ANISOTROPIC DEFORMATION-STRENGTH CHARACTERISTICS OF AN ASSEMBLY OF SPHERICAL PARTICLES UNDER THREE DIMENSIONAL STRESSES

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ABSTRACT

In order to clarify the influence of inherent anisotropy on the deformation-strength behaviours for an assembly consisting of spherical particles, isotropic compression tests and radial shear stress tests were performed on cubical specimens consisting of glass beads under the condition of independent stress control. In all the tests, the direction of specimen deposition was identical with that of gravity. The test results are summarized as follows. For random packing of spheres deposited by free falling under the action of gravity, an inherent anisotropy was considered to have been caused by the requirements of sphere-stabilities. In the direction of specimen deposition, the specimens showed a compressibility which was lower than that in the direction perpendicular to that of specimen deposition. The deformation-strength behaviours were consequently affected by the relation between the direction of specimen deposition and that of the radial shear stress path.

Key words: anisotropy, deformation, drained shear, granular material, sand, shear strength, soil structure, special shear test, yield

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INTRODUCTION

Due to the fact that, in naturally occurring deposits, the grain shape and the mode of deposition induce clearly a grain orientation, in-situ soils may substantially have an inherent anisotropy. Because of difficulties with sampling of an undisturbed sand, recent studies on the stress-strain behaviours of sands have been confined to the anisotropy of sands investigated by plane strain tests and independent stress-control tests on cubical specimens prepared in laboratories. There are, for example, experimental studies by Arthur and Menzies (1972) and Oda et al. (1978) with tilted samples and by Green (1971), Reades and Green (1976), Arthur et al. (1977a) and Yamada and Ishihara (1979) with samples subjected to different shear directions, including theoretical studies by Tatsuoka (1976) and Matsuoka et al. (1980). The methods of genuinely triaxial loading for cubical specimens can be grouped into the strain-controlled system with rigid platen boundary-conditions, the stress-controlled system with flexible boundary-conditions and mixed bound-

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ary-conditions (Sture and Desai, 1979). The boundary-condition of apparatus employed in this study is a flexible boundary using a rubber bag cell. Genuinely triaxial tests using an independent stress-control cell with flexible rubber bags were performed by Ko and Scott (1968), Moroto et al. (1975), Miyamori (1976), Arthur et al. (1977a) and Yamada and Ishihara (1979).

Oda (1972) investigated influences of grain shape and of specimen preparation method on the initial fabric of sands, and showed that sands composed of elongated grains indicated strong anisotropic feature of fabric character. Although it may be considered that an assembly of spherical particles behaves like an isotropic material, no clear explanation of this problem has been given. The purpose of the present paper is to discuss the results of an experimental study performed to clarify the anisotropic deformation-strength behaviours of an assembly of glass beads deposited by free falling through air under the gravity. Isotropic compression tests and radial shear tests under a constant mean principal stress were performed in the condition of independent stress control. As shown in the previous study, an inherent anisotropy in the specimen disappears at large strains causing failure of the specimen, therefore the present paper deals with the test results within a range of small deformations.

**STRESS AND STRAIN PARAMETERS**

Fig. 1 shows a stress space bounded by three orthogonal axes representing three principal stresses \( \sigma_x, \sigma_y \), and \( \sigma_z \), wherein the \( \sigma_z \)-axis coincides with the direction of gravity and the other two axes lie horizontally. All the points along the space diagonal defined by \( \sigma_x = \sigma_y = \sigma_z \) represent spherical stress conditions, and all the points in the deviatoric plane defined by \( \sigma_x + \sigma_y + \sigma_z = \text{constant} \), perpendicular to the space diagonal, represent deviatoric stress conditions. Accordingly, the state of stress at one point can be expressed by the spherical stress, \( N \), the deviatoric stress, \( S \), and the angle of shear direction, \( \theta \). In terms of the principal stress, they are

\[
N = \frac{1}{\sqrt{3}} (\sigma_x + \sigma_y + \sigma_z) \tag{1}
\]

\[
S = \frac{1}{\sqrt{3}} \sqrt{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2} \tag{2}
\]

\[
\tan \theta = \frac{\sqrt{3} (\sigma_x - \sigma_y)}{2\sigma_z - \sigma_x - \sigma_y} \tag{3}
\]

