AN APPLICATION OF MOGAMI'S STRENGTH FORMULA TO THE CLASSIFICATION OF GRANULAR SOILS

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ABSTRACT

A plasticity chart has been utilized for the material classification of cohesive soils. However, such a chart has not been used for cohesionless soils. Therefore, the author presents a new classification diagram of $k-e_{\min}$ for granular soils such as rockfill materials and sands. This diagram can be drawn provided the minimum void ratio $e_{\min}$ and Mogami's strength coefficient k obtained from the limit density test and the shear tests, respectively, are known. The equi-internal friction angle lines are inserted on the $k-e_{\min}$ diagram. On this diagram, the engineering materials take their positions according to the particle characters such as grading and angularity of particles. At a higher confining pressure level, there can be observed the tendency for the coefficient k of different soils to take on similar values irrespectively of the particle characteristics of the granular materials. This finding leads to the basic fact that the compaction and compactibility of material are very important and thus well graded and rounded materials are more suitable for use in high dams.

Key words : angle of internal friction, coarse-grained soil, compaction, density, drained shear, earthfill, gravel, granular material, particle breakage, rockfill, sand, sandy soil, shear strength, soil classification, triaxial compression test, void ratio (IGC : D 6)

INTRODUCTION

A plasticity chart has been utilized for the material classification of cohesive soils. However, such a chart has not been used for cohesionless soils. Leslie (1969) emphasized the importance of establishing a classification system to correlate engineering properties at a speciality session entitled "Mechanical Properties of Rockfill and Gravel Materials" of the International Conference on Soil Mechanics and Foundation Engineering held in Mexico. Therefore, the author presented a new classification diagram ($k-e_{\min}$ diagram) for granular soils such as gravels and sands. This diagram can be drawn provided the minimum void ratio, $e_{\min}$ and Mogami's strength coefficient, k obtained from the limit

density test and the shear tests, respectively, are known (Moroto, 1975, 1976).

k and $e_{\min}$

The behaviour of soils has been studied with respect to shear and consolidation. The limiting state in the consolidation of granular soils is defined as the minimum void ratio, $e_{\min}$. On the other hand, the limiting state of soil in the shearing condition is represented by the internal friction angle, ϕ . These two state variables can be interrelated through a strength-void ratio equation.

There are various kinds of strength formula for granular soils. Among these, Caquot's and Mogami's formulae are often referred to where,

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$$e \tan \phi = \text{constant} \quad (\text{Caquot's}) \quad (1)$$

$$\sin \phi = \frac{k}{1+e} \quad (\text{Mogami's, Mogami (1965)}) \quad (2)$$

e : void ratio

k : strength coefficient of Mogami

The author employs Mogami's equation in this paper since (1) it has the clear physical meaning that shearing strength is proportional to density and (2) the coefficient k plays an important role in Mogami's theory. In the densest state, Eq. (2) can be written

$$\sin \phi_M = \frac{k}{1+e_{\min}} \quad (3)$$

ϕ_M : ϕ at the maximum density

$e_{\min}$: void ratio in the densest state

The value of $e_{\min}$ can be determined by the limit density test. As is well known, $e_{\min}$ increases with increasing angularity and decreases as the uniformity coefficient, U_c increases. The strength coefficient, k can be obtained by the shear test of the granular soil sample with the initial void ratio e_0 as,

$$k = (1+e_0) \sin \phi \quad (5)$$

When the values of ϕ are obtained on a number of granular soil samples, the average value of k can be calculated as,

$$\bar{k} = \frac{1}{n} \sum_{i=1}^n (1+e_{0i}) \sin \phi_i \quad (6)$$

where, the subscript i shows the i -th numbered data.

