BEARING CAPACITY OF STRIP FOOTING ON c - φ SOILS EXHIBITING ANISOTROPY AND NONHOMOGENEITY IN COHESION

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ABSTRACT

Limit analysis (upper bound) has been applied for bearing capacity of strip footing on c - φ soils exhibiting anisotropy and nonhomogeneity in cohesion. The failure mechanism assumed is similar to Prandtl-Terzaghi type, but with varying boundary wedge angles. The results are presented in the form of charts in nondimensional form for the various parameters considered, viz., angle of internal friction, depth to width ratio of foundation, degree of nonhomogeneity, coefficient of anisotropy and unit weight of soil. The results are compared with those of other methods. The study reveals that anisotropy and nonhomogeneity have considerable influence on the bearing capacity.

Key words : anisotropy, bearing capacity, plasticity, shallow foundation (IGC : E 3)

INTRODUCTION

The ultimate bearing capacity of strip footing on homogeneous and isotropic soils has been extensively studied by several investigators. However, it is recognized that natural soil deposits are nonhomogeneous and anisotropic with respect to shear strength (Bishop, 1966; Casagrande and Carillo, 1944; Livneh and Komornik, 1967; Lo, 1965). Considerable work has been done with regard to influence of anisotropy and nonhomogeneity on bearing capacity of clays (Davis and Christian, 1971; Davis and Brooker, 1973; Livneh and Greenstein, 1973; Raymond, 1967; Reddy and Srinivasan, 1967, and 1971; Salencon, 1974 a and 1974 b). On the other

hand, very few attempts have been made for studying the effect of anisotropy and nonhomogeneity on the bearing capacity of c - φ soils. Reddy and Srinivasan (1970) using the method of characteristics obtained the bearing capacity of c - φ soils exhibiting nonhomogeneity in cohesion. Meyerhof (1978) has obtained the bearing capacity for soils exhibiting anisotropy in friction by the conventional Terzaghi's type approach using two extreme values of φ for the outer zones and an equivalent φ for the radial shear zone. Shklarsky and Livneh (1961 a) found from tests made on asphalt paving mixtures that the ratio of strengths with load parallel and perpendicular to the direction of compaction was greater than one. It was shown by them

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Fig. 2. Velocity diagram

continuity, V_1 and V_r are given by $V_0 \cos \varphi / \cos (\xi - \varphi)$ and $V_0 \sin \xi / \cos (\xi - \varphi)$, respectively (Fig. 2). Rate of external work done by the foundation load

$$= \frac{1}{2} \frac{q \cdot B \cdot V_0 \cos \varphi}{\cos (\xi - \varphi)} \quad (4)$$

Rate of energy dissipation along bc

$$= \text{Average } c_i \text{ along } bc \cdot \text{length } bc \cdot V_r \cos \varphi \quad (5)$$

The average c_i along bc taken as the average of c_i at b and c_i at c

$$\text{At } b, \quad c_i = [c_{Hs} + \beta D][1 + (k-1)\sin^2 i] \quad (6)$$

$$\begin{aligned} \text{At } c, \quad c_i &= \left[c_{Hs} + \beta D + \frac{1}{2} \beta B \tan \xi \right] \\ &\times [1 + (k-1)\sin^2 i] \end{aligned} \quad (7)$$

where

$$i = \xi + \mu \text{ and } \mu = 45^\circ - \varphi/2. \quad (8)$$

Substituting for average c_i , V_r and length bc , Eq. (5) reduces to

$$\begin{aligned} &\left[c_{Hs} + \beta D + \frac{\beta B}{4} \tan \xi \right] [1 + (k-1)\sin^2 (\xi + \mu)] \\ &\times \frac{\tan \xi \cos \varphi B V_0}{\cos (\xi - \varphi) 2} \end{aligned} \quad (9)$$

