HYDRO-DYNAMIC BEHAVIOR OF GROUND WATER IN CONFINED AND UNCONFINED LAYERS WITH CUT-OFF WALL

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ABSTRACT

This paper presents some theoretical and experimental results on hydro-dynamic behavior of ground water in horizontally confined and unconfined layers with the cut-off wall, based on Dupuit's assumption and approximate equation in cases of steady flows. Hydraulic quantities obtained theoretically and experimentally are the change of piezometric head at both boundaries of the cut-off wall, change of flow rate and storage volume change in both upstream and downstream aquifers.

Key words : alluvial deposit, cutoff, dynamic, ground water, seepage (IGC : E 7)

INTRODUCTION

In recent years, a large-scale cut-off wall has been constructed to cut the seepage flow of aquifers in those cases as the excavation works, the underground works and the underground dam to store the water resources in this country. It is very important to know several hydraulic effects of the cut-off wall, such as the relationships between the seepage flow rate and the permeability of wall, and the change of free surface after the construction of cut-off wall in the aquifer. The cut-off wall itself is made from the grouting, the steel pile wall and the concrete wall for various purposes.

Hitherto, studies of this kind have been carried out by Matsuo et al. (1970), and Sato (1977), which are concerned with the storage effect of underground dam. Hydraulic analy-

sis of different kind was presented by Thirriot (1979) by means of the potential mapping method.

This paper presents some theoretical and experimental results concerning the hydro-dynamic behavior of ground water in horizontally confined and unconfined layers with the cut-off wall, based on Dupuit's assumption and a successive steady state method in cases of steady and unsteady flows. Hydraulic quantities obtained by this theory and experiments are intensively concerned with the change of piezometric head near the cut-off wall, change of flow rate through the wall and storage volume change in both upstream and downstream aquifers of cut-off wall. Experiments are done by Hele-Shaw's model, and their results are compared with theoretical ones.

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FUNDAMENTAL THEORY OF FLOW

Steady State Flow

Two kinds of layer models containing the cut-off wall are introduced as shown in Fig.1 and Fig.2. One is unconfined flow model having the free surface, and the other one is confined flow model having the constant thickness aquifer. Referring to Figs. 1 and 2, the governing equations can be written by using Dupuit's uniform flow assumption for unconfined and confined flows, respectively,

$$\left. \begin{aligned} q &= -Kh \frac{dh}{dx}, \\ \tilde{q} &= -Km \frac{dh}{dx}, \end{aligned} \right\} \quad (1)$$

in which q and $\tilde{q}$ are flow rate for unconfined and confined flows respectively, K is permeability of layer, m is thickness of confined layer, h is head, and x is axis along the bottom of layer.

Then, by integrating Eq. (1) we have,

$$\left. \begin{aligned} qx &= -\frac{K}{2} h^2 + C, \\ \tilde{q}x &= -Kmh + \tilde{C}, \end{aligned} \right\} \quad (2)$$

Fig. 1. Layer model of unconfined flow

Fig. 2. Layer model of confined flow

where C and $\tilde{C}$ are constants.

If there is no the cut-off wall in aquifer, the geometric shape of free surface of unconfined flow is parabolic, and that of the piezometric head of confined flow is straight as shown by dotted lines in Figs. 1 and 2, respectively. In those cases the flow rate q_0 of unconfined flow and $\tilde{q}_0$ of confined one can be written by,

$$\left. \begin{aligned} q_0 &= k_0 \frac{(h_0^2 - h_3^2)}{2l}, \\ \tilde{q}_0 &= k_0 m \frac{(h_0 - h_3)}{l}, \end{aligned} \right\} \quad (3)$$

under the conditions of $x=0$, $h=h_0$, $\tilde{h}=h_0$ and $x=l$, $h=h_3$, $\tilde{h}=h_3$, in which k_0 is the permeability of aquifers, l is the horizontal length of aquifer ($l=l_1+l_2+l_3$), and h_0 and h_3 are boundary water levels.

Furthermore, in the case there is not the cut-off wall, the piezometric heads h_{10} , h_{20} of unconfined flow and $\tilde{h}_{10}$, $\tilde{h}_{20}$ of confined one at the distances $x=l_1$, l_1+l_2 , can be given by,

$$\left. \begin{aligned} h_{10} &= \sqrt{h_0^2 - \frac{2q_0 l_1}{k_0}}, \\ \tilde{h}_{10} &= h_0 - \frac{\tilde{q}_0 l_1}{k_0 m}, \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} h_{20} &= \sqrt{h_0^2 - \frac{2q_0 (l_1 + l_2)}{k_0}}, \\ \tilde{h}_{20} &= h_0 - \frac{\tilde{q}_0 (l_1 + l_2)}{k_0 m}, \end{aligned} \right\} \quad (5)$$

from Eqs. (2) and (3).