Based on the experimentally obtained data by shear tests (mainly the triaxial compression tests) on sands, gravels and volcanic ash sands termed 'Shirasu', the author has made it clear that

Fig. 1. Relationship between k and e_0 (data from Nash (1953), tri-axial compression)

Fig. 2. Relationship between k and e_0 (data from Asao reported by Mogami and Yoshikoshi (1968), direct shear)

Fig. 3. Relationship between k and e_0 (data from Haruyama (1972), triaxial compression)

Fig. 4. Relationship between k and e_0 (data from Penman (1953), triaxial compression)

(1) For granular soil samples of a given material with different initial densities, the constancy of k is good as shown in Figs. 1~4

(2) The value of the coefficient, k changed considerably depending on particle characters, ranging from $k=0.7 \sim 2.0$,

where $k=0.9 \sim 1.1$ (sands)

$k=0.7 \sim 1.3$ (gravels)

Fig. 5. Relationship between $\bar{k}$ and $e_{\min}$ (data used for this plotting are listed in Table 1, triaxial compression)

Fig. 6. Relationship between $\bar{k}$ and $e_{\min}$ (SAND from Fukumoto (1969), GRAVEL from Leslie and GLASS BEADS from author's, triaxial compression)

$k=1.0 \sim 2.0$ (volcanic ash sands)

(3) Roughly speaking, From Fig. 5 a linear relationship can be seen between $\bar{k}$ and $e_{\min}$. The value of $\bar{k}$ increases with an increase in $e_{\min}$.

(4) After a more detailed investigation by

Angularity by Lees (A)

material	A	material	A
Crashed Stone	2500	Tuff (1.3-1.4)	1800-2400
Angular Sand (Shiroyama sand)		Beach Gravel (1.3-1.4)	200-280
Beach Gravel (3.3-7.3)	200-280	River Gravel	800-1500
Tuff (53)	1800-2400	Tuff (19)	1700-2100
Tuff (14)	1700-2100	Crystalline Shist	3300
Weathered Sand Stone	1600-2100		

Fig. 7. Relationship between $\bar{k}$ and $e_{\min}$ (data from Murao & Mishina (1971), tri-axial compression)

using Fig. 6 where so the mean particle size rises, the angularity decreases, the following characteristics were found. When a series of shear tests were carried out on a material having similar grading and varying angularity, it was found that the $k-e_{\min}$ relationship could be represented as

$$k = a_1 e_{\min} + b_1 \quad a_1 \text{ and } b_1 : \text{constants} \quad (7)$$

But, based on shear tests on a material with similar angularity and various grading, the following formula was established:

$$k = a_2 e_{\min} + b_2 \quad a_2 \text{ and } b_2 : \text{constants} \quad (8)$$

Here, generally, the constant a_1 is larger than a_2 (Moroto, 1976). These constants may change their values for different materials. Fig. 7 shows a similar characteristics to Fig. 6.

CHARACTERISTICS OF $k-e_{\min}$ DIAGRAM

From an engineering point of view, the value of the internal friction angle, ϕ of granular soil take on a greater importance than the value of k for design purposes. This explains the insert 'equi- ϕ_M lines' on the diagram of $k-e_{\min}$ as shown in Figs. 6, 7. These lines are straight.

The engineering granular materials take every possible position according to the particle characteristics. For materials with similar uniformity coefficient, an angular material is located to the upper right of the rounded one (for example, line-a in Fig. 6, compare the beach gravel with the crashed stone in Fig. 7). Rounded material is plotted on a lower spot (for example, line-a in Fig. 6, glass beads in Fig. 6). This indicates less quality of shear strength, because the material is located on a smaller $\text{equi-}\phi_M$ line.

Angular material with well graded takes a position on the left upper side of rounded material with uniform grading (Fig. 7). This material shows excellent shear strength, because it is located on higher $\text{equi-}\phi$ line. Volcanic ash sand (Shirasu) with high angularity is located on an upper right spot irrespective of its grading (Fig. 5, Fig. 8).

