HYDRO-DYNAMIC BEHAVIOR OF GROUND WATER IN CONFINED AND UNCONFINED LAYERS WITH CUT-OFF WALL

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ABSTRACT

This paper presents some theoretical and experimental results on hydro-dynamic behavior of ground water in horizontally confined and unconfined layers with the cut-off wall, based on Dupuit's assumption and approximate equation in cases of steady flows. Hydraulic quantities obtained theoretically and experimentally are the change of piezometric head at both boundaries of the cut-off wall, change of flow rate and storage volume change in both upstream and downstream aquifers.

Key words: alluvial deposit, cutoff, dynamic, ground water, seepage (IGC : E7)

INTRODUCTION

In recent years, a large-scale cut-off wall has been constructed to cut the seepage flow of aquifers in those cases as the excavation works, the underground works and the underground dam to store the water resources in this country. It is very important to know several hydraulic effects of the cut-off wall, such as the relationships between the seepage flow rate and the permeability of wall, and the change of free surface after the construction of cut-off wall in the aquifer. The cut-off wall itself is made from the grouting, the steel pile wall and the concrete wall for various purposes.

Hitherto, studies of this kind have been carried out by Matsuo et al. (1970), and Sato (1977), which are concerned with the storage effect of underground dam. Hydraulic analysis of different kind was presented by Thirriot (1979) by means of the potential mapping method.

This paper presents some theoretical and experimental results concerning the hydro-dynamic behavior of ground water in horizontally confined and unconfined layers with the cut-off wall, based on Dupuit's assumption and a successive steady state method in cases of steady and unsteady flows. Hydraulic quantities obtained by this theory and experiments are intensively concerned with the change of piezometric head near the cut-off wall, change of flow rate through the wall and storage volume change in both upstream and downstream aquifers of cut-off wall. Experiments are done by Hele-Shaw's model, and their results are compared with theoretical ones.

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Manuscript was received for review on April 22, 1981.
Written discussions on this paper should be submitted before January 1, 1983.
FUNDAMENTAL THEORY OF FLOW

Steady State Flow

Two kinds of layer models containing the cut-off wall are introduced as shown in Fig. 1 and Fig. 2. One is unconfined flow model having the free surface, and the other one is confined flow model having the constant thickness aquifer. Referring to Figs. 1 and 2, the governing equations can be written by using Dupuit's uniform flow assumption for unconfined and confined flows, respectively,

\[
\begin{align*}
q &= -K \frac{dh}{dx}, \\
\bar{q} &= -Km \frac{dh}{dx},
\end{align*}
\]

where \( q \) and \( \bar{q} \) are flow rate for unconfined and confined flows respectively, \( K \) is permeability of layer, \( m \) is thickness of confined layer, \( h \) is head, and \( x \) is axis along the bottom of layer.

Then, by integrating Eq. (1) we have,

\[
\begin{align*}
qx &= -\frac{K}{2} h^2 + C, \\
\bar{q}x &= -Kmh + \bar{C},
\end{align*}
\]

under the conditions of \( x=0, \ h=h_0, \ \bar{h}=\bar{h}_0 \) and \( x=l, \ h=h_3, \ \bar{h}=\bar{h}_3 \), in which \( k_0 \) is the permeability of aquifers, \( l \) is the horizontal length of aquifer \( (l=l_1+l_2+l_3) \), and \( h_0 \) and \( h_3 \) are boundary water levels.

Furthermore, in the case there is not the cut-off wall, the piezometric heads \( h_{10}, h_{20} \) of unconfined flow and \( \bar{h}_{10}, \bar{h}_{20} \) of confined one at the distances \( x=l_1, l_1+l_2 \), can be given by,

\[
\begin{align*}
\bar{h}_{10} &= \frac{\bar{h}_0}{k_0} - \frac{2 \bar{q} l_1}{k_0}, \\
\bar{h}_{20} &= \frac{\bar{h}_0}{k_0} - \frac{2 \bar{q} (l_1+l_2)}{k_0},
\end{align*}
\]

from Eqs. (2) and (3).