If three layers; the upstream aquifer, the cut-off wall and the downstream aquifer, are independent of each other, the flow rate q_1 , q_2 and q_3 for each layer can be written by Eq. (2) as,

$$\left. \begin{aligned} q_1 &= k_0 \frac{(h_0^2 - h_1^2)}{2l_1}, & q_2 &= k \frac{(h_1^2 - h_2^2)}{2l_2}, \\ q_3 &= k_0 \frac{(h_2^2 - h_3^2)}{2l_3}, \\ \tilde{q}_1 &= k_0 m \frac{(h_0 - \tilde{h}_1)}{l_1}, & \tilde{q}_2 &= km \frac{(\tilde{h}_1 - \tilde{h}_2)}{l_2}, \\ \tilde{q}_3 &= k_0 m \frac{(\tilde{h}_2 - h_3)}{l_3}, \end{aligned} \right\} \quad (6)$$

in which k is the permeability of cut-off wall

and $h_1, h_2, \tilde{h}_1$ and $\tilde{h}_2$ are the piezometric heads at the distances $x=l_1$ and l_1+l_2 corresponding to the boundaries of cut-off wall for unconfined and confined flows, respectively.

Accordingly, the flow rate q of steady unconfined flow and $\tilde{q}$ of steady confined one are given in Eqs. (8) and (9) by using the continuity conditions of Eq. (7),

$$\left. \begin{aligned} q &= q_1 = q_2 = q_3, \\ \tilde{q} &= \tilde{q}_1 = \tilde{q}_2 = \tilde{q}_3, \end{aligned} \right\} \quad (7)$$

$$\frac{q}{k_0(h_0+h_3)} = \frac{(h_0-h_3)}{2\{(l_1+l_3) + (k_0/k)l_2\}}, \quad (8)$$

$$\frac{\tilde{q}}{k_0(h_0-h_3)} = \frac{m}{\{(l_1+l_3) + (k_0/k)l_2\}}. \quad (9)$$

Ratio of flow rate q to q_0 , and that of $\tilde{q}$ to $\tilde{q}_0$ are obtained from Eqs. (3), (8) and (9) as,

$$q/q_0 = \tilde{q}/\tilde{q}_0 = \frac{1}{\left\{ \frac{(l_1+l_3)}{l} + (k_0/k) \frac{(l_2/l)}{l} \right\}}, \quad (10)$$

for both unconfined and confined flows.

From Eq. (10) we have,

$$\left. \begin{aligned} q/q_0 &= \tilde{q}/\tilde{q}_0 = 1; \text{ when } k_0/k = 1, \\ q/q_0 &= \tilde{q}/\tilde{q}_0 < 1; \text{ when } k_0/k > 1. \end{aligned} \right\} \quad (11)$$

Eq. (10) is one of the most important relations, which shows a hydraulic effect of cut-off wall in aquifer. This fundamental relation means that the term $(k_0/k)(l_2/l)$ will determine the cut-off effect, since the term $(l_1+l_3)/l \doteq l$ because of $l_2 \ll l$ holds usually in fields. Accordingly, the cut-off effect is governed by permeability ratio (k_0/k) in denominator of Eq. (10) because the non-dimensional quantity (l_2/l) is very small. For example, we often use the steel sheet pile in order to cut-off the seepage flow in usual excavation works. In that case we fail to have the cut-off effect on the seepage if there is the leakage through the junctions of a series of pile walls; so the term (k_0/k) is not so large as the term (l_2/l) is smaller.

On the other hand, the piezometric heads $h_1, h_2, \tilde{h}_1$ and $\tilde{h}_2$ at the distances $x=l_1$ and l_1+l_2 for unconfined and confined flows are given in the following equations by using Eq. (6) respectively,

$$\left. \begin{aligned} h_1 &= \sqrt{\frac{(kl_3+k_0l_2)h_0^2+kl_1h_3^2}{k(l_1+l_3)+k_0l_2}}, \\ h_2 &= \sqrt{\frac{kl_3h_0^2+(kl_1+k_0l_2)h_3^2}{k(l_1+l_3)+k_0l_2}}, \end{aligned} \right\} \quad (12)$$

$$\left. \begin{aligned} \tilde{h}_1 &= \frac{(kl_3+k_0l_2)h_0+kl_1h_3}{k(l_1+l_3)+k_0l_2}, \\ \tilde{h}_2 &= \frac{kl_3h_0+(kl_1+k_0l_2)h_3}{k(l_1+l_3)+k_0l_2}. \end{aligned} \right\} \quad (13)$$

