between τ/σ_N and W is approximated by the following equation.

$$W = m(\tau/\sigma_N)^r \quad (7)$$

where m and r ; parameters depending on the void ratio, fabric of particulate material and so on.

In Fig.11 data seem to be a little apart from the linear relation expressed by Eq. (7) at small stress ratio, but difference between the measured and the calculated energy increments, ΔW , are almost negligible in numerical experiments because the absolute values of ΔW are very small at small stress ratio. The values of these model parameters a, b, m and r which are used in the numerical experiments are summarized in Table 2.

The parameter R_η is derived in the previous paper (Kitamura, 1981 b) as follows.

$$R_\eta = \frac{\sum_{\beta_1=0}^{\pi/2} \sum_{\beta_2=0}^{2\pi} \{n_\eta(x_{\eta,1} + x_{\eta,2})\}}{2 \cdot \Delta W_e} \quad (8)$$

It is necessary to obtain the elastic component of energy increment ΔW_e in order to determine the parameter R_η in Eq. (8). But it is impossible to measure ΔW_e directly at each contact point. As the first approximation, ΔW_e is assumed to be determined by using the slopes of initial tangents $E_i (i=1, 2, 3)$ in the stress ratio-strain relations, $\tau/\sigma_N \sim \epsilon_i (i=1, 2, 3)$. These experimentally determined values $E_i (i=1, 2, 3)$ are also listed up in Table 2.

Coefficient of inter-particle friction, μ ; It is clear that the frictional resistance at contact points of particles plays an important role in the non-linear stress-strain relation of

particulate material. As already mentioned (Kitamura, 1981 a, b), it is, however, a really difficult task to determine the coefficient of inter-particle friction μ between two particles. In this study we take the way to deduce the inter-particle coefficient μ from the global friction of particulate material. Matsuoka and Nakai (1974) proposed the SMP to explain the global friction of particulate material and derived the following equation as a stress-dilatancy equation based on the SMP.

$$\tau/\sigma_N = \lambda' \left(-\frac{d\epsilon_N}{d\gamma} \right) + \mu \quad (9)$$

where $d\epsilon_N$ and $d\gamma$; normal and shear strain increments on the SMP.

Fig. 12. $\tau/\sigma_N \sim (-d\epsilon_N/d\gamma)$ relations for monotonous shearing tests

They regarded the value of τ/σ_N at the normal strain increment $d\epsilon_N=0$ on the SMP as the frictional coefficient of particulate material.

The relations between τ/σ_N and $-d\epsilon_N/d\gamma$ for the shearing tests of Toyoura sand are given in Fig.12. It is apparent from this figure that the relation between τ/σ_N and $-d\epsilon_N/d\gamma$ on the SMP satisfies the stress-dilatancy equation, Eq. (9), and does not depend on the initial void ratio and stress path. Thus, it is reasonable to regard μ in Eq. (9) as the global frictional coefficient for particulate material. From Fig.12 the value of μ for Toyoura sand is found to be approximately 0.25. This obtained value is used as the inter-particle frictional coefficient in the numerical experiments.

Initial distribution of contact angles; The initial distribution of contact angles at contact points should

be given as an initial condition of particulate material in order to solve the basic equation of Markov process. Concerning the initial distribution of contact angles, experimental researches have been carried out by Oda (1972) and Matsuoka (1974). According to the results obtained by them who directly measured the contact angles of sands or rods by microscope or photographs, it is found that the initial distribution of contact angles is approximated by triangle or trapezium in form in the two dimensional space if β_1 is adopted to measure the angle of contact plane. This shows that under the gravity field the probability that the normals to contact planes are in the vertical direction is the largest. In the numerical experiments the initial distribution of contact angles is assumed to be cone shape in form in the three dimensional space.

Ratios of disappearance and appearance of contact points; It has been pointed out in the previous paper (Kitamura, 1981 a) that the discontinuous motions of particles at contact points play very important roles in the deformation process of particulate material. These motions are similar to the dislocation of crystal solid under plastic deformation process. By introducing the concepts of the disappearance and the appearance of contact points, the discontinuous motions have been prescribed as follows in the previous paper (Kitamura, 1981 a). When the direction of the maximum principal stress is vertical, the probability of the disappearance, $P_d(\beta_1)$, is given by the following equation.

$$P_d(\beta_1) = \left(\frac{2}{\pi} \beta_1 \right)^\lambda \quad (10)$$

where λ ; parameter depending on the stress state, void ratio, fabric of particulate material and so on.