If three layers; the upstream aquifer, the cut-off wall and the downstream aquifer, are independent of each other, the flow rate \( q_1, q_2 \) and \( q_3 \) for each layer can be written by Eq. (2) as,

\[
\begin{align*}
q_1 &= k_0 \frac{(h_0^2-h_1^2)}{2l_1}, \\
q_2 &= k_0 \frac{(h_2^2-h_3^2)}{2l_2}, \\
q_3 &= k_0 \frac{(h_3^2-h_0^2)}{2l_3},
\end{align*}
\]

\[
\begin{align*}
\bar{q}_1 &= k_0 m \frac{(h_0-\bar{h}_1)}{l_1}, \\
\bar{q}_2 &= k_0 m \frac{(\bar{h}_1-\bar{h}_2)}{l_2}, \\
\bar{q}_3 &= k_0 m \frac{(\bar{h}_2-h_3)}{l_3},
\end{align*}
\]

in which \( k \) is the permeability of cut-off wall.
and \( h_1, h_2, \bar{h}_1 \) and \( \bar{h}_2 \) are the piezometric heads at the distances \( x = l_1 \) and \( l_1 + l_2 \) corresponding to the boundaries of cut-off wall for unconfined and confined flows, respectively. Accordingly, the flow rate \( q \) of steady unconfined flow and \( \bar{q} \) of steady confined one are given in Eqs. (8) and (9) by using the continuity conditions of Eq. (7),

\[
\begin{align*}
q &= q_1 = q_2 = q_t, \\
\bar{q} &= \bar{q}_1 = \bar{q}_2 = \bar{q}_t,
\end{align*}
\]

(7)

\[
\frac{q}{k_0(h_0 + h_0)} = \frac{(h_0 - h_0)}{2[(l_1 + l_2) + (k_0/k)l_2]},
\]

(8)

\[
\frac{\bar{q}}{k_0(h_0 + h_0)} = \frac{m}{(l_1 + l_2) + (k_0/k)l_2}.
\]

(9)

Ratio of flow rate \( q \) to \( q_0 \), and that of \( \bar{q} \) to \( \bar{q}_0 \) are obtained from Eqs. (3), (8) and (9) as,

\[
q/q_0 = \frac{q/\bar{q}_0}{(l_1 + l_2) + (k_0/k)l_2}.
\]

(10)

for both unconfined and confined flows.

From Eq. (10) we have,

\[
q/q_0 = \frac{q/\bar{q}_0}{(l_1 + l_2) + (k_0/k)l_2} = 1; \text{ when } k_0/k = 1,
\]

(11)

\[
q/q_0 = \frac{q/\bar{q}_0}{(l_1 + l_2) + (k_0/k)l_2} < 1; \text{ when } k_0/k > 1.
\]

Eq. (10) is one of the most important relations, which shows a hydraulic effect of cut-off wall in aquifer. This fundamental relation means that the term \((k_0/k)(l_1/l)\) will determine the cut-off effect, since the term \((l_1 + l_2)/l = l\) because of \(l_2 < l\) holds usually in fields. Accordingly, the cut-off effect is governed by permeability ratio \((k_0/k)\) in denominator of Eq. (10) because the non-dimensional quantity \((l_1/l)\) is very small. For example, we often use the steel sheet pile in order to cut-off the seepage flow in usual excavation works. In that case we fail to have the cut-off effect on the seepage if there is the leakage through the junctions of a series of pile walls; so the term \((k_0/k)\) is not so large as the term \((l_1/l)\) is smaller.