Thus, by using Eqs. (10), (11), (12), and (13) the ratios of $h_1/h_{10}, h_2/h_{20}, \tilde{h}_1/\tilde{h}_{10}$ and $\tilde{h}_2/\tilde{h}_{20}$ are obtained as,

$$\left. \begin{aligned} (h_1/h_{10}) &= \sqrt{\frac{(k_0/k) \frac{(l_2/l)}{l} + \frac{(l_1/l)}{l} (h_3/h_0)^2 + (l_3/l)}{\{(1-l_1/l) + (l_1/l) (h_3/h_0)^2\} \times \{(k_0/k) \frac{(l_2/l)}{l} + (l_1+l_3)/l\}}}, \\ (h_2/h_{20}) &= \sqrt{\frac{\{(l_1/l) + (k_0/k) \frac{(l_2/l)}{l}\} \times (h_3/h_0)^2 + (l_3/l)}{\{(1-l_3/l) (h_3/h_0)^2 + (l_3/l)\} \times \{(k_0/k) \frac{(l_2/l)}{l} + (l_1+l_3)/l\}}}, \end{aligned} \right\} \quad (14)$$

$$\left. \begin{aligned} (\tilde{h}_1/\tilde{h}_{10}) &= \frac{(k_0/k) \frac{(l_2/l)}{l} + \frac{(l_1/l)}{l} (h_3/h_0) + l_3/l}{\{(1-l_1/l) + (l_1/l) (h_3/h_0)\} \times \{(k_0/k) \frac{(l_2/l)}{l} + (l_1+l_3)/l\}}, \\ (\tilde{h}_2/\tilde{h}_{20}) &= \frac{\{(l_1/l) + (k_0/k) \frac{(l_2/l)}{l}\} \times (h_3/h_0) + l_3/l}{\{(1-l_3/l) (h_3/h_0) + (l_3/l)\} \times \{(k_0/k) \frac{(l_2/l)}{l} + (l_1+l_3)/l\}}. \end{aligned} \right\} \quad (15)$$

From Eqs. (14) and (15), we have,

$$\begin{aligned} (h_1/h_{10}) &= (h_2/h_{20}) = (\tilde{h}_1/\tilde{h}_{10}) \\ &= (\tilde{h}_2/\tilde{h}_{20}) = 1; \text{ when } (k_0/k) = 1, \end{aligned} \quad (16)$$

$$\begin{aligned} (h_1/h_{10}) &> 1, \quad (\tilde{h}_1/\tilde{h}_{10}) > 1, \\ (h_2/h_{20}) &< 1, \quad (\tilde{h}_2/\tilde{h}_{20}) < 1, \end{aligned} \quad ; \text{ when } (k_0/k) > 1. \quad (17)$$

We can calculate the piezometric heads $h_1, h_2, \tilde{h}_1$ and $\tilde{h}_2$ after the construction of cut-off wall for unconfined and confined steady flows by using Eqs. (14) and (15), if the initial hydraulic boundary conditions, the permeability ratio, and horizontal length of layer are given. It can be found that the piezometric heads h_1 and $\tilde{h}_1$ become h_0 , and h_2 and $\tilde{h}_2$ become h_3 , if permeability ratio k_0/k is infinite from Eqs. (12) and (13). At the same time, the seepage flow rate q and $\tilde{q}$ are zero by $k_0/k \rightarrow \infty$ from Eq. (10).

Unsteady State Flow

After the cut-off wall is constructed, the ground water is stored in the upstream aquifer of cut-off wall, and is withdrawn in the downstream aquifer as described at early paragraph. In both aquifers the motion of ground water is unsteady, the piezometric head can be changed gradually with time. Analytic solution of this unsteady movement is troublesome and tedious. For the sake of practical use, an approximate solution of unsteady flow caused by installing the cut-off wall will be proposed by means of the method of successive steady states, at which obey Dupuit's flow (Aravin and Numerov, 1965).

Referring to Figs. 1 and 2, the ground water volume of upstream aquifer V and the volume of downstream one V_0 before construction of cut-off wall are easily known. Then, we have two storage equations for unconfined and confined flow, respectively,

$$\left. \begin{aligned} (V_1 - V_{10})/t &\doteq (h_1 - h_{10}) l_1 \lambda_0 / 2t = q_1 - q_2, \\ (V_{30} - V_3)/t &\doteq (h_{20} - h_2) l_3 \lambda_0 / 2t = q_3 - q_2, \end{aligned} \right\} \quad (18)$$

$$\left. \begin{aligned} (\tilde{V}_1 - \tilde{V}_{10})/t &\doteq (\tilde{h}_1 - \tilde{h}_{10}) l_1 S / 2t = \tilde{q}_1 - \tilde{q}_2, \\ (\tilde{V}_{30} - \tilde{V}_3)/t &\doteq (\tilde{h}_{20} - \tilde{h}_2) l_3 S / 2t = \tilde{q}_3 - \tilde{q}_2, \end{aligned} \right\} \quad (19)$$