Rate of energy dissipation along log spiral cd

$$= \int_0^{\theta_1} c_i r d\theta V \quad (10)$$

c_i at angle θ on log spiral is given by

$$\begin{aligned} c_i &= [c_{Hs} + \beta D + \beta r \sin(\theta + \xi)] \\ &\times [1 + (k-1)\sin^2 i] \end{aligned} \quad (11)$$

where

$$\begin{aligned} r &= r_0 e^{\theta \tan \varphi}; \quad V = V_0 e^{\theta \tan \varphi}; \\ i &= (90 + \varphi + \mu - \xi) - \theta = x - \theta; \text{ and} \\ \theta_1 &= \pi + \beta_1 - \xi - \eta \end{aligned} \quad (12)$$

Substituting these values in Eq. (10), it reduces to

$$\begin{aligned} &\int_0^{\theta_1} \left[c_{Hs} + \beta D + \frac{1}{2} \frac{\beta B}{4 \cos \xi} e^{\theta \tan \varphi} \sin(\theta + \xi) \right] \\ &\times [1 + (k-1)\sin^2 (x - \theta)] \end{aligned}$$

$$\times \left[\frac{B V_0 e^{2\theta \tan \varphi}}{2 \cos \xi} d\theta \right] \quad (13)$$

Eq. (13) on further simplification reduces to

$$\begin{aligned} &\frac{B V_0}{2 \cos \xi} [c_{Hs} + \beta D][I_1 + (k-1)I_2] \\ &+ \frac{\beta B^2 V_0}{4 \cos^2 \xi} [I_3 + (k-1)I_4] \end{aligned} \quad (14)$$

where I_1 , I_2 , I_3 and I_4 are integrals given by

$$I_1 = \int_0^{\theta_1} e^{2\theta \tan \varphi} d\theta, \quad (15)$$

$$I_2 = \int_0^{\theta_1} e^{2\theta \tan \varphi} \sin^2 (x - \theta) d\theta, \quad (16)$$

$$I_3 = \int_0^{\theta_1} e^{3\theta \tan \varphi} \sin(\theta + \xi) d\theta, \text{ and} \quad (17)$$

$$I_4 = \int_0^{\theta_1} e^{3\theta \tan \varphi} \sin^2 (x - \theta) \sin(\theta + \xi) d\theta \quad (18)$$

Rate of energy dissipation in the radial shear zone bcd

$$\begin{aligned} &= \int_0^{\theta_1} \text{Average } c_{iav} \text{ along any radial} \\ &\text{line} \cdot r d\theta \cdot V \end{aligned} \quad (19)$$

c_{iav} , the average c_i along any radial line is taken as the average of c_i at b given by Eq. (6) and c_i given by Eq. (11), but with $i = \theta + \xi + \mu$. Substituting for average c_i and other quantities of Eq. (19), it reduces to

$$\begin{aligned} &\int_0^{\theta_1} \left[c_{Hs} + \beta D + \frac{1}{\varphi} \frac{\beta B}{4 \cos \xi} e^{\theta \tan \varphi} \sin(\theta + \xi) \right] \\ &\times [1 + (k-1)\sin^2 (\theta + \xi + \mu)] \\ &\times \frac{B V_0}{2 \cos \xi} e^{2\theta \tan \varphi} d\theta \end{aligned} \quad (20)$$

Eq. (20) on further simplification reduces to

$$\begin{aligned} &\frac{B V_0}{2 \cos \xi} [c_{Hs} + \beta D][II_1 + (k-1)II_2] \\ &+ \frac{\beta B V_0}{8 \cos^2 \xi} [II_3 + (k-1)II_4] \end{aligned} \quad (21)$$

where II_1 , II_2 , II_3 and II_4 are integrals given by

$$II_1 = \int_0^{\theta_1} e^{2\theta \tan \varphi} d\theta \quad (22)$$

$$II_2 = \int_0^{\theta_1} e^{2\theta \tan \varphi} \sin^2 (\theta + \xi + \mu) d\theta \quad (23)$$

$$II_3 = \int_0^{\theta_1} e^{3\theta \tan \varphi} \sin(\theta + \xi + \mu) d\theta, \text{ and} \quad (24)$$

$$II_4 = \int_0^{\theta_1} e^{3\theta \tan \varphi} \sin^2(\theta + \xi + \mu) \sin(\theta + \xi) d\theta \quad (25)$$