When the direction of the maximum principal stress is horizontal, $P_d(\beta_1)$ becomes in the

following form.

$$P_d(\beta_1) = \left\{ \frac{2}{\pi} \left(\frac{\pi}{2} - \beta_1 \right) \right\}^\lambda \quad (11)$$

The ratios of the disappearance and the appearance are derived by the probability of the disappearance, $P_d(\beta_1)$, as follows.

$$N_{s,i}/N_{s-ds,i} = 1 - \frac{1}{N_c} \sum_{\beta_1=0}^{\pi/2} \sum_{\beta_2=0}^{2\pi} \{P_d(\beta_1) \cdot n_\eta\} \quad (12)$$

for the disappearance,

$$N_{s,i}/N_{s-ds,i} = 1 + \frac{1}{N_c} \sum_{\beta_1=0}^{\pi/2} \sum_{\beta_2=0}^{2\pi} \{\kappa P_d(\beta_1) \cdot n_\eta\} \quad (13)$$

for the appearance,

where κ ; parameter depending on the stress state, void ratio, fabric of particulate material and so on.

It is required to determine the values of λ and κ defined in Eqs. (10), (11), (12) and (13) in order to derive the strains of particulate material. In the proposed model these parameters are assumed to be simply the linear functions of octahedral shearing stress τ_{oct} as the first approximation. Then, the following equation for λ is given.

$$\lambda = \lambda_1 + \lambda_2 \cdot \tau_{oct} \quad (14)$$

where λ_1 and λ_2 ; parameters depending on the mechanical properties of particulate material and its shearing stress path.

Fig. 13. Schematic relation between $P_d(\beta_1)$ and β_1

Fig.13 schematically shows how λ effects on the function $P_d(\beta_1)$ defined in the previous paper (Kitamura, 1981a). Fig.13(a) is for the case when the vertical principal stress σ_1 is the maximum and Fig.13(b) is for the case when the horizontal stress σ_2 or σ_3 is the maximum. From Fig.13 it is found that the probability of the disappearance of contact points decrease when the parameter λ is larger.

In the following discussions, let's denote the appearance ratios of new contact points by κ_1 , κ_2 and κ_3 in the directions of X_1 -, X_2 - and X_3 - axes respectively. The values of λ_1 , λ_2 , κ_1 , κ_2 and κ_3 used in numerical experiments are chosen to fit the stress-strain relations obtained by numerical experiments to those obtained by experiments described in the previous chapter. These values are summarized in Table 2. κ_1 has non-zero value only for the case of the conventional triaxial extension test.

Let's consider the discontinuous motions of particles under the shearing process based on the parameters λ and κ . As shown in Table 2 the parameter λ_2 is found to be positive in all experiments. Thus, λ in Eq. (14) increases when the octahedral shearing stress τ_{oct} increases. This means that the probability of disappearance at contact points always decreases for the shear loading process. It may be considered that the fabric of particulate material is transformed so as to mobilize and stabilize against the shear stress at the particle scale.

It is found from the results of numerical experiments in Table 2 that the parameter κ in Eq. (13) is constant, although κ is considered to generally be the function of stress state. This fact shows that the number of contact points where the appearance occurs is linearly related to the number of contact points where the disappearance occurs. Let's discuss what the parameter κ means in the case of the test under the constant confining pressure, $3 \times 10^2 \text{ kN/m}^2$, for which $\kappa_2 = \kappa_3 = 0.5$. $\kappa_2 = \kappa_3 = 0.5$ means that the number of contact points

increase in the horizontal paths by 50% of the number of disappeared contact points in the vertical path. Such motions of particles are schematically shown in Fig.6 of the previous paper (Kitamura, 1981a). It is reasonable to consider that κ_2 is always equal to κ_3 for the conventional compression test because the stress and the fabric in the specimen are both axi-symmetric. In the case of the test with $\theta=15^\circ$, $\kappa_2=0.2$ and $\kappa_3=1.2$ mean that the numbers of contact points increase in the X_2 - and X_3 directions by 20% and 120% of the number of disappeared contact points in the vertical path. Negative values of κ_2 for the test of $\theta=30^\circ$, 45° and 60° means that the disappearance of contact points occurs in the direction of X_2 - axis. For example, $\kappa_2=-0.2$ for the test of $\theta=30^\circ$ represents that the number of contact points disappearing along the horizontal path in the direction of X_2 - axis is 20% of the number of contact points disappearing along the vertical path. And $\kappa_2=-2.0$ for the test of $\theta=60^\circ$ means that the number of contact points disappearing along the horizontal path in the direction of X_2 - axis is twice as large as the number of contact points disappearing in the vertical path. The test of $\theta=60^\circ$ is the extension test where the stresses in the directions of X_1 - and X_2 - axes are both the maximum principal stresses. Thus, $\kappa_2=-1.0$ should be theoretically given if the fabric of material is isotropic. Therefore, this discrepancy for κ_2 reflects the fabric anisotropy of particulate material. As shown Table 2 the sign of κ_2 changes from positive to negative between the stress paths of $\theta=15^\circ$ and 30° . This means that the discontinuous motion of disappearance changes to the discontinuous motion of appearance in the direction of X_2 - axis and corresponds to the fact described in the previous chapter that the stress path under plane strain condition is one between $\theta=15^\circ$ and 30° .