in which t is time, λ_0 is effective porosity, S is the storage coefficient of aquifer, $V_1, \tilde{V}_1$ are ground water volumes of upstream aquifer, $V_3, \tilde{V}_3$ are ground water volumes of downstream aquifer, for unconfined and confined flows, respectively, after the construction of cut-off wall, $V_{10}, \tilde{V}_{10}$ are water volumes of upstream aquifer, and $V_{30}, \tilde{V}_{30}$ are water volumes of downstream aquifer, for unconfined and confined flows, respectively, before the construction of cut-off wall.

Eq. (18) are valid for only case where the water level difference between the upstream water level h_0 and the downstream one h_3 at aquifer boundaries is small. This assumption is ordinarily satisfied, because a horizontal length of aquifer l will be large in many fields.

Then, by substituting Eq. (6) into Eqs. (18) and (19), we have two simultaneous equations for unconfined and confined flows, including

unknown quantities $h_1, h_2, \tilde{h}_1$ and $\tilde{h}_2$. Their solutions are given by,

$$\left. \begin{aligned} a_1 H_1 + b_1 H_2 &= C_1 \\ a_2 H_1 + b_2 H_2 &= C_2 \end{aligned} \right\} \quad (20)$$

$$\left. \begin{aligned} \tilde{a}_1 \tilde{H}_1 + \tilde{b}_1 \tilde{H}_2 &= \tilde{C}_1 \\ \tilde{a}_2 \tilde{H}_1 + \tilde{b}_2 \tilde{H}_2 &= \tilde{C}_2 \end{aligned} \right\} \quad (21)$$

where,

$$\left. \begin{aligned} a_1 &= 1/\tau + \kappa\alpha + \varepsilon, \quad b_1 = -\varepsilon \\ C_1 &= \kappa\alpha + H_{10}/\tau, \\ a_2 &= -\varepsilon, \quad b_2 = 1/\tau' + \kappa\beta + \varepsilon, \\ C_2 &= \kappa\beta H_3 + H_{20}/\tau', \\ \tilde{a}_1 &= 1/\tilde{\tau} + \kappa\tilde{\alpha} + \tilde{\varepsilon}, \quad \tilde{b}_1 = -\tilde{\varepsilon}, \\ \tilde{C}_1 &= \kappa\tilde{\alpha} + \tilde{H}_{10}/\tilde{\tau}, \\ \tilde{a}_2 &= -\tilde{\varepsilon}, \quad \tilde{b}_2 = 1/\tilde{\tau}' + \kappa\tilde{\beta} + \tilde{\varepsilon}, \\ \tilde{C}_2 &= \kappa\tilde{\beta} H_3 + \tilde{H}_{20}/\tilde{\tau}', \end{aligned} \right\} \quad (22)$$

and,

$$\begin{aligned} \tau &= 2tk/l_1 \lambda_0, \quad \tilde{\tau} = 2tk/l_1 S, \\ \tau' &= 2tk/l_3 \lambda_0, \quad \tilde{\tau}' = 2tk/l_3 S, \\ \kappa &= k_0/k, \\ \alpha &= (h_0 + h_{10})/2l_1, \quad \tilde{\alpha} = m/l_1, \\ \beta &= (h_{20} + h_3)/2l_3, \quad \tilde{\beta} = m/l_3, \\ \varepsilon &= (h_{10} + h_{20})/2l_2, \quad \tilde{\varepsilon} = m/l_2, \\ H_1 &= h_1/h_0, \quad \tilde{H}_1 = \tilde{h}_1/h_0, \quad H_2 = h_2/h_0, \\ \tilde{H}_2 &= \tilde{h}_2/h_0, \quad H_{10} = h_{10}/h_0, \quad \tilde{H}_{10} = \tilde{h}_{10}/h_0, \\ H_3 &= h_3/h_0, \quad H_{20} = h_{20}/h_0, \quad \tilde{H}_{20} = \tilde{h}_{20}/h_0, \end{aligned}$$

$$\left. \begin{aligned} H_1 &= \frac{b_2 C_1 - b_1 C_2}{a_1 b_2 - a_2 b_1}, \\ H_2 &= \frac{a_2 C_1 - a_1 C_2}{a_2 b_1 - a_1 b_2}, \end{aligned} \right\} \quad (24)$$

$$\left. \begin{aligned} \tilde{H}_1 &= \frac{\tilde{b}_2 \tilde{C}_1 - \tilde{b}_1 \tilde{C}_2}{\tilde{a}_1 \tilde{b}_2 - \tilde{a}_2 \tilde{b}_1}, \\ \tilde{H}_2 &= \frac{\tilde{a}_2 \tilde{C}_1 - \tilde{a}_1 \tilde{C}_2}{\tilde{a}_2 \tilde{b}_1 - \tilde{a}_1 \tilde{b}_2}. \end{aligned} \right\} \quad (25)$$