On the other hand, the piezometric heads \( h_1, h_2, \bar{h}_1 \) and \( \bar{h}_2 \) at the distances \( x = l_1 \) and \( l_1 + l_2 \) for unconfined and confined flows are given in the following equations by using Eq. (6) respectively,

\[
h_1 = \sqrt{\frac{(k_0/k)h_0^2 + kl_1h_0^2}{k(l_1 + l_2) + k_0l_2}},
\]

(12)

\[
h_2 = \sqrt{\frac{kl_1h_0^2 + (k_0/k)h_0^2}{k(l_1 + l_2) + k_0l_2}},
\]

(13)

Thus, by using Eqs. (10), (11), (12), and (13) the ratios of \( h_1/h_{10}, h_2/h_{20}, \bar{h}_1/\bar{h}_{10} \) and \( \bar{h}_2/\bar{h}_{20} \) are obtained as,

\[
(h_1/h_{10}) = \left\{ \frac{(k_0/k)l_1(l_1/l)}{(l_1 + l_2) + (k_0/k)l_2} \right\} \left( \frac{(l_1/l) + (k_0/k)(l_1/l)}{(h_0/h_{10})^2 + (l_0/l)} \right)\}
\]

(14)

\[
(h_2/h_{20}) = \left\{ \frac{(k_0/k)l_1(l_1/l)}{(l_1 + l_2) + (k_0/k)l_2} \right\} \left( \frac{(l_1/l) + (k_0/k)(l_1/l)}{(h_0/h_{10})^2 + (l_0/l)} \right)\}
\]

(15)

From Eqs. (14) and (15), we have,

\[
(h_1/h_{10}) = (h_2/h_{20}) = (\bar{h}_1/\bar{h}_{10}) = (\bar{h}_2/\bar{h}_{20}) = 1; \text{ when } (k_0/k) = 1,
\]

(16)

\[
(h_1/h_{10}) > 1, \quad (\bar{h}_1/\bar{h}_{10}) > 1; \text{ when } (k_0/k) > 1.
\]

(17)

We can calculate the piezometric heads \( h_1, h_2, \bar{h}_1 \) and \( \bar{h}_2 \) after the construction of cut-off wall for unconfined and confined steady flows by using Eqs. (14) and (15), if the initial hydraulic boundary conditions, the permeability ratio, and horizontal length of layer are given. It can be found that the piezometric heads \( h_1 \) and \( \bar{h}_1 \) become \( h_0 \) and \( h_2 \) and \( \bar{h}_2 \) become \( h_0 \) if permeability ratio \( k_0/k \) is infinite from Eqs. (12) and (13). At the same time, the seepage flow rate \( q \) and \( \bar{q} \) are zero by \( k_0/k \rightarrow \infty \) from Eq. (10).
Unsteady State Flow

After the cut-off wall is constructed, the ground water is stored in the upstream aquifer of cut-off wall, and is withdrawn in the downstream aquifer as described at early paragraph. In both aquifers the motion of ground water is unsteady, the piezometric head can be changed gradually with time. Analytic solution of this unsteady movement is troublesome and tedious. For the sake of practical use, an approximate solution of unsteady flow caused by installing the cut-off wall will be proposed by means of the method of successive steady states, at which obey Dupuit's flow (Aravin and Numerov, 1965).

Referring to Figs. 1 and 2, the ground water volume of upstream aquifer $V$ and the volume of downstream one $V_s$ before construction of cut-off wall are easily known. Then, we have two storage equations for unconfined and confined flow, respectively,

$$
\begin{align*}
(V_1 - V_{10})/t & \equiv (h_1 - h_{10}) l_1 \lambda_0/2 t = q_1 - q_{10} \\
(V_{s0} - V_s)/t & \equiv (h_{s0} - h_s) l_s \lambda_0/2 t = q_{s0} - q_s
\end{align*}
$$

in which $t$ is time, $\lambda_0$ is effective porosity, $S$ is the storage coefficient of aquifer, $V_1, V_{s0}$ are ground water volumes of upstream aquifer, $V_s, V_{s0}$ are ground water volumes of downstream aquifer, for unconfined and confined flows, respectively, after the construction of cut-off wall, $V_{10}, V_{s0}$ are water volumes of upstream aquifer, and $V_{s0}, V_{s0}$ are water volumes of downstream aquifer, for unconfined and confined flows, respectively, before the construction of cut-off wall.