\( N \) and \( S \) represent the distance, \( OO' \), along the space diagonal from the origin to the deviatoric plane and the size of radius, \( O'P \), in the deviatoric plane, respectively. The clockwise angle, \( \theta \), measured from the \( \sigma_z \)-axis gives the direction of shear stress on the deviatoric plane. If the state of stress is expressed by the mean principal stress, \( p \), and by the octahedral shear stress, \( \tau_{oct} \), the relationships between \( N \) and \( p \), and between \( S \) and \( \tau_{oct} \) can be connected, respectively, as follows:

\[
p = N/\sqrt{3} \tag{4}
\]

and

\[
\tau_{oct} = S/\sqrt{3} \tag{5}
\]

Then, the stress ratio, \( \eta \), expressed by \( S/N \) is equal to \( \tau_{oct}/p \).
If the deformation corresponding to the above stress states has three principal strains, \( \varepsilon_x, \varepsilon_y, \varepsilon_z \) in the \( \sigma_x, \sigma_y, \sigma_z \)-directions, then, in the same manner, the strain conditions are characterized by the volumetric strain, \( \nu \), the distortion strain, \( \tau \), and the direction angle of distortion strain, \( \omega \), related to the principal strains as shown in the following equations:

\[
\nu = \frac{1}{\sqrt{3}} (\varepsilon_x + \varepsilon_y + \varepsilon_z) \tag{6}
\]

\[
\tau = \frac{1}{\sqrt{3}} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2} \tag{7}
\]

\[
\tan \omega = \frac{3(\varepsilon_x - \varepsilon_y)}{2\varepsilon_x - \varepsilon_y - \varepsilon_z} \tag{8}
\]

The actual unit volume change, \( v_a \), and the octahedral shear strain, \( \tau_{out} \), are expressed, respectively, as follows:

\[
v_a = \sqrt{3} \nu \tag{9}
\]

and

\[
\tau_{out} = 2\nu / \sqrt{3} \tag{10}
\]

Stress and strain parameters in this paper are defined as positive when representing a compressive state.

**TRIAXIAL TESTING APPARATUS**

The testing apparatus consists mainly of a cubical triaxial cell, an independent stress-control system and a volume change measurement system as shown in Fig. 2. A cross sectional view of the bottom- and side wall-units of the cubical triaxial cell with the internal dimension of 100 mm x 100 mm x 100 mm is shown in Fig. 3. The cubical specimen is placed in a cell formed by six wall- assemblies whose opposed pairs of faces, the vertical pair for \( \sigma_z \)-axis and the two lateral pairs for \( \sigma_x \)- and \( \sigma_y \)-axes, are inter-connected, which in turn are connected to an independent stress-control system. The four vertical walls are first tightened by two horizontal clamp-bands, and then the whole units are assembled by tightening the bottom and top clamp-plates. Each wall unit is made of a flexible rubber membrane, 0.8 mm in thickness, a pressure chamber, an adjustable plate and an adjustable piston with a nut.

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**Fig. 2. Triaxial testing apparatus**

**Fig. 3. Cross sectional view of cubical triaxial cell**
The application of load through the rubber membranes containing water under pressure generated by three independent loading systems results in uniform and known stresses on all six faces of the specimen. Adjustable plates in the pressure chambers shown in Fig.3 can be shifted by adjustable pistons in a stroke of 10 mm. At the time of specimen preparation and before the application of stress to the specimen, adjustable plates retain the specimen surrounded by membranes. After the specimen is loaded, adjustable plates are kept apart from membranes. Three independent principal stress-control systems are made possible by means of a compressor, air pressure-regulators, air-fluid interfaces, Bourdon tube pressure-gauges and pressure chambers in the cubical triaxial cell. The desired stress-path for a test is followed by simultaneous operation of the three pressure-regulators.

Four medical hypodermic needles inserted at the four top corners of cubical box allow air to flow in and out of the specimen. The volume change of specimen under a pressure increment is measured by observing changes of water level in four coloured paraffin volume change gauges connected to the pressure chambers. Changes of the three principal strains are then estimated from the measured volume changes in the $\sigma_x$, $\sigma_y$ and $\sigma_z$ directions. An influence of membrane penetration on the volume change was neglected, considering the thickness of membrane, the stress level and the grading of sample. The volume change, however, was corrected for the effect of expansion of all the tubing system under pressure.