We can calculate unknown piezometric heads $h_1, h_2, \tilde{h}_1$ and $\tilde{h}_2$ at the distances $x=l_1$ and $l_1 + l_2$ for unconfined and confined unsteady flows by using Eqs. (24) and (25). Eqs. (24) and (25) are equal to Eqs. (14) and (15) respectively, when non-dimensional time $\tau, \tilde{\tau}, \tau'$ and $\tilde{\tau}'$ become infinite at time $t \rightarrow \infty$; steady state flow.

Then, non-dimensional flow rate of unsteady flow for unconfined and confined flows can be

obtained by Eqs. (3), (6), (24) and (25) as,

$$\left. \begin{aligned} q_1/q_0 &= \frac{(1-H_1^2)}{L_1(1-H_3^2)}, \\ q_2/q_0 &= \frac{H_1^2}{\kappa L_2} \frac{\{1-(H_2/H_1)^2\}}{(1-H_3^2)}, \\ q_3/q_0 &= \frac{H_2^2}{L_3} \frac{\{1-(H_3/H_2)^2\}}{(1-H_3^2)}, \end{aligned} \right\} \quad (26)$$

and,

$$\left. \begin{aligned} \tilde{q}_1/\tilde{q}_0 &= \frac{1}{L_1} \frac{(1-\tilde{H}_1)}{(1-\tilde{H}_3)}, \\ \tilde{q}_2/\tilde{q}_0 &= \frac{\tilde{H}_1}{\kappa L_2} \frac{\{1-(\tilde{H}_2/\tilde{H}_1)\}}{(1-\tilde{H}_3)}, \\ \tilde{q}_3/\tilde{q}_0 &= \frac{\tilde{H}_2}{L_3} \frac{\{1-(H_3/\tilde{H}_2)\}}{(1-\tilde{H}_3)}, \end{aligned} \right\} \quad (27)$$

by putting $L_1=l_1/l$, $L_2=l_2/l$ and $L_3=l_3/l$.

For only case where the water level difference between the upstream water level h_0 and the downstream one h_3 at both aquifer boundaries is small, the storage volumes of each aquifer for unconfined flow are written by,

$$\left. \begin{aligned} V_1 &\doteq l_1 \lambda_0 (h_0 + h_1)/2 = \frac{h_0}{2} (1 + h_1/h_0) l_1 \lambda_0, \\ V_{10} &\doteq l_1 \lambda_0 (h_0 + h_{10})/2 = \frac{h_0}{2} (1 + h_{10}/h_0) l_1 \lambda_0, \end{aligned} \right\}$$

and, the ratio of both volumes is,

$$V_1/V_{10} = \frac{(1+H_1)}{(1+H_{10})}, \quad V_1/V_{10} > 1, \quad \text{so } H_1 > H_{10}, \quad (28)$$

and similarly,

$$\left. \begin{aligned} V_3 &\doteq l_3 \lambda_0 (h_2 + h_3)/2 = \frac{h_2}{2} (1 + h_3/h_2) l_3 \lambda_0, \\ V_{30} &\doteq l_3 \lambda_0 (h_{20} + h_3)/2 = \frac{h_0}{2} (h_{20}/h_0 + h_3/h_0) l_3 \lambda_0, \\ V_3/V_{30} &= \frac{(H_2 + H_3)}{(\tilde{H}_{20} + H_3)}, \quad V_3/V_{30} < 1, \quad \text{so } H_2 < H_{20}, \end{aligned} \right\} \quad (29)$$

in which V_1 and V_3 are the storage volumes of upstream flow and downstream one, respectively and V_{10} and V_{30} are those of initial condition for case without the cut-off wall. Similarly, the volume ratios of confined flow are given by,

$$\tilde{V}_1/\tilde{V}_{10} = \frac{(1+\tilde{H}_1)}{(1+\tilde{H}_{10})}, \quad \tilde{V}_1 > \tilde{V}_{10}, \quad \text{so } \tilde{H}_1 > \tilde{H}_{10}, \quad (30)$$

$$\tilde{V}_3/\tilde{V}_{30} = \frac{(\tilde{H}_2 + H_3)}{(\tilde{H}_{20} + H_3)}, \quad \tilde{V}_3 < \tilde{V}_{30}, \quad \text{so } \tilde{H}_2 < \tilde{H}_{20}, \quad (31)$$

in which $\tilde{V}_1$ and $\tilde{V}_3$ are the storage volumes

of upstream flow and downstream one, respectively and $\tilde{V}_{10}$ and $\tilde{V}_{30}$ are those of initial condition.