Eq. (18) are valid for only case where the water level difference between the upstream water level $h_1$ and the downstream one $h_s$ at aquifer boundaries is small. This assumption is ordinarily satisfied, because a horizontal length of aquifer $l$ will be large in many fields.

Then, by substituting Eq. (6) into Eqs. (18) and (19), we have two simultaneous equations for unconfined and confined flows, including unknown quantities $h_1, h_s, h_1$ and $h_s$. Their solutions are given by,

$$
\begin{align*}
a_1 H_1 + b_1 H_s &= C_1 \\
a_2 H_1 + b_2 H_s &= C_2 \\
a_3 h_1 + b_3 h_s &= \bar{C}_1 \\
a_4 h_1 + b_4 h_s &= \bar{C}_2
\end{align*}
$$

where,

$$
\begin{align*}
a_1 &= 1/\tau + \kappa \alpha + \varepsilon, \quad b_1 = -\varepsilon \\
C_1 &= \kappa \alpha + H_{10}/\tau, \\
C_2 &= \kappa \beta H_s + H_{s0}/\tau', \\
\bar{C}_1 &= 1/\bar{\tau} + \kappa \bar{\alpha} + \bar{\varepsilon}, \quad \bar{b}_1 = -\bar{\varepsilon}, \\
\bar{C}_2 &= \kappa \bar{\beta} H_s + H_{s0}/\bar{\tau}', \\
\bar{C}_3 &= \kappa \bar{\beta} H_s + H_{s0}/\bar{\tau}', \\
\bar{C}_4 &= \kappa \bar{\beta} H_s + H_{s0}/\bar{\tau}',
\end{align*}
$$

and,

$$
\begin{align*}
\tau &= 2 t/k_1 l_1, \quad \bar{\tau} = 2 t/k_1 l_1 S, \\
\tau' &= 2 t/k_1 l_1, \quad \bar{\tau}' = 2 t/k_1 l_1 S, \\
\kappa &= k_0/k_1, \\
\alpha &= (h_s + h_{10})/2 l_1, \quad \bar{\alpha} = m/l_1, \\
\beta &= (h_{s0} + h_s)/2 l_1, \quad \bar{\beta} = m/l_1, \\
\varepsilon &= (h_{10} + h_{s0})/2 l_1, \quad \bar{\varepsilon} = m/l_1,
\end{align*}
$$

we can calculate unknown piezometric heads $h_1, h_s, h_1$ and $h_s$ at the distances $x = l_1$ and $l_1 + l_2$ for unconfined and confined unsteady flows by using Eqs. (24) and (25). Eqs. (24) and (25) are equal to Eqs. (14) and (15) respectively, when non-dimensional time $\tau, \bar{\tau}$, $\tau'$ and $\bar{\tau}'$ become infinite at time $t \to \infty$; steady state flow.

Then, non-dimensional flow rate of unsteady flow for unconfined and confined flows can be
obtained by Eqs. (3), (6), (24) and (25) as,

\[ \frac{q_l}{q_0} = \frac{(1-H_s^0)}{L_1(1-H_s^0)} \],
\[ \frac{q_l}{q_0} = \frac{H_s^2}{kL_2} \frac{[1-(H_s/H_1)^{2}]}{(1-H_s^2)}, \]
\[ \frac{q_l}{q_0} = \frac{H_s^2}{L_3} \frac{[1-(H_s/H_2)^{2}]}{(1-H_s^2)}, \]

and,
\[ \bar{q}_l/q_0 = \frac{1}{L_1} \frac{(1-H_s)}{(1-H_s^0)}, \]
\[ \bar{q}_l/q_0 = \frac{\bar{H}_1}{kL_2} \frac{[1-(\bar{H}_1/\bar{H}_1)^{2}]}{(1-\bar{H}_s^2)}, \]
\[ \bar{q}_l/q_0 = \frac{\bar{H}_2}{L_3} \frac{[1-(\bar{H}_2/\bar{H}_2)^{2}]}{(1-\bar{H}_s^2)}, \]

by putting \( L_1 = l_1/l, L_2 = l_2/l, \) and \( L_3 = l_3/l. \)