**PREPARATION OF SPECIMEN AND TESTING PROCEDURE**

Glass beads with grain diameters ranging from 0.125 mm to 0.420 mm and with a specific gravity of 2.476 were used in this test. The maximum void ratio determined by placing quietly glass beads using a spoon into a cylindrical mold with the inner diameter of 50 mm and with the volume of 100 ml was 0.731. The minimum void-ratio compacted by tapping sides of the mold with a wooden hammer was 0.491. Dry specimens were prepared by pouring calmly glass beads using a spoon into the cubical triaxial cell, having a void ratio of 0.719. On the specimens prepared in such a manner, the direction of specimen deposition coincided with that of the principal stress, $\sigma_x$.

Isotropic compression tests along the space diagonal and radial shear tests in the deviatoric plane were performed under drained condition. For performing isotropic compression tests, specimens were loaded to an effective confining pressure of 245 kPa, followed by unloading to 2.45 kPa and then were reloaded to 98 kPa. For radial shear tests, specimens were first isotropically compressed to a pressure of 98 kPa, and then were subjected to shear stress under the constant value of $\sigma_x + \sigma_y + \sigma_z = 294$ kPa. The radial shear stress paths (Ko and Scott, 1968) were given at a 15° interval on the deviatoric plane and were designated as a $\theta$-path. Each of the loading stages along the radial shear stress paths was conducted with an increment of 4.9 kPa in the major or the minor principal stresses, while maintaining $\sigma_x + \sigma_y + \sigma_z = 294$ kPa.

The stress state during the tests can also be expressed by the parameter, $b$, introduced by Habib (1953):

$$b = \frac{\sigma_x - \sigma_z}{\sigma_1 - \sigma_3}$$  \hspace{1cm} (11)

where $\sigma_x$, $\sigma_y$, and $\sigma_z$ are the major, intermediate and minor principal stresses, respectively. The parameter, $b$, is related to Lode's parameter, $\mu$, by the relation of $2b = (1 + \mu)$. Therefore, the states of stresses employed in the tests were classified into five groups: $b$-value is zero for triaxial compression, $\theta$0°- and $\theta$120°-paths, $b=0.268$ for $\theta$15°-, $\theta$105°- and $\theta$135°-paths, $b=0.5$ for a constant value of the intermediate principal stress, $\theta$30°-, $\theta$90°- and
\( \theta 150^\circ \)-paths, \( b=0.732 \) for \( \theta 45^\circ \), \( \theta 75^\circ \) and \( \theta 165^\circ \)-paths and \( b=1.0 \) for triaxial extension, \( \theta 90^\circ \) and \( \theta 180^\circ \)-paths.

**ISOTROPIC COMPRESSION TESTS**

Regular packings of uniform spheres such as hexagonal close-packing and face-centered cubic packing have clearly an inherent anisotropy through their geometry of structure. Isotropic compression tests were performed in order to check the anisotropy of cubical specimens consisting of glass beads. Three principal strains, \( \varepsilon_x \), \( \varepsilon_y \), and \( \varepsilon_z \), and the actual unit volume change, \( \nu \), plotted against hydrostatic pressure, \( p \), are shown in Fig. 4. The total deformation under any stress condition can be divided into recoverable and irrecoverable deformations which are, respectively, referred to as elasticity and sliding. Now, strains, \( \varepsilon_x \) and \( \varepsilon_y \), in the directions perpendicular to that of specimen deposition and the actual unit volume change, \( \nu \), are plotted in Fig. 5 against strain, \( \varepsilon_z \), in the direction of specimen deposition. Loading, unloading and reloading paths were expressed by the same linear relation. If the cubical specimen showed isotropic behaviour, principal strains, \( \varepsilon_x \), \( \varepsilon_y \) and \( \varepsilon_z \), in the three perpendicular directions should be equal mutually, and \( \nu \) should be equal to \( 3 \varepsilon_z \). However, Fig. 5 shows \( \varepsilon_x = \varepsilon_y = 2.2 \varepsilon_z \) and \( \nu = 5.4 \varepsilon_z \). It was confirmed that no difference between horizontal strains, \( \varepsilon_x \) and \( \varepsilon_y \), was recognized, and that the horizontal compressibility was higher than the vertical one. Therefore, it was concluded that the specimen prepared under the gravitational field behaved anisotropically in the relationship between vertical and horizontal directions and behaved isotropically in the horizontal directions.