From the characteristics of Eq. (3) and

Fig. 8. Relationship between $\bar{k}$ and $e_{\min}$ (data from Haruyama (1972) and Fujimoto (1966), in triaxial compression)

Fig. 9. Relationship between $\bar{k}$ and uniformity coefficient U_c (data of Fig. 8)

Fig. 10. $\bar{k}-e_{\min}$ diagram

$k-e_{\min}$ diagram, it can be said that if there are two materials of similar value of $e_{\min}$, the one has a higher value of k shows a greater internal friction angle ϕ_M ; if there are two materials of similar value of k , the one has a less value of $e_{\min}$ shows a greater internal friction angle ϕ_M ; if there are two materials of different value of $e_{\min}$, the one located on a higher $\text{equi-}\phi_M$ line shows a greater value of internal friction angle, ϕ_M .

Terzaghi (1925) defined compactability as

$$F = \frac{e_{\max} - e_{\min}}{e_{\min}} \quad (9)$$

In well graded cohesionless soils, $e_{\max} - e_{\min}$ is large and $e_{\min}$ is small, hence F is large. These soils are easily compacted. In uniform granular soil, $e_{\max} - e_{\min}$ is small and $e_{\min}$ is large, hence F is small and the soil is more difficult to compact (Hilf, 1975). Therefore, materials having smaller values of $e_{\min}$ show higher compactability.

These things said above lead to a promising suggestion that $k-e_{\min}$ diagram would be used for material classification of granular soils.

Based on the data available to the author, Fig. 10 was prepared as a rough standard. This figure helps to provide a quantitative

understanding of the data. The $\bigcirc$ mark shows the approximate position of material. Up to now, the shearing strength under normal confining pressure range (not higher than about 7 kg/cm^2 ($\div 700 \text{ kN/m}^2$)) has been treated.

CHARACTERISTICS OF $k-e_{\min}$ IN HIGH CONFINING PRESSURE RANGES

When a granular soil sample is sheared under high confining pressure, the value of k becomes smaller than when a normal range of confining pressure. This decrease in the shearing strength is due to particle breakdown in the clear and becomes a serious problem for materials in high fill dams, and in other cases of granular material under high pressure. Features of $k-e_{\min}$ relationships changed by the elevation of the confining pressure are shown in Figs. 11~14. Table 2, 3 list up the data Figs. 11, 12, Figs. 13, 14, respectively. In these figures, where the confining pressures are relatively low, the plotted points show a tendency that the relationship rises as the value of $e_{\min}$ increases. However, when the confining pressure increases, the gradient of the relationship

Fig. 11. Relationship between k and $e_{\min}$ (data from Marsal (1973), triaxial compression on rockfill materials)

Fig. 12. Relationship between k and $e_{\min}$ (data from Marsal (1973), triaxial compression on rockfill materials)

Fig. 13. Relationship between k and $e_{\min}$ (data from Marsal (1971), triaxial compression on rockfill materials)

Fig. 14. Relationship between k and $e_{\min}$ (data from Marsal (1971), triaxial compression on rockfill materials)

gradually decreases and finally drops and becomes parallel to the $e_{\min}$ axis. At a higher confining pressure level, there can be observed the tendency for the coefficient k of different soils to take on similar values irrespectively of the particle characteristics of granular soils. It can be seen from Eq. (2) that the material with a smaller initial void ratio (a higher density)

has a greater shearing strength at high confining pressure levels. This finding leads to the basic fact that compaction of cohesionless materials is very important for earth work in the field. The granular soil consisting of well graded and well rounded materials has superior engineering properties since these materials can be most densely compacted. This important conclusion is consistent with