Rate of energy dissipation along de
 = Average c_i along de · length de · $V_3 \cos \varphi$ (26)

At e , $c_i = c_{Hs} [1 + (k-1) \sin^2 i]$ (27)

At d , $c_i = [c_{Hs} + \beta de \sin \beta_2]$
 $\times [1 + (k-1) \sin^2 i]$ (28)

where $i = \beta_2 - \mu$, $V_3 = V_0 e^{\theta_1 \tan \varphi}$, β_2 is the angle made by the line de with horizontal and length de is given by geometry as

$$= \frac{Be^{\theta_1 \tan \varphi} \sin \eta}{2 \cos \xi \cos(\varphi + \eta)} \quad (29)$$

Substituting these quantities in Eq. (26), it reduces to

$$\left[c_{Hs} + \frac{\beta B}{4 \cos \xi} \frac{e^{\theta_1 \tan \varphi} \sin \eta \sin \beta_2}{\cos(\varphi + \eta)} \right] \times [1 + (k-1) \sin^2(\beta_2 - \mu)] \times \left[\frac{BV_0}{2 \cos \xi} e^{2\theta_1 \tan \varphi} \frac{\sin \eta \cos \varphi}{\cos(\varphi + \eta)} \right] \quad (30)$$

Rate of work done by the soil weight in triangle abc and taking only one half

$$= \frac{1}{8} \frac{\gamma B^2 V_0 \sin \xi \cos \varphi}{\cos \xi \cos(\xi - \varphi)} \quad (31)$$

where γ = unit weight of soil.

Rate of work done by the weight of soil in portion bcd

$$= \frac{1}{2} \gamma \int_0^{\theta_1} r^2 V_0 \cos(\theta + \xi) d\theta \quad (32)$$

Eq. (32) on simplification reduces to

$$= -\frac{1}{8} \frac{\gamma B^2 V_0}{\cos^2 \xi (1 + 9 \tan^2 \varphi)} \{ 3 \tan \varphi \cos \xi + \sin \xi + [3 \tan \varphi \cos(\beta_1 - \eta) + \sin(\beta_1 - \eta)] \times e^{3(\pi + \beta_1 - \xi - \eta) \tan \varphi} \} \quad (33)$$

Rate of work done by the weight of soil in

portion $bdef$ is given by

$$= -\frac{1}{8} \frac{\gamma V_0 B^2}{\cos^2 \xi} \left[\frac{\sin \eta \cos \varphi}{\cos(\varphi + \eta)} + \frac{\sin \beta_1 \cos \beta_1 \cos^2 \varphi}{\cos^2(\varphi + \eta)} \right] \times \cos(\beta_1 - \eta) e^{3[\pi + \beta_1 - \xi - \eta] \tan \varphi} \quad (34)$$

(Eqs. (33) and (34) are same as given by Chen (1975)). Equating the rate of external work done (Eq. (4)) including the rate of work done by the weight of the soil (Eqs. (31), (33), and (34)) to the rate of energy dissipation (Eqs. (9), (14), (21) and (30)), and introducing the nondimensional parameters, $d' = d/B$, $\nu = \beta B/c_{Hs}$, $q' = q/c_{Vs}$, and $G = \gamma B/c_{Hs}$, the expression for q' is obtained as

$$q' = N_c + G N_{rq} \quad (35)$$

where N_c and N_{rq} are functions of ξ and η for given values of parameters, d' , ν , k , G and μ . For the minimum value of q' , ξ and η should satisfy

$$\frac{\partial q'}{\partial \xi} = 0 \text{ and } \frac{\partial q'}{\partial \eta} = 0. \quad (36)$$

RESULTS AND DISCUSSION

Values of q' have been obtained for $\varphi = 10, 20, 30$, and 40 degrees, $D/B = 0.0$, and 1.0 , $G = 0.0$ and 2.0 , $k = 0.8, 1.0, 1.2, 1.6$ and 2.0 , and $\nu = 0.0, 0.40, 0.80$ and 1.2 .