According to the tests on glass beads by Kallstenius and Bergau (1961), the number of spheres within a vertical cross-sectional unit area for the random packing of equal spheres was greater than that within a horizontal cross-sectional unit area. Also, Borowicka

![Fig. 4. Loading and reloading curves for isotropic compression test](image1)

![Fig. 5. Relationships between strains under isotropic compression test](image2)
(1973) showed that for a loose coarse sand before shearing the number of points of contact
was greater in the vertical direction in comparison with other directions. It is clear from
these studies that the difference in arrangements of spheres between vertical and horizontal
directions causes the specimen to show an inherent anisotropy, due to the requirements
of stability under the gravity acting on spheres, and that a typical sphere rests in the
hollow formed by the spheres below it. The manner of deposition prepared by free falling
of the glass beads through air might cause the specimen to have an inherent anisotropy,
wherein every grain was permitted to find an individual stable position.

According to El-Sohby and Andrawes (1973), the loading path plotted $v_u$ against $\varepsilon_z$
a greater deviation from the isotropic behaviour for loose sand, and the unloading path
nearly coincided with the isotropic relationship, $v_u=3\varepsilon_z$, independent of sample density.
Rowe (1971) showed that the slope of unloading path was nearly 3, notwithstanding the
differences in shape of grain, in grading and in porosity. However, in this study, both
loading and unloading processes, as shown in Fig.5, exhibited anisotropic behaviour, and
recoverable and irrecoverable deformations were consequently anisotropic. The deformation
characteristics reported herein are apparently different from the facts reported by Rowe
and El-Sohby et al. In their test results, when specimens were subjected to isotropic
compression until a hydrostatic pressure of 620 kPa was attained, the effects of anisotropy
in the specimens tended to disappear as the pressure became large. A main difference
between the present test and their tests lies in the magnitude of hydrostatic pressure.
The anisotropic behaviour shown in Fig.5 may have resulted from the fact that the difference
in geometry of structure between the vertical and horizontal directions would not
disappear in the range of stress level attained in this isotropic compression test.

RADIAL SHEAR TESTS

Principal Strain

The measured relationships between three principal strains, $\varepsilon_x$, $\varepsilon_y$ and $\varepsilon_z$, and the stress
ratio, $\eta$, obtained from the tests along the radial shear stress paths on the deviatoric
plane are shown in Fig.6. For a reference purpose, the result of a conventional triaxial
compression test is plotted in Fig.6(a). This test was performed on a cylindrical speci-
men, maintaining the constant value of a given mean principal stress. In this test,
strains, $\varepsilon_x$ and $\varepsilon_y$, were calculated by applying the measured volume change, $v_u$, and
axial strain, $\varepsilon_z$, to the relation of $\varepsilon_x=\varepsilon_y=(v_u-\varepsilon_z)/2$. It may be considered that the results
of a radial shear test and of a conventional triaxial compression test are approximately
the same within the limits of scatter normally experienced.