 Table 1. The value of $\bar{k}$

Material	Specimen dia. (cm)	$e_{\min}$	$\bar{k}$	σ_3 (kg/sq. cm)
Eldin & Bishop (1935), 3rd ICSMFE, Vol.1, p.100				
Brasted sand-saturated	3.81	0.497	1.04	0.37-2.1
Nash (1953), 3rd ICSMFE, Vol.1, p.160				
a river sand-saturated	—	0.613	1.02	0.35-5.25
a river sand-dry	—	0.613	1.02	0.35-5.25
Wu (1953), ASCE, SM Div. paper 1161				
a sand-saturated	3.57	0.57	1.099	1.4, 2.8
a sand-saturated	3.57	0.445	1.017	1.4, 2.8
a sand-saturated	3.57	0.38	0.965	1.4, 2.8
Bjerrum et al (1961), 5th ICSMFE, Vol.1, p.29				
Valgrinda sand-saturated	3.57	0.644	1.03	
de Beer (1965), 6th ICSMFE, Vol.1, p.165				
G-12	—	0.493	1.036	0.51, 5.12
Takaoka & Moriya (1966), JSSM, Tsuchi to Kiso, Nov., p.5				
Shiroyama sand-dry	7.0	0.681	1.167	0.5, 1.0, 2.0, 3.0
Shiroyama sand-dry	7.0	0.694	1.186	0.5, 1.0, 2.0, 3.0
Shiroyama sand-dry	7.0	0.719	1.201	0.5, 1.0, 2.0, 3.0
Shiroyama sand-dry	7.0	0.753	1.203	0.3, 1.0, 2.0, 3.0
Lee & Seed (1967), ASCE, SM 6, p.117				
Sacramento River sand-saturated	3.57	0.61	1.05	1.0
Ottawa sand-saturated	3.57	0.49	0.92	1.0
Conforth, ASTM, STP 523, p.281				
Guinea sand-saturated	10.2	0.518	1.02	1.41, 2.81, 4.22
Portland sand-saturated	10.2	0.633	1.20	1.76
Holbec & D'Appolonia, ASTM, STP 523, p.304				
Ottawa sand-saturated	5.08	0.482	0.90	—
Southport sand-saturated	5.08	0.530	1.06	—
Olivine sand-saturated	5.08	0.628	1.11	—
Ruishenko (1968), 3rd Budapest Conf., p.275				
Gravel-saturated	30	0.56	1.05	1-3
Gravel-saturated	30	0.26	0.94	1-3
Gravelly sand-saturated	30	0.30	0.93	1-3
Gravel-saturated	30	0.31	0.97	0.8-3
Boughton (1970), ASCE, SM 5, p.1715				
Quartzite (HE-1)-dry	—	0.23	1.00	1.76
Rhyolite (SMA-2)-saturated	—	0.22	0.97	2.8
Haruyama (1966:1972), JSSM, Tsuchi to Kiso, Aug., 1966:Feb. 1972)				
Toso Shirasu-saturated	5.0	0.800	1.328	—
Shitoura Shirasu-saturated	5.0	0.825	1.328	—
Shitoura Shirasu-saturated	5.0	0.638	1.085	—
Shitoura Shirasu-saturated	5.0	1.243	1.602	—
Syomyozi Shirasu-saturated	5.0	1.585	1.884	—

Table 2. The values of k and $e_{\min}$ of rockfill materials (data from Marsal, 1973)