Next let's investigate the parameters λ and κ for the dense and the loose specimen. $((\lambda_2)_{\text{dense}}=10.60) > ((\lambda_2)_{\text{loose}}=3.18)$ for the test of $\theta=0^\circ$ means that the disappearance of contact points for the loose specimen tends to occur along the vertical path more than for the dense sample. $((\kappa_2=\kappa_3)_{\text{dense}}=0.5) > ((\kappa_2=\kappa_3)_{\text{loose}}=0.35)$ shows that the appearance of contact points for the dense sample tends to occur along the horizontal paths more than that for the loose specimen.

Comparing λ_2 for loose specimen with λ_2 for the dense sample, it is found that the discontinuous motions of disappearance tend to occur more in the loose sample. These facts mean that the larger number of particles for loose specimens does not contribute to the deformation process than that for the dense specimen.

In the repeated loading tests, $(\lambda_2)_{1\text{st}}$ which is the value of λ_2 in the first loading process is smaller than $(\lambda_2)_{3\text{rd}}$ which is the values in the third loading process. κ , however, is almost constant under each loading process. For example, in the test under the constant confining pressure $\sigma_r=3 \times 10^2 \text{ kN/m}^2$, $((\lambda_2)_{1\text{st}}=6.36) < ((\lambda_2)_{3\text{rd}}=8.49)$ and $(\kappa)_{1\text{st}}=(\kappa)_{3\text{rd}}=0.5$. This means that as the particulate material experiences the repeated loading, its fabric is transformed so as to resist more against the shear stress and the discontinuous motions of particles decrease gradually in the repeated loading. According to the parameters λ and κ in Table 2 for the unloading process of repeated loading tests, it is supposed that the discontinuous motions of particles become more active than those for the loading process. Especially, this phenomena become remarkable near zero deviatoric stress state.

Stress-strain Relations in the Numerical Experiments

The stress-strain relations which are obtained by the numerical experiments are shown by $\circ$ marks in Fig.6. These relations are obtained by using the model parameters which are explained in the previous section and summarized in Table 2. That is to say, the parameters a and b in Eq. (6) are necessary to calculate the energy transferred into the specimen for the monotonous or the repeated virgin loading process. The parameters m and r in Eq. (7) are used for the unloading and the reloading processes. The elastic

component of energy is calculated by the parameters E_1 , E_2 and E_3 . The parameters λ and κ are chosen in order that the calculated stress-strain curves fit those obtained by the experimental results with Toyoura sand.

It is concluded from Fig.6 that the proposed mechanical model can versatily represent the mechanical behaviours of particulate material by taking account of the inherent anisotropy due to the fabric, the extrinsic anisotropy due to the stress history and the non-linearity of stress-strain relationships.

CONCLUSIONS

In this paper the mechanical properties of particulate material are investigated by carrying out the shearing tests with Toyoura sand and the numerical experiments with the proposed mechanical model under general stress conditions.

The main results obtained in this paper may be summarized as follows.

(1) The newly modified triaxial apparatus is introduced which can generate three different principal stresses. In this apparatus the horizontal loading system with rigid plates is installed in the conventional triaxial chamber.

(2) The saturated Toyoura sand is used in the shearing tests. The void ratios are adjusted in order to obtain the medium dense and loose specimens. All the tests are carried out under the drained condition. The stress conditions of shearing tests are the constant confining pressure and the constant mean effective principal stress with radial stress path on Π -plane. One-way and two-way repeated loading tests are also performed.

(3) The relations between the work, W , done in the particulate material and the stress ratio, τ/σ_N , are represented by the hyperbolic equations for monotonous loading, the virgin loading in the one-way repeated loading and the loading in the two-way repeated loading tests. For unloading and reloading processes the relation between W and τ/σ_N are approximated by the linear equations on the logarithmic scale.