The specimens contracted in the direction of the major principal stress while they
expanded in that of the minor principal stress. The intermediate principal strain changed
from extension to compression with an increase of $b$-value located between zero and
unity. The $0^\circ$- and $180^\circ$-paths are exactly equal to the stress conditions employed in
the conventional triaxial compression and extension tests, respectively. The $120^\circ$-path
has the same stress condition as the $0^\circ$-path. In the $0^\circ$-path, $\sigma_x=\sigma_y$, the principal
stresses increased vertically in the $\sigma_x$-direction and decreased horizontally in the $\sigma_x$-
and $\sigma_y$-directions, while in the $120^\circ$-path, $\sigma_x=\sigma_y$, the principal stresses increased in one
horizontal $\sigma_x$-direction and decreased in the other horizontal $\sigma_y$- and the vertical $\sigma_z$-
directions. The difference in condition between both the paths lay whether the specimens
were compressed in the direction of specimen deposition or in the direction perpendicular
to that of specimen deposition. From Figs. 6(a) and 6(i), for the $0^\circ$-path expansive
strains, $\varepsilon_x$ and $\varepsilon_y$, were nearly equal; but, for the $120^\circ$-path the expansive strain, $\varepsilon_z$,
became larger than the expansive strain, $\varepsilon_y$, with an increase in $\eta$. The $60^\circ$-path has
Fig. 6. Stress ratio–principal strain curves along radial shear stress paths
the same stress condition as the $\theta 180^\circ$-path. The $\theta 60^\circ$-path, $\sigma_x=\sigma_y>\sigma_z$, and the $\theta 180^\circ$-path, $\sigma_x=\sigma_y>\sigma_z$, had the decreasing of $\sigma_y$ and $\sigma_z$, respectively; and had the increase of stresses in the other two principal stresses. From Figs. 6(e) and 6(m), the two compressive strains, $\varepsilon_x$ and $\varepsilon_y$, for the $\theta 180^\circ$-path were exactly equal, the compressive strain, $\varepsilon_z$, for the $\theta 60^\circ$-path, however, was remarkably larger than the corresponding strain, $\varepsilon_z$. Such anisotropic behaviours as mentioned above indicate that the specimen had a low compressibility and had a high extensibility in the direction of specimen-deposition in comparison with the direction perpendicular to its direction. Yamada and Ishihara (1979) presented the similar test results for a loose sand. None of the specimens for the $\theta 30^\circ$, $\theta 90^\circ$- and $\theta 150^\circ$-paths of $b=0.5$ shown in Fig.6 developed the intermediate principal strain up to the start of yielding, but the strain was observed after the yielding. The stress state in which no intermediate principal strain develops corresponds to a plane strain condition.

The relationships between principal strains and $b$-values at yielding point defined later are shown in Fig.7. In this figure, points A and C where $b=0$ correspond to the $\theta 0^\circ$- and $\theta 120^\circ$-paths, respectively, and points B and D where $b=1.0$ correspond to the $\theta 60^\circ$- and $\theta 180^\circ$-paths, respectively. The coincidence between $\varepsilon_x$ and $\varepsilon_y$ and the noncoincidence between $\varepsilon_x$ and $\varepsilon_y$ (or $\varepsilon_z$) seemed clear for the stress conditions where $b=0$ and $b=1.0$. The principal strain versus $b$-value curves in the $\theta$-value ranging from $0^\circ$ to $60^\circ$ appeared similar to the curves from $\theta=120^\circ$ to $180^\circ$, the major principal strain curve, however, showed a different tendency in the $\theta$-value ranging from $60^\circ$ to $120^\circ$. This property was considered dependent on the interrelation between the shear direction and the direction of specimen deposition. The directions of major and minor principal stresses were parallel and normal to the direction of specimen deposition in the ranges of $\theta=0^\circ$ to $60^\circ$ and $\theta=120^\circ$ to $180^\circ$, respectively; the directions of major and minor principal stresses in the range of $\theta=60^\circ$ to $120^\circ$, however, were both perpendicular to the direction of specimen deposition. $\varepsilon_x$ changed from compression to extension as the $\theta$-value increased from zero to $180^\circ$. Generally speaking, it was noted that the yielding strain was the function of a $b$-value, the major and minor principal strains showed a lower compressibility with an increase in $b$-value, and the intermediate principal strain showed a higher compressibility with the increase in $b$-values.

![Fig. 7. Relationships between principal strains and $b$-value at yielding](image)

**Distortion Strain and Strength**

Fig. 8 shows $\eta$ versus $\tau$ curves for each $b$-value. After $\tau$ of about 0.2% was attained, the larger the $\theta$-value was, the larger became the strain for the equal increment of $\eta$.