FUNDAMENTAL CHARACTERS OF THEORIES

It is very important to examine several fundamental characters of theoretic results based on simplified Dupuit's assumptions with respect to uniform flow. The most important equation, which shows a hydraulic effect of cut-off wall, can be presented by Eq. (10). Fig. 3 shows relations between q/q_0 and k_0/k , where l_2/l is parameter. It can be found that the flow rate ratio q/q_0 becomes 1 when $k_0/k = 1$, and q/q_0 approaches gradually to 0 as k_0/k increases. Furthermore, the hydraulic effect of cut-off wall may be expected with non-dimensional thickness of wall l_2/l .

Ratios of piezometric heads at the distances $x=l_1$ and l_1+l_2 corresponding to the boundaries of upstream and downstream sides of cut-off wall for unconfined and confined flow can be demonstrated, depending on the parameter h_3/h_0 as shown in Fig. 4. Fig. 4 indicates that piezometric heads h_1 and $\tilde{h}_1$ of upstream aquifer rise as the permeability ratio k_0/k increases, and at the same time those of downstream aquifer h_2 and $\tilde{h}_2$ withdraw according to Eqs. (14) and (15). It must be noted that the increment in the piezometric head of upstream aquifer caused by the cut-off wall is equal to the decrement of downstream aquifer withdrawn by wall, when the ratio of both boundary water levels h_0 and h_3 is small. However, the withdrawal of downstream aquifer can be distinguished when the

Fig. 3. Theoretical relationships between q/q_0 and k_0/k

Fig. 4. Theoretical relationships between h_1/h_0 , $\tilde{h}_1/h_0$, h_2/h_0 , $\tilde{h}_2/h_0$ and k_0/k

Fig. 5. Theoretical relationships between H_1 , $\tilde{H}_1$, H_2 , $\tilde{H}_2$ and t ($h_3/h_0=0.8$, $k_0/k=10^2$, $h_0=30$ m, $k_0=8.640$ m/d, $l_1/l=0.5$, $l_2=0.01$, $l_3/l=0.49$)

parameter h_3/h_0 is large, since the seepage flow is decreased with the cut-off effect.

For unsteady flow due to the cut-off wall theoretical relations between non-dimensional piezometric heads $H_1(=h_1/h_0)$, $\tilde{H}_1(=\tilde{h}_1/h_0)$, $H_2(=h_2/h_0)$, $\tilde{H}_2(=\tilde{h}_2/h_0)$ and time t (day) are given as shown in Figs. 5 and 6. Fig. 5 shows a case of $h_3/h_0=0.8$, $k_0/k=10^2$, $h_0=30$ m, $k_0=8.640$ m/d, $l_1=500$ m, $l_2=10$ m, $l_3=490$ m, the effective porosity $\lambda_0=0.2$ and the storage coefficient $S=10^{-5}$. Fig. 6 is the case of $h_3/h_0=0.5$, and the other quantities are same with Fig. 5. Non-dimensional piezometric heads H_{10} and H_{20} in the case of initial condition, there is no the cut-off wall, are calculated from Eqs. (4) and (5) for unconfined flow, and also $\tilde{H}_{10}$ and $\tilde{H}_{20}$ are obtained by the same equations for confined flow. In Fig. 5, H_1 and $\tilde{H}_1$ of both unconfined and confined flows rise with time, and H_2 and $\tilde{H}_2$ withdraw gradually with time. However, it must

Fig. 6. Theoretical relationships between H_1 , $\tilde{H}_1$, H_2 , $\tilde{H}_2$ and t ($h_3/h_0=0.5$, Other conditions are the same with Fig. 5)

Fig. 7. Theoretical relationships between flow rates and time (All conditions are the same with Fig. 5)

be noted that $\tilde{H}_1$ and $\tilde{H}_2$ of confined flow change quickly in a few days since the storage coefficient S is much smaller than the effective porosity λ_0 in nature. This fact can be often experienced in actual fields on the occasion of construction of the cut-off wall. In addition to this fact, it is interesting that two relations between H_1 , $\tilde{H}_1$ and t , as well as H_2 , $\tilde{H}_2$ and t , are very similar, because the water difference at boundaries of upstream and downstream aquifers is not so large: $h_3/h_0=0.8$.