(4) The stress-dilatancy relations on the SMP are shown by using the shearing tests under general stress conditions. It is found that the stress ratio at $d\epsilon_N=0$ is almost same and independent of void ratio, stress path and confining pressures. Thus, this value is adopted as the inter-particle frictional coefficient in the proposed model.

(5) The values of the ratios of the disappearance and the appearance are chosen so that the stress-strain relations obtained by numerical experiments are fit to those obtained by the shearing tests with Toyoura sand. The deformation mechanisms of particulate material are discussed based on these values which estimate the discontinuous motions of particles.

(6) The stress-strain relations obtained by the numerical experiments are compared with those by the shearing tests. From these comparisons it is apparent that the proposed mechanical model can versatily represent the various mechanical behaviours of particulate material such as sand. Thus, it is found that the proposed model is a unified and simple one that takes account of the inherent anisotropy due to the fabric, extrinsic anisotropy due to the stress history and the non-linearity of stress-strain relationships.

ACKNOWLEDGEMENTS

This study forms a part of the work on "Construction of constitutive equations for particulate material" carried out in the Disaster Prevention Research Institute, Kyoto Univ. under the supervisions of Professors S. Murayama and K. Akai. The author wishes to express his deepest gratitude to their continuous guidances and encouragements.

The author is also indebted to Professor T. Shibata and Associate Professor T. Adachi

for their helpful suggestions concerning the performance of this research.

NOTATION

- A_i and B_{ii} =coefficients in the basic equation of proposed model
 a and b =parameters of hyperbolic equation for $\tau/\sigma_N \sim W$ relation
 $E_i, (i=1, 2, 3)$ =slope of initial tangent in the stress-strain relation, $\tau/\sigma_N \sim \epsilon_i$
 m and r =parameters of power function for $\tau/\sigma_N \sim W$ relation
 $N_{s,i}/N_{s-A s,i}$ =ratio of disappearance or appearance
 n_η =number of contact points with the contact angle η
 $P_d(\beta_1)$ =probability of disappearance
 R_η =coefficient expressing the amount of activation energy at the contact points where the recoverable change in contact angles occurs
 W =mechanical energy transferred due to external action per unit volume
 ΔW =increment of W
 ΔW_e =elastic component of energy transferred into the particulate material
 $x_{\eta,1}$ and $x_{\eta,2}$ =minimum and maximum heights of the potential barrier at each contact angle η
 γ_{oct} =octahedral shearing strain
 ϵ_1, ϵ_2 and ϵ_3 =principal strains
 ϵ_v =volumetric strain
 $d\epsilon_1, d\epsilon_2$ and $d\epsilon_3$ =principal strain increments
 $d\epsilon_N/d\gamma$ =shear-normal strain increment ratio on the SMP
 $\eta=(\beta_1, \beta_2)$ =contact angle
 θ =angle of radial stress path on the octahedral plane
 κ =parameter for appearance
 λ =parameter for disappearance
 λ' =coefficient of stress-dilatancy equation on the SMP
 μ =frictional coefficient for particulate material
 σ_1, σ_2 and σ_3 =principal stresses
 σ_m =mean effective principal stress
 σ_r =confining pressure
 τ_{oct} =octahedral shearing stress
 τ/σ_N =shear-normal stress ratio on the SMP

REFERENCES

- 1) Adachi, T. and Kitamura, R. (1979): "Modified triaxial apparatus," Proc. 24th Symposium of JSSMFE, pp.137-142 (in Japanese).
- 2) Kitamura, R. (1981 a): "A mechanical model of particulate material based on stochastic process," Soils and Foundations, Vol.21, No.2, pp.63-72.
- 3) Kitamura, R. (1981 b): "Analysis of deformation mechanism of particulate material at particle scale," Soils and Foundations, Vol.21, No.2, pp.85-98.
- 4) Lade, P. V. and Duncan, J. M. (1973): "Cubical triaxial tests on cohesionless soil," Proc., ASCE, Vol. 99, No. SM 10, pp.793-812.
- 5) Matsuoka, H. (1974): "A microscopic study on shear mechanism of granular materials," Soils and Foundations, Vol. 14, No.1, pp.29-43.
- 6) Matsuoka, H. and Nakai, T. (1974): "Stress-deformation and strength characteristics of soil under three different principal stresses," Proc., JSCE, No.232, pp.59-70.
- 7) Oda, M. (1972): "Initial fabrics and their relations to mechanical properties of granular material," Soils and Foundations, Vol. 12, No.1, pp.17-36.

(Received June 5, 1980)