For only case where the water level difference between the upstream water level \( h_0 \) and the downstream one \( h_1 \) at both aquifer boundaries is small, the storage volumes of each aquifer for unconfined flow are written by,

\[ V_1 = l_1 h_0 (h_0 + h_1)/2 = \frac{h_0}{2} (1 + h_1/h_0) l_1 h_0, \]
\[ V_{10} = l_1 h_0 (h_0 + h_{10})/2 = \frac{h_0}{2} (1 + h_{10}/h_0) l_1 h_0, \]

and, the ratio of both volumes is,
\[ V_1/V_{10} = \frac{(1+H_1)}{(1+H_{10})}, \quad V_1/V_{10}>1, \quad \text{so} \quad H_1>H_{10}, \]

and similarly,
\[ V_2 = l_2 h_0 (h_2 + h_3)/2 = \frac{h_2}{2} (1 + h_3/h_2) l_2 h_0, \]
\[ V_{20} = l_2 h_0 (h_{20} + h_3)/2 = \frac{h_2}{2} (h_{20}/h_2 + h_3/h_2) l_2 h_0, \]
\[ V_2/V_{20} = \frac{(H_2 + H_3)}{(H_{20} + H_3)}, \quad V_2/V_{20}<1, \quad \text{so} \quad H_2<H_{20}, \]

in which \( V_1 \) and \( V_2 \) are the storage volumes of upstream flow and downstream one, respectively and \( V_{10} \) and \( V_{20} \) are those of initial condition.

**FUNDAMENTAL CHARACTERS OF THEORIES**

It is very important to examine several fundamental characters of theoretic results based on simplified Dupuit's assumptions with respect to uniform flow. The most important equation, which shows a hydraulic effect of cut-off wall, can be presented by Eq. (10). Fig. 3 shows relations between \( q/q_0 \) and \( k_s/k \), where \( l_0/l \) is parameter. It can be found that the flow rate ratio \( q/q_0 \) becomes 1 when \( k_s/k = 1 \), and \( q/q_0 \) approaches gradually to 0 as \( k_s/k \) increases. Furthermore, the hydraulic effect of cut-off wall may be expected with non-dimensional thickness of wall \( l_0/l \).

Ratios of piezometric heads at the distances \( x = l_1 \) and \( l_1 + l_2 \) corresponding to the boundaries of upstream and downstream sides of cut-off wall for unconfined and confined flow can be demonstrated, depending on the parameter \( h_3/h_0 \) as shown in Fig. 4. Fig. 4 indicates that piezometric heads \( h_1 \) and \( \bar{h}_1 \) of upstream aquifer rise as the permeability ratio \( k_s/k \) increases, and at the same time those of downstream aquifer \( h_2 \) and \( \bar{h}_2 \) withdraw according to Eqs. (14) and (15). It must be noted that the increment in the piezometric head of upstream aquifer caused by the cut-off wall is equal to the decrement of downstream aquifer withdrawn by wall, when the ratio of both boundary water levels \( h_0 \) and \( h_3 \) is small. However, the withdrawal of downstream aquifer can be distinguished when the
parameter \( h_3/h_0 \) is large, since the seepage flow is decreased with the cut-off effect.

For unsteady flow due to the cut-off wall, theoretical relations between non-dimensional piezometric heads \( H_1 (= h_1/h_0) \), \( H_2 (= h_2/h_0) \), \( H_3 (= h_3/h_0) \), \( H_4 (= h_4/h_0) \) and time \( t \) (day) are given as shown in Figs.5 and 6. Fig.5 shows a case of \( h_3/h_0 = 0.8 \), \( k_0/k = 10^4 \), \( h_0 = 30 \text{ m} \), \( k_0 = 8.649 \text{ m/d} \), \( l_1/l = 0.5 \), \( l_2 = 0.01 \), \( l_3/l = 0.49 \). Fig.6 is the case of \( h_3/h_0 = 0.5 \), and the other quantities are same with Fig.5. Non-dimensional piezometric heads \( H_{10} \) and \( H_{20} \) in the case of initial condition, there is no the cut-off wall, are calculated from Eqs. (4) and (5) for unconfined flow, and also \( H_{10} \) and \( H_{20} \) are obtained by the same equations for confined flow. In Fig.5, \( H_1 \) and \( H_1 \) of both unconfined and confined flows rise with time, and \( H_2 \) and \( H_2 \) withdraw gradually with time. However, it must