On the other hand, Fig. 6 is the case of $h_3/h_0=0.5$, and initial piezometric heads H_{10} , $\tilde{H}_{10}$ are somewhat different, compared with those of Fig. 5. For this case all piezometric heads will change somewhat rapid after the construction of cut-off wall, and approaches gradually to constant final heads at time $t \rightarrow \infty$. Figs. 7 and 8 show the changes in the seepage flow rate q_1/q_0 , q_2/q_0 , and q_3/q_0 with time for unconfined flow, and also $\tilde{q}_1/q_0$, $\tilde{q}_2/q_0$ and $\tilde{q}_3/q_0$ for confined flow, corre-

Fig. 8. Theoretical relationships between flow rates and time
(All conditions are the same with Fig. 6)

sponding to $h_3/h_0=0.8$ and 0.5 , respectively. In both Figs. 7 and 8 the steady flow rate q/q_0 at time $t \rightarrow \infty$ becomes constant from Eqs. (26) and (27), which is the same with Eq. (10). Both upstream flow rates q_1/q_0 , $\tilde{q}_1/q_0$ and downstream flow rates q_3/q_0 , $\tilde{q}_3/q_0$ decrease gradually with time, on the contrary, flow rates q_2/q_0 and $\tilde{q}_2/q_0$ in cut-off wall increase with time, and all flow rates approach to $q/q_0=0.503$; const.. It can be understood evidently that q_2/q_0 and $\tilde{q}_2/q_0$ are 0 at time $t=0$, and they increase gradually as time elapses, since the head difference between upstream side and downstream one of cut-off wall becomes large all at once.

EXPERIMENTAL VERIFICATION OF THEORY

Experimental Apparatus and Procedure

In order to verify the theory by experiments, the flow rate and the piezometric head at both upstream and downstream sides of cut-off wall were measured and examined by Hele-Shaw apparatus of viscous fluid. Hele-Shaw apparatus is shown in Fig. 9, which has 130 cm in length and 35 cm in height, and made of the steel body having transparent resin plate of 1.93 cm in thickness. Ground water flow can be appeared in an interstice of 102 cm in length and 0.2 cm in width. In this experiments three kinds of cut-off wall were installed in the middle of the apparatus: steel plates of 0.05, 0.12 and 0.18 cm in thickness and 5.0 and 1.15 cm in width. Flow rate was measured by a graduated cylinder

Fig. 9. Hele-Shaw model apparatus
(Unit ; cm)

at downstream over-flow tank, and the free surface was observed by photograph with time. All experiments were concerned with the unconfined flows which have the free surface.

Experimental procedure is as follows; at first, the unconfined flow can be made under required conditions by adjusting of upstream and downstream over-flow tanks, in the next place, the flow rate will be measured by graduated cylinder at the downstream over-flow tank, and observed by photograph at the same time. All experimental conditions are summarized as shown in Table 1. Two unconfined flow of tail water level 18 cm and 13.5 cm were experienced for each cut-off wall at constant temperature $T=20^\circ\text{C}$. Kinematic viscosity of used fluid is $3.410 \text{ cm}^2/\text{s}$, and the specific weight is 0.892 at the temperature $T=20^\circ\text{C}$.

Experimental Results and Comparison with Theoretical Results

Several case of these experiments are demonstrated in Photo. 1, 2, 3 and 4 as examples. Photo. 1 shows the unconfined flow without the cut-off wall under the condition that the boundary water level of upstream aquifer h_0 is 25 cm and that of downstream aquifer h_3 is 18 cm. In this case flow rate q_0 is $1.406 \text{ cm}^2/\text{s}$ by experiment ($1.414 \text{ cm}^2/\text{s}$ by theory), and the permeability of aquifer k_0 is 0.958 cm/s . Piezometric heads of upstream side h_{10} and downstream one h_{20} of cut-off wall are 22.100 cm and 21.5 cm by experiment, respectively.

Table 1. Comparison with theoretical results and experiments

Exp. No.	Length of wall, aquifer				Water level		Width of interstile		Permeability	
	l_1	l_2	l_3	l	h_0	h_3	d_0	d	k_0	k
Exp.-1	48.500	5.000	48.500	102	25	18.0	0.2	0.15	0.958	0.539
	"	"	"	"	"	13.5	"	"	"	"
Exp.-2	48.500	5.000	48.500	102	25	18.0	0.2	0.08	0.958	0.153
	"	"	"	"	"	13.5	"	"	"	"
Exp.-3	55.425	1.150	45.425	102	25	18.0	0.2	0.02	0.958	0.00958
	"	"	"	"	"	13.5	"	"	"	"