Table 1 compares the values of N_c for isotropic and homogeneous soils, obtained by present analysis with those of others. As could be seen from table, the values of N_c obtained by the present method are practically same as those obtained by Sokolovsky, Salencon and Chen.

Table 2 compares the ratios of N_c non-homogeneous to N_c homogeneous, obtained by the present analysis and those of Salencon (1974b). The ratios obtained by the present analysis are slightly higher than those of

Table 1. Comparison of values of N_c for $G=0.0$, $k=1$ and $\nu=0.0$

φ , in degrees (1)	Present analysis (2)	Sokolovsky ⁺ (3)	Salencon ⁺⁺ (4)	Chen ⁺⁺⁺ (5)	Reddy and Srinivasan ⁺ (6)	Terzaghi ⁺ (7)	Meyerhof ⁺ (8)
10	8.34	8.34	8.35	8.34	9.30	9.30	8.00
20	14.83	14.80	14.84	14.80	16.00	17.00	14.50
30	30.14	30.10	30.15	30.10	32.00	37.00	31.00
40	75.32	75.30	75.34	75.31	82.00	97.50	73.00

⁺ From Reddy and Srinivasan (1970)

⁺⁺ From Salencon (1974b)

⁺⁺⁺ From Chen (1975)

Fig. 3. Comparison of N_c values for $G=0.0$ and $\nu=0.0$

Fig. 4. Comparison of N_c values for $G=0.0$ and $\nu=0.4$

Fig. 5. Comparison of N_c values for $G=0.0$ and $\nu=1.2$

Table 2. Comparison of ratios of N_c nonhomogeneous to N_c homogeneous for $k=1$, $G=0.0$ and $\nu=1.2$

φ , in degrees (1)	Present analysis (2)	Salencon (1974b) (3)
10	1.527	1.45
20	1.699	1.60
30	1.965	1.85
40	3.437	3.25

Salencon. Values of present analysis are 5.3 percent higher for $\varphi=10$ degrees and 8.3 percent higher for $\varphi=40$ degrees.

Fig. 3, 4 and 5 compare the values of N_c obtained for $G=0.0$, by the present analysis with those of Reddy and Srinivasan (1970). It can be seen from these figures, the values of N_c by the present analysis are slightly lower for $\nu=0.0$ and are slightly higher for $\nu \neq 0$ cases. Figs. 6 to 9 compare the values of q' obtained by the present analysis and those obtained by Reddy and Srinivasan (1970). It can be seen from these figures, that the

present analysis gives values higher than those obtained by Reddy and Srinivasan (1970). The difference in the values of q' obtained by the two methods, increases with increase in φ . Figs. 10 to 25 show the q' variation with ν for various parameters considered. These figures reveal that the variation of q' with ν is almost linear for the range of parameters considered. q' increases with ν , and decreases with increase in k . Table 3 shows the percentage change in q' with respect to isotropic and homogeneous values for the extreme values of parameters considered. As could be seen from the table, the change in q' when compared to its values for $k=1$ and $\nu=0$ varies from -48 percent to +305 percent.

CONCLUSION

For the range of parameters considered, numerical values have been presented in the