be noted that \( H_1 \) and \( H_2 \) of confined flow change quickly in a few days since the storage coefficient \( S \) is much smaller than the effective porosity \( \lambda_0 \) in nature. This fact can be often experienced in actual fields on the occasion of construction of the cut-off wall. In addition to this fact, it is interesting that two relations between \( H_1 \), \( H_1 \), and \( t \), as well as \( H_2 \), \( H_2 \), and \( t \), are very similar, because the water difference at boundaries of upstream and downstream aquifers is not so large: \( h_3/h_0 = 0.8 \).

On the other hand, Fig.6 is the case of \( h_3/h_0 = 0.5 \), and initial piezometric heads \( H_{10}, H_{20} \) are somewhat different, compared with those of Fig.5. For this case all piezometric heads will change somewhat rapid after the construction of cut-off wall, and approaches gradually to constant final heads at time \( t \rightarrow \infty \). Figs.7 and 8 show the changes in the seepage flow rate \( q_1/q_0 \), \( q_2/q_0 \) and \( q_3/q_0 \) with time for unconfined flow, and also \( q_1/q_0 \), \( q_2/q_0 \) and \( q_3/q_0 \) for confined flow, corre-
Fig. 8. Theoretical relationships between flow rates and time
(All conditions are the same with Fig. 6)

According to $h_s/h_0 = 0.8$ and 0.5, respectively. In both Figs. 7 and 8 the steady flow rate $q/q_0$ at time $t \rightarrow \infty$ becomes constant from Eqs. (26) and (27), which is the same with Eq. (10). Both upstream flow rates $q_s/q_0$, $q_s/q_0$ and downstream flow rates $q_d/q_0$, $q_d/q_0$ decrease gradually with time, on the contrary, flow rates $q_s/q_0$ and $q_d/q_0$ in cut-off wall increase with time, and all flow rates approach to $q/q_0 = 0.503$; const. It can be understood evidently that $q_s/q_0$ and $q_d/q_0$ are 0 at time $t = 0$, and they increase gradually as time elapses, since the head difference between upstream side and downstream one of cut-off wall becomes large all at once.

EXPERIMENTAL VERIFICATION OF THEORY
Experimental Apparatus and Procedure
In order to verify the theory by experiments, the flow rate and the piezometric head at both upstream and downstream sides of cut-off wall were measured and examined by Hele-Shaw apparatus of viscous fluid. Hele-Shaw apparatus is shown in Fig. 9, which has 130 cm in length and 35 cm in height, and made of the steel body having transparent resin plate of 1.93 cm in thickness. Ground water flow can be appeared in an interstice of 102 cm in length and 0.2 cm in width. In this experiments three kinds of cut-off wall were installed in the middle of the apparatus: steel plates of 0.05, 0.12 and 0.18 cm in thickness and 5.0 and 1.15 cm in width. Flow rate was measured by a graduated cylinder at downstream over-flow tank, and the free surface was observed by photograph with time. All experiments were concerned with the unconfined flows which have the free surface.

Experimental procedure is as follows; at first, the unconfined flow can be made under required conditions by adjusting of upstream and downstream over-flow tanks, in the next place, the flow rate will be measured by graduated cylinder at the downstream over-flow tank, and observed by photograph at the same time. All experimental conditions are summarized as shown in Table 1. Two unconfined flow of tail water level 18 cm and 13.5 cm were experienced for each cut-off wall at constant temperature $T=20^\circ C$. Kinematic viscosity of used fluid is 3.410 cm$^2$/s, and the specific weight is 0.892 at the temperature $T=20^\circ C$.