Initial level				Initial flow rate		Flow rate		Cut-off water level			
h_{10}	$h_{10 \text{ exp}}$	h_{20}	$h_{20 \text{ exp}}$	q_0	$q_{0 \text{ exp}}$	q	q_{exp}	h_1	$h_{1 \text{ exp}}$	h_2	$h_{2 \text{ exp}}$
21.480	22.10	21.400	21.50	1.414	1.406	1.362	1.322	22.071	22.5	21.491	21.5
20.132	20.00	19.561	19.50	2.170	2.090	2.003	1.975	20.548	21.0	19.623	19.7
"	"	"	"	1.414	1.406	1.124	1.100	22.611	23.4	20.923	20.5
				2.170	2.090	1.653	1.500	21.393	21.4	18.700	18.5
"	"	"	"	1.414	1.406	0.668	0.65	23.400	23.8	19.670	20.0
				2.170	2.090	0.983	0.99	22.620	22.5	16.570	16.5

Note : suffix "exp." means experimental results

Photo. 1. Experiment in the case of no-cut-off wall
($h_0=25$ cm, $h_3=18$ cm)

Photo. 3. Experimental free surface by cut-off wall
($h_0=25$ cm, $h_3=18$ cm in case of Exp.-2)

Photo. 2. Experimental free surface by cut-off wall
($h_0=25$ cm, $h_3=13.5$ cm in case of Exp.-1)

Photo. 4. Experimental free surface by cut-off wall
($h_0=25$ cm, $h_3=18$ cm in case of Exp.-3)

Fig. 10. Comparison with theoretical results and experiments for unsteady flow having the cut-off wall ($h_0=25$ cm, $h_3=18$ cm of Exp.-3)

Photo. 2 shows the case with the cut-off wall on the conditions to $h_0=25$ cm, $h_3=13.5$ cm, $q=1.975$ cm²/s (2.003 cm²/s by theory), and the permeability of cut-off wall $k=0.539$ cm/s. Photo. 3 shows the case of $h_0=25$ cm, $h_3=18$ cm, $q=1.100$ cm²/s (1.124 cm²/s by theory) and $k=0.153$ cm/s. Photo. 4 shows the case of $h_0=25$ cm, $h_3=18$ cm, $q=0.650$ cm²/s, and $k=0.00958$ cm/s. Photos. 2, 3 and 4 show that the free surface is approximately linear in its shape for both upstream and downstream aquifers. All the experimental results are summarized as shown in Table 1, compared with theoretical results calculated from Eqs. (4), (5) for h_{10} and h_{20} , Eq. (3) for q_0 , Eq. (8), and Eq. (12). It can be concluded that the theoretical results agree with experimental ones, and the theories developed above may be valid for steady state flow with the cut-off wall.

For unsteady flow the change of heads h_1 and h_2 with time can be compared with theoretical ones by Eq. (24) for the values of $k_0=0.958$ cm/s, $k=0.00958$ cm/s, $\lambda_0=1.0$, $l_1=55.425$ cm, $l_2=1.150$ cm, $l_3=45.425$ cm, $h_0=25$ cm, $h_3=18$ cm, $h_{10}=21.48$ cm and $h_{20}=21.40$ cm, as shown in Fig. 10. Good agreement between the results of experiments and those obtained theoretically can be recognized by Fig. 10, while a little discrepancy exists and the validity of the theory can be confirmed. Accordingly, it can be concluded that an approximate treatment of unsteady flow, proposed above, by means of the method of

successive steady states will be useful for all practical purposes unless the water level difference h_0-h_3 at aquifer boundaries is not so large.

CONCLUSION

Theoretical approaches to analyze the hydraulic effect of the cut-off wall on ground water movement were proposed according to Dupuit's uniform flow assumption and the method of successive steady states with respect to unsteady motion for horizontally unconfined and confined layers. Theoretical results were verified experimentally by using Hele-Shaw apparatus. In this paper it can be concluded that non-dimensional flow rate q/q_0 is expressed by Eq. (10), and non-dimensional piezometric heads h_1/h_{10} , h_2/h_{20} at upstream and downstream sides of cut-off wall are given by Eq. (14) for unconfined flow. At the same time, $\tilde{h}_1/\tilde{h}_{10}$, $\tilde{h}_2/\tilde{h}_{20}$ for confined flow are given by Eq. (15). On the other hand, non-dimensional heads $H_1(=h_1/h_0)$, $H_2(=h_2/h_0)$ of unconfined flow are obtained by Eq. (24) for unsteady flow, and also $\tilde{H}_1(=\tilde{h}_1/h_0)$, $\tilde{H}_2(=\tilde{h}_2/h_0)$ of confined flow are by Eq. (25). Furthermore, non-dimensional flow rates in the upstream and downstream aquifers with cut-off wall are summarized by Eqs. (26) and (27) in conclusion. In addition, it becomes important to discuss the storage capacity of water resources by means of underground wall, which is called the underground dam, on the basis of Eqs. (28), (29), (30) and (31).

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