Material	Symbol	U_c	$e_{\min}$	e_0	σ_3		ϕ in degree	k
					kg/cm ²	kN/m ²		
El Infiernillo silicified conglomerate angular	○	10	0.40	0.61	1.88	184	45.8	1.154
				0.59	4.88	479	41.2	1.047
				0.46	9.98	979	39.5	0.929
				0.57	16.9	1660	36.1	0.925
				0.51	25.0	2450	35.5	0.877
El Infiernillo diorite angular	●	5	0.48	0.60	1.88	184	45.0	1.131
				0.53	4.88	479	39.0	0.963
				0.56	9.98	979	37.0	0.939
				0.51	16.9	1660	35.3	0.873
				0.49	24.9	2440	34.5	0.844
Pinzandarán sand and gravel rounded	×	100	0.29	0.29	1.88	184	48.9	0.972
				0.28	5.60	549	43.0	0.873
				0.29	10.5	1030	41.7	0.858
				0.31	16.9	1660	39.3	0.830
				0.29	24.9	2440	38.9	0.810
Malpaso conglomerate subangular	+	63	0.31	0.43	1.88	184	48.3	1.068
				0.34	4.88	479	45.2	0.951
				0.45	9.98	979	38.6	0.905
				0.37	16.9	1660	39.2	0.866
				0.39	24.5	2400	37.0	0.837
San Francisco basalt, 1 angular	△	11	0.33	0.39	4.72	463	45.6	0.993
				0.33	9.98	979	40.6	0.866
				0.38	24.7	2420	38.0	0.850
San Francisco basalt, 2 angular	▲	18	0.29	0.39	4.71	462	46.2	1.003
				0.33	9.80	961	42.1	0.892
				0.35	24.8	2430	38.3	0.837
Mica granitic-gneiss, 1 subangular	□	14	0.31	0.33	4.70	461	37.2	0.804
				0.36	9.67	949	34.0	0.760
				0.38	16.5	1620	32.7	0.746
				0.36	24.7	2420	32.3	0.726
Mica granitic-gneiss, 2 subangular	■	2.5	0.58	0.66	4.59	450	33.8	0.923
				0.63	9.80	961	33.2	0.893
				0.68	16.9	1660	31.7	0.883
				0.67	24.7	2420	27.7	0.776
Mica granitic-gneiss, 1 +30% schist	▽	19	0.29	0.36	4.52	443	35.4	0.788
				0.34	9.81	962	34.5	0.759
				0.36	16.5	1620	32.5	0.731
				0.34	24.6	2410	32.5	0.720
Mica granitic-gneiss, 2 +30% schist	▼	2.5	0.60	0.64	4.70	461	34.3	0.924
				0.61	9.69	951	29.3	0.788
				0.59	16.5	1620	30.3	0.802
				0.67	24.6	2410	28.8	0.805
El Granero slate, 1 angular	◇	10	0.45	0.55	1.69	166	45.5	1.106
				0.55	4.63	454	40.6	1.009
				0.55	9.84	965	35.0	0.889
				0.53	16.7	1640	36.5	0.910
				0.42	24.7	2420	34.3	0.800
El Granero slate, 2 angular	⊗	4.3	0.64	0.64	1.76	173	45.0	1.160
				0.68	4.64	455	39.0	1.057
				0.63	8.24	808	37.5	0.992
				0.58	15.3	1500	34.5	0.895
				0.59	23.3	2290	32.9	0.864

Table 3. The values of k and $e_{\min}$ of rockfill materials (data from Marsal, 1971)

material	Symbol	U_c	e_0	lateral pressure σ_3		ϕ	k
				kg/cm ²	kN/m ²		
El Granero slate	○	10	0.46	0.90	88	50.7	1.130
			0.50	4.68	459	40.8	0.981
			0.41	24.73	2430	33.9	0.786
angular	●	4.3	0.63	0.90	88	47.3	1.198
			0.64	4.79	470	38.5	1.021
			0.54	23.00	2260	32.7	0.832
Chivor flite	□	7.1	0.59	0.78	77	43.7	1.098
			0.58	5.08	498	38.5	0.984
			0.58	22.08	2170	30.7	0.807
angular	■	2.0	0.74	0.78	77	41.3	1.148
			0.75	5.08	498	33.5	0.966
			0.74	22.08	2170	27.1	0.793
	◼	10.3	0.30	0.78	77	43.7	0.898
			0.32	5.08	498	41.1	0.868
			0.33	22.08	2170	37.3	0.806
La Angostura lime stone	△	7.1	0.36	0.78	77	41.9	0.908
			0.37	5.08	498	36.2	0.809
			0.38	22.08	2170	30.6	0.702
angular	▲	2.0	0.49	0.93	91	32.5	0.801
			0.56	5.08	498	30.0	0.780
			0.64	22.08	2170	26.2	0.724

Fig. 15. Relationship between k and $e_{\min}$ (data from Iwakata (1973), triaxial compression on rockfill materials)

Casagrande's conclusive statement that well graded and well rounded materials are better than the uniformly graded and angular material with respect to mechanical properties and thus are more suitable for use in high dams.

FUTURE STUDY OBJECTIVES

(1) The correlation between the position of material in the $k-e_{\min}$ diagram and mechanical properties of granular soils.

(2) Insert of characteristic lines such as the A- and B-lines on the plasticity chart of the cohesive soil classification.

(3) The need for more data.

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