Now, the point of maximum curvature and the peak point of an $\eta$-$\tau$ curve are defined as the yielding and the failure points, respectively. Fig. 9 shows the loading, unloading
Fig. 8. Stress ratio-distortion strain curves of radial shear tests

and reloading curves of the radial shear test along the \( \theta 0^\circ \)-path and it is used to describe the definition of yielding point. By general definition, the current yielding point is a boundary which separates the plastic behaviour from the elastic one and is the limit within which recoverable deformation takes place. In reality, however, it is very difficult to determine the boundary for the specimen of glass beads, as gradual transition from elastic to plastic behaviour is observed. On the reloading curve of the specimen having a stress history of loading, irrecoverable strains does not develop until the stress-ratio of the first loading is reached. The yielding strength in the reloading curve, \( bac \), in Fig.9 may be the same as the strength at the end of the first loading and be given by the point, \( a \). When the reloading curve, \( bac \), is plotted from the same origin as the first loading curve, \( oa \), a smooth reloading curve, \( oaa'c' \), is obtained. Namely, the curve, \( oaa'c' \), is plotted from the origin of \( \tau-\gamma \) co-ordinates on the basis of increments of each strain-component, \( \Delta \varepsilon_x \), \( \Delta \varepsilon_y \) and \( \Delta \varepsilon_z \), caused by an increase in the corresponding principal stresses from the starting point of reloading. The new reloading curve obtained has the maximum curvature at point, \( a' \), corresponding to point, \( a \). From the similarity between the first loading curve,
\( oac \), and the reloading curve, \( oad'c' \), the yielding point in the first loading is defined as the point of maximum curvature of the \( \eta - \gamma \) curve and is shown by the point, \( Y \).

The specimen of glass beads is not ideally elastic, since measurable irrecoverable strains may occur during the loading which is less than the stress-ratio of the point, \( Y \). According to this definition of yielding, the specimen might well be treated as a quasi-elastic material. This method of defining the yielding point from an experimental stress-strain curve appears similar to the method introduced by Taylor and Quinney (1931). On the stress level of a starting point of large strain, the good agreement is found between the radial shear test along \( \theta 0^\circ \)-path and the conventional triaxial compression test, as shown in Fig.8(a).

The relationship between \( \gamma \) and the \( b \)-value at the yielding point is shown in Fig.10. \( \gamma \) at yielding decreased with the increase in the \( b \)-value in the ranges of \( \theta = 0^\circ \) to \( 60^\circ \) and \( \theta = 120^\circ \) to \( 180^\circ \) and was independent of the \( b \)-value in the range of \( \theta = 60^\circ \) to \( 120^\circ \).

\( \gamma \) at the failure strength was determined by the hyperbolic relation proposed by Kondner (1963), \( \gamma = \gamma/(m+n\eta) \), which can be expressed as a linear form with an intercept \( m \) and a slope \( n \) when \( \gamma/\eta \) is plotted as a function of \( \gamma \):

\[
\frac{\gamma}{\eta} = m + n\gamma
\]  \hspace{1cm} (12).

The reciprocals of \( m \) and \( n \) for the material tested refer to the initial tangent modulus of the \( \eta - \gamma \) curve and the ultimate value of \( \eta \), respectively. Yamada and Ishihara (1979) found that the inherent anisotropic structure formed in the specimen exerted a strong influence on the stress-strain behaviour within a range of small shear stresses, and that, when the shear stress became large enough to cause a failure in the specimen, the inherent anisotropic behaviour disappeared. According to their test results, when the shear strain was less than 1.0%, deformation of the specimen was highly anisotropic. Arthur et al. (1977a) showed from the test results free from any influence of inherent anisotropy that an induced anisotropy could have a large influence on the strain required to achieve a given stress ratio. From the above discussion, it is at present difficult to determine the failure strength of the specimen having an inherent anisotropy. Therefore, it is postulated in this study that the failure strength may be estimated by applying Eq. (12) to the deformation characteristics in the early stage of the radial shear test.

The plot of \( \gamma/\eta \) against \( \gamma \) for the \( \theta 120^\circ \)-path as a typical representation is shown in Fig.11. The applicability of a hyperbolic stress-strain relation was indicated by the agreement between the straight line and the experimental data.