Experimental Results and Comparison with Theoretical Results
Several case of these experiments are demonstrated in Photo.1, 2, 3 and 4 as examples. Photo.1 shows the unconfined flow without the cut-off wall under the condition that the boundary water level of upstream aquifer $h_0$ is 25 cm and that of downstream aquifer $h_s$ is 18 cm. In this case flow rate $q_s$ is 1.406 cm$^2$/s by experiment (1.414 cm$^2$/s by theory), and the permeability of aquifer $k_0$ is 0.968 cm/s. Piezometric heads of upstream side $h_s$ and downstream one $h_0$ of cut-off wall are 22.100 cm and 21.5 cm by experiment, respectively.
### Table 1. Comparison with theoretical results and experiments

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Length of wall, aquifer</th>
<th>Water level</th>
<th>Width of intersticium</th>
<th>Permeability</th>
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<tr>
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<td>$l_2$</td>
<td>$l_3$</td>
<td>$l$</td>
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<tr>
<td>Exp.-1</td>
<td>48.500</td>
<td>5.000</td>
<td>48.500</td>
<td>102</td>
</tr>
<tr>
<td>Exp.-2</td>
<td>48.500</td>
<td>5.000</td>
<td>48.500</td>
<td>102</td>
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<tr>
<td>Exp.-3</td>
<td>55.425</td>
<td>1.150</td>
<td>45.425</td>
<td>102</td>
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<table>
<thead>
<tr>
<th>Initial level</th>
<th>Initial flow rate</th>
<th>Flow rate</th>
<th>Cut-off water level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{10}$</td>
<td>$q_{10 EXP}$</td>
<td>$h_{20}$</td>
<td>$q_{20 EXP}$</td>
</tr>
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</table>

Note: suffix "exp." means experimental results.

Photo 1. Experiment in the case of no-cut-off wall
($h_2=25$ cm, $h_3=18$ cm)

Photo 2. Experimental free surface by cut-off wall
($h_2=25$ cm, $h_3=13.5$ cm in case of Exp.-1)

Photo 3. Experimental free surface by cut-off wall
($h_2=25$ cm, $h_3=18$ cm in case of Exp.-2)

Photo 4. Experimental free surface by cut-off wall
($h_2=25$ cm, $h_3=18$ cm in case of Exp.-3)
successive steady states will be useful for all practical purposes unless the water level difference $h_0 - h_3$ at aquifer boundaries is not so large.

CONCLUSION

Theoretical approaches to analyze the hydraulic effect of the cut-off wall on ground water movement were proposed according to Dupuit's uniform flow assumption and the method of successive steady states with respect to unsteady motion for horizontally unconfined and confined layers. Theoretical results were verified experimentally by using Hele-Shaw apparatus. In this paper it can be concluded that non-dimensional flow rate $q_0/h_0$ is expressed by Eq. (10), and non-dimensional piezometric heads $h_1/h_{10}$, $h_2/h_{10}$ at upstream and downstream sides of cut-off wall are given by Eq. (14) for unconfined flow. At the same time, $h_1/h_{10}$, $h_2/h_{10}$ for confined flow are given by Eq. (15). On the other hand, non-dimensional heads $H_1(=h_1/h_0)$, $H_2(=h_2/h_0)$ of unconfined flow are obtained by Eq. (24) for unsteady flow, and also $H_1(=\tilde{h}_1/h_0)$, $H_2(=\tilde{h}_2/h_0)$ of confined flow are by Eq. (25). Furthermore, non-dimensional flow rates in the upstream and downstream aquifers with cut-off wall are summarized by Eqs. (26) and (27) in conclusion. In addition, it becomes important to discuss the storage capacity of water resources by means of underground wall, which is called the underground dam, on the basis of Eqs. (28), (29), (30) and (31).

REFERENCES