Fig. 6. Comparison of q' values for $\varphi=10$ degrees and $G/k=4$

Fig. 7. Comparison of q' values for $\varphi=20$ degrees and $G/k=4$

Fig. 8. Comparison of q' values for $\varphi=30$ degrees and $G/k=4$

Fig. 9. Comparison of q' values for $\varphi=40$ degrees and $G/k=4$

Fig. 10. Values of q' for $\phi=10$ degrees, $D/B=0.0$ and $G=0.0$

Fig. 11. Values of q' for $\phi=10$ degrees, $D/B=0.0$ and $G=2.0$

Fig. 12. Values of q' for $\phi=10$ degrees, $D/B=1.0$ and $G=0.0$

Fig. 13. Values of q' for $\phi=10$ degrees, $D/B=1.0$ and $G=2.0$

Fig. 14. Values of q' for $\varphi=20$ degrees, $D/B=0.0$ and $G=0.0$

Fig. 15. Values of q' for $\varphi=20$ degrees, $D/B=0.0$ and $G=2.0$

Fig. 16. Values of q' for $\varphi=20$ degrees, $D/B=1.0$ and $G=0.0$

Fig. 17. Values of q' for $\varphi=20$ degrees, $D/B=1.0$ and $G=2.0$

Fig. 18. Values of q' for $\varphi=30$ degrees, $D/B=0.0$ and $G=0.0$

Fig. 19. Values of q' for $\varphi=30$ degrees, $D/B=0.0$ and $G=2.0$

Fig. 20. Values of q' for $\varphi=30$ degrees, $D/B=1.0$ and $G=0.0$

Fig. 21. Values of q' for $\varphi=30$ degrees, $D/B=1.0$ and $G=2.0$

Fig. 22. Values of q' for $\phi=40$ degrees, $D/B=0.0$ and $G=0.0$

Fig. 23. Values of q' for $\phi=40$ degrees, $D/B=0.0$ and $G=2.0$

Fig. 24. Values of q' for $\phi=40$ degrees, $D/B=1.0$ and $G=0.0$

Fig. 25. Values of q' for $\phi=40$ degrees, $D/B=1.0$ and $G=2.0$

Table 3. Percentage change in values of q' with respect to homogeneous and isotropic values

φ , in degrees (1)	d' (2)	G (3)	$k=0.8$		$k=2.0$		$k=1.0$
			$\nu=0.0$ (4)	$\nu=1.2$ (5)	$\nu=0.0$ (6)	$\nu=1.2$ (7)	$\nu=1.2$ (8)
10	0.0	0.0	+14.33	+74.91	-28.72	+8.00	+52.69
		2.0	+15.83	+65.89	-31.71	-8.44	+43.66
	1.0	0.0	+15.81	+197.00	-32.33	+74.92	+156.80
		2.0	+19.33	+123.50	-39.50	+22.32	+90.10
20	0.0	0.0	+16.41	+98.02	-32.82	+13.69	+69.91
		2.0	+18.93	+73.54	-37.89	-6.52	+46.86
	1.0	0.0	+18.08	+183.50	-36.30	+73.60	+169.10
		2.0	+21.91	+110.20	-43.46	+5.25	+75.22
30	0.0	0.0	+18.41	+132.60	-36.92	+24.15	+96.47
		2.0	+23.52	+78.60	-43.09	-12.43	+48.25
	1.0	0.0	+19.81	+248.40	-39.69	+79.13	+192.00
		2.0	+23.27	+99.06	-46.54	-6.99	+63.72
40	0.0	0.0	+20.11	+191.08	-40.58	+46.94	+143.68
		2.0	+23.33	+79.08	-46.73	-18.22	+46.67
	1.0	0.0	+21.05	+305.20	-42.37	+98.56	+235.50
		2.0	+24.08	+88.24	-47.18	-16.29	+53.43

form of charts in nondimensional form. The results show that anisotropy and nonhomogeneity have considerable influence on bearing capacity.

NOTATION

- B = width of foundation
 c = cohesion parameter of soil
 c_H, c_V = cohesions corresponding to horizontal and vertical directions, respectively
 c_{Hs}, c_{Vs} = cohesions at surface corresponding to horizontal and vertical directions, respectively
 c_i = cohesion for an inclination i of major principal stress with horizontal direction
 c_{lav} = average cohesion along a line
 D = depth of foundation
 $d' = d/B$ = nondimensional depth factor
 h = depth of a point below the surface
 k = coefficient of anisotropy
 q = bearing capacity
 $q' = q/c_{Vs}$ = nondimensional bearing capacity
 β = variation of cohesion with depth
 β_1 = angle made by the line be with horizontal (Fig. 1)
 β_2 = angle made by the line de with horizontal (Fig. 1)
 γ = unit weight of soil
 ξ, η = boundary wedge angles that define failure mechanism (Fig. 1)
 θ_1 = angle cbd (Fig. 1)
 $\nu = \beta B/c_{Hs}$ = nondimensional parameter expressing the variation of cohesion with depth
 φ = angle of internal friction of soil
 $\mu = 45 - \varphi/2$

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