The relationships between \( \gamma \) and the \( b \)-value at the yielding and failure strengths are shown in Fig.12. Resulting from the inherent anisotropy in the specimen, \( \gamma \) at yielding and at failure were dependent on both \( \theta \)- and \( b \)-values. Yamada and Ishihara (1979) showed that \( \gamma \) at failure was independent of the \( \theta \)-value and was determined only as a function of the \( b \)-value. This discrepancy can be explained by the difference in the magnitude of strains between the present study and their test results. The discussion in this study is based on the results within a range of small deformation in the early stage of radial shear tests. Then, the angle of internal friction, \( \phi \), determined by the equation \( \sin \phi = (\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3) \) at yielding and at failure was plotted against the \( b \)-value as shown in Fig.13. It is interesting to note that \( \phi \) is also a function of the \( b \)- and \( \theta \)-values in the same manner as \( \gamma \). Under the same stress condition specified by the \( b \)-value, both \( \phi \) and \( \gamma \) decreased with an increase in the \( \theta \)-value within the range of \( \theta = 0^\circ \) to \( 180^\circ \).
Assuming the isotropic strength for the material, the $\phi$-$b$ relation in Fig.13 should be expressed by a single curve independently of the $\theta$-value, and then the yielding and failure surfaces on the deviatoric plane in Fig.14 should be expressed symmetrically with respect to the $\theta=60^\circ$ and $\theta=120^\circ$-paths in the areas specified by $\theta=0^\circ$, $60^\circ$, $120^\circ$ and $180^\circ$ on that plane. The $\phi$-$b$ relations ascertained by previous studies using the flexible rubber bag cell have been presented by a single curve (Ko and Scott, 1968; Miyamori, 1976; Yamada and Ishihara, 1979). According to Moroto et al. (1975), $\phi$ in the range of $\theta=0^\circ$ to $60^\circ$ was by about $2^\circ$ larger than that in the range of $\theta=120^\circ$ to $180^\circ$. The difference in $\phi$ obtained by Arthur et al. (1977b) was $2^\circ$ to $3^\circ$. In Fig.13 the difference between the curve $AB$ for $\theta=0^\circ$ to $60^\circ$ and the curve $CD$ for $\theta=120^\circ$ to $180^\circ$ was $4.5^\circ$ to $9^\circ$ at failure and was $4.5^\circ$ to $6.5^\circ$ at yielding. By dividing the $\phi$-$b$ relation in Fig.13 into the curves $AB$, $BC$ and $CD$, it was ascertained that $\phi$ in the stress condition of conventional triaxial compression was always smaller than $\phi$ in that of conventional triaxial extension. The curves $AB$ and $CD$ showed similar property, and the curve $BC$ seemed to be a transitional part from the curve $AB$ to $CD$. Furthermore, $\phi$ showed a maximum value at the $\theta$-value of $15^\circ$ and showed a minimum value.
at the $\theta$-value of 120°.

The $\phi$-values are shown also on the deviatoric plane in Fig. 14 together with Mohr-Coulomb and Lade-Duncan (1975) failure criteria for the isotropic soil. It is considered that $\phi$ when $\theta=0^\circ$ to $60^\circ$ and also when $\theta=120^\circ$ to $180^\circ$ fitted nearly with the criterion proposed by Lade and Duncan, and that the inherent anisotropy in the specimen exerted a strong influence on the $\phi$-values when $\theta=60^\circ$ to $120^\circ$, especially when $\theta=60^\circ$ to $90^\circ$. Such a change of $\phi$ which was dependent on the radial shear stress path also resulted from the relation between the direction of specimen deposition and the direction of increments of principal stresses. Namely, the direction of increase of the principal stress in the range of $\theta=0^\circ$ to $75^\circ$ and the direction of decrease of the principal stress in the range of $\theta=115^\circ$ to $180^\circ$ coincided with that of specimen deposition. Any change of the principal stress in the direction of specimen deposition did not occur in the $\theta=90^\circ$-path.

Fig. 15 shows the relationship between the directions of radial shear stress paths in the principal stress space and the increment vectors of strain caused by the increment of stress-ratio of 0.05 from the current stress level of the distortion strain of 0.05%, 0.1% and 0.3% in the principal strain space. Accompanied by a given increase of stress ratio, the increments of three principal stresses develop increments of corresponding principal strains, $\Delta \varepsilon_z$, $\Delta \varepsilon_y$ and $\Delta \varepsilon_x$. The increment of distortion strain, $\Delta \varepsilon$, and the increment of strain direction angle, $\Delta \omega$, can be calculated by substituting $\Delta \varepsilon_z$, $\Delta \varepsilon_y$ and $\Delta \varepsilon_x$ into Eqs. (7) and (8), respectively. $\Delta \varepsilon$ is equal to the length of strain vector in the point of current stress state along the radial shear stress paths, and $\Delta \omega$ is equal to the angle of deviation from the corresponding radial shear stress path. The yielding points in each radial shear stress path were noted between $\tau=0.1\%$ and 0.3\%. On the $\theta=0^\circ$- and $\theta=180^\circ$-paths, the strain vectors coincided with the direction of stress path, whilst on the other stress paths the strain vectors had always a discrepancy in the clockwise direction, dependent on the inherent anisotropy.

**Volumetric Strain**

On the associated volume change of a specimen during radial shear, the volumetric strain, $\eta$, versus the stress ratio, $\tau$, curves for each $b$-value are shown in Fig. 16. Under the same stress condition as specified by an equal $b$-value, the volumetric strains changed from expansion to contraction with the increase in the $\theta$-value in the range of $\theta=0^\circ$ to $180^\circ$. The observed volumetric strains showed expansion for the stress paths in the range of $\theta=0^\circ$ to $30^\circ$, showing contraction for $\theta=60^\circ$ to $180^\circ$. There was really little or no volume change of the specimen in the $\theta=45^\circ$-path. In spite of the looseness of the specimen, the expansive tendency of the specimen which was noted in the range of small $\theta$-values is considered due to both the inherent anisotropy in the specimen and the loading condition under a constant value of the mean principal stress. However, the volume changes were only nominal. When the specimen approached failure beyond the yielding point, the volume change of the specimen during a conventional triaxial compression test indicated rapid expansion.
CONCLUSIONS

The main results are summarized as follows:

(1) Due to the requirement of grain stabilities, an assembly of spherical particles deposited by free falling under the gravitational field showed an inherent anisotropy.

(2) Deformations during loading, unloading and reloading processes in the isotropic compression test exhibited an anisotropic behaviour.

(3) The specimen prepared under the gravity had a compressibility in the direction of its deposition which was lower than that in the direction perpendicular to that of deposition. The strain vectors in all the radial shear stress paths excepting the $\theta = 0^\circ$ and $\theta = 180^\circ$ paths generally had a discrepancy in the clockwise direction.

(4) It is not possible to discuss the yielding criterion of anisotropic materials only within the range $\theta = 0^\circ$ to $60^\circ$. Deformation and strength behaviours were dependent on the relationship between the direction of specimen deposition and the direction of shear stress path. When the direction of specimen deposition coincided with the direction where $\theta = 0^\circ$, a deformation occurring at a given stress level increased the tendency of contraction with the increase the $\theta$-value under the equal stress state specified by the $b$-value. A remarkable influence of anisotropy on the strength occurred when $\theta = 60^\circ$ to $120^\circ$, especially when $\theta = 60^\circ$ to $90^\circ$. 

Fig. 16. Volumetric strain-stress ratio curves along radial shear stress paths
NOTATION

\[ b = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3) \]

\[ N = \text{spherical stress} \]

\[ \rho = \text{mean principal stress} \]

\[ S = \text{deviatory stress} \]

\[ v = \text{volumetric strain} \]

\[ \nu_a = \text{actual unit volume change} \]

\[ \gamma = \text{distortion strain} \]

\[ \varepsilon_{\text{oct}} = \text{octahedral shear strain} \]

\[ \varepsilon_{x}, \varepsilon_{y}, \text{and } \varepsilon_{z} = \text{three principal strains} \]

\[ \gamma = \text{stress ratio defined by } S/N \]

\[ \theta = \text{direction of radial shear stress path on deviatory plane} \]

\[ \sigma_1, \sigma_2 \text{ and } \sigma_3 = \text{major, intermediate and minor principal stresses} \]

\[ \sigma_{x}, \sigma_{y} \text{ and } \sigma_{z} = \text{three principal stresses} \]

\[ \tau_{\text{oct}} = \text{octahedral shear stress} \]

\[ \phi = \text{angle of internal friction} \]

\[ \omega = \text{angle of strain direction} \]

REFERENCES


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