CYCLIC UNDRAINED STRESS-STRAIN BEHAVIOR
OF DENSE SANDS BY TORSIONAL
SIMPLE SHEAR TEST

FUMIO TATSUOKA*, MASASHIGE MURAMATSU** and TSUTOMU SASAKI***

ABSTRACT

In the first part of the article a review of laboratory cyclic undrained test methods is presented. The features of the cyclic torsional simple shear device which was newly developed to assess the resistance of saturated dense sand against cyclic undrained loading are described. The test results are represented by the relationship between the relative density $D_r$ and the cyclic shear stress amplitude normalized by the effective mean principal stress at consolidation $\tau_{cy}/\sigma'_{ms}$ for which a certain value of double amplitude shear strain is observed at a certain number of loading cycles. Cyclic undrained tests were performed for 1) isotropically and anisotropically consolidated specimens, 2) specimens initially sheared and not sheared during consolidation and 3) specimens prepared by different methods, using two different sands, Toyoura Sand, a clean, uniform, fine sand having no fine particles and Sengenyama Sand, a medium fine sand including fine particles to some extent. For any test condition, specimens denser than some critical densities showed extremely high resistances which were much larger than expected from the concept that the cyclic undrained strength be proportional to relative density. For identical values of relative dinsity, loose Sengenyama Sand specimens had strengths similar to loose Toyoura Sand specimens, while dense Sengenyama Sand specimens had strengths much less than dense Toyoura Sand specimens. Loose specimens prepared by the air pluviation method had strengths similar to ones prepared by the wet tamping method, while air-pluviated dense specimens had strengths much lower than wet tamped dense specimens. Other factors affecting the cyclic undrained strength of sands were also investigated. Effects of any factor investigated on the cyclic undrained strength were very large for dense specimens, but only slight for loose specimens.

Key words: earthquake, liquefaction, pore pressure, repeated load, sandy soil, special shear test, torsion (IJC : D 6/D 7)

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INTRODUCTION

Needs to evaluate precise cyclic undrained stress-strain behavior of saturated dense sands have been increased for seismic designs of critical structures such as nuclear power plants, large storage tanks or high earthfill dams which should be stable even for large earthquake motions. When a design acceleration level is low, it is reasonable to consider that dense sands have enough liquefaction strengths, without having to perform critical evaluations of the liquefaction strengths of dense sands. In the past, some considered that strength of sand against liquefaction was proportional to relative density up to a certain value of relative density, say 80%. For example, a linear relationship have been proposed as

\[
\frac{\sigma_{\text{ref}}}{2 \sigma'_c} = 0.0042 D_r
\]

(1)

in which \( \sigma_{\text{ref}}/2 \sigma'_c \) is the stress ratio in cyclic undrained triaxial tests which induces liquefaction at the number of cycles of 20 and \( D_r \) is the relative density of the specimen (Ishihara, 1977; Tatsuoka et al., 1978). However, it is understood at present that the resistance of denser saturated sand against cyclic undrained loading is higher than evaluated by postulating that the liquefaction strength is proportional to relative density.

Several types of cyclic undrained simple shear tests on sands have been performed in the early stage (Peacock and Seed, 1968; Finn et al., 1971). Recently, Silver et al. (1980) performed a series of cyclic triaxial and NGI-type simple shear tests on Monterey No. 0 Sand specimens reconstituted by air-pluviation and wet tamping methods. Fig. 1 is a reproduction from Silver et al. (1980). It can be clearly seen from this figure that the relationship is not linear between the relative density and the stress ratio which induces a 15 percent shear strain double amplitude at the number of loading cycles of 10. Another important point which can be seen from this figure is that the cyclic undrained triaxial strength of sand prepared by tamping moist soil may give an overestimate of the liquefaction strength in horizon-
DeAlba et al. that the amount of shear strain was limited to a certain value for dense specimens, with limited amount of shear strain being decreased with the increase in density. Since in this type of large simple shear test a specimen is placed on a shaking table, it is rather difficult to keep the cyclic shear stress amplitude constant in the course of liquefaction because of a large variation of the natural frequency of the specimen due to a large reduction in the rigidity of specimen. Furthermore, due to other two features, very expensive to perform and having a large system compliance, this type of test will not be used as a routine laboratory test to assess liquefaction characteristics.

To overcome the disadvantages inevitable in conventional type of simple shear test which has end boundaries, torsional simple shear devices have been developed in which hollow cylindrical specimens are used (Ishihara and Li, 1972; Ishihara and Yasuda, 1975; Yoshiimi and Oh-Oka, 1973; Ishibashi and Sherif, 1974). Since a hollow cylindrical specimen has no end boundaries, the specimen can be considered free from stress concentration problems. Ishihara and Yasuda (1975) showed that the cyclic torsional simple shear strength of a loose sand specimen prepared by pluviating through water is similar to the cyclic triaxial strength when the cyclic shear stress amplitude is normalized based on the effective mean principal stress at consolidation.

Presented herein are the cyclic torsional simple shear test results which were performed to assess the liquefaction characteristics of sands having a wide range of density ($D_s = 35\%$ to near $100\%$). It was found that specimens having densities larger than the critical value which was a function of several factors had considerably larger resistances against liquefaction. It was also found that, for dense specimens, clean, uniform, fine sand tested had a cyclic undrained strength larger than medium fine sand tested which had some fine contents.

**TEST PROGRAM**

Two kinds of sand were selected for this study. The first one is Toyoura sand, a commercially available washed and sieved

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<th>Table 1. Physical properties of sands tested</th>
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* by the method proposed by the Japanese Society of Soil Mechanics and Foundation Engineering (1980).
** The values of $e_{max}$ and $e_{min}$ by the method proposed by Yoshiimi and Toh-no (1972) were as follows.
  $e_{max} = 0.96$
  $e_{min} = 0.64$

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<th>Table 2. Test program</th>
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<td><strong>Sand</strong></td>
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<td><strong>Toyougra Sand</strong></td>
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* AP: Air-Pluviation, WT: Wet-Tamping, WV: Wet-Vibration
beach sand. This is a uniform, subangular, fine sand which has been widely used for liquefaction studies in Japan. The other sand is Sengenyama Sand from deposits of diluvial origin, which has been widely used for hydraulic fill reclamation projects in the Tokyo Bay area. The physical properties of these sands are listed in Table 1.

To prepare specimens, three different specimen preparation methods were adopted (Table 2). The air pluviation method consists of pluviating air dry sand into a mold from a tube keeping the height of fall constant. Densification of the specimen was accomplished by increasing the height of fall. Major part of the tests was performed for the specimens reconstituted by the air pluviation method. The wet tamping method adopted by Ladd (1978) and Silver et al. (1978) is a method of compacting moist coarse grained material in which the material is placed in layers with each layer compacted to a prescribed dry unit weight. The density of each layer is controlled by adjusting the height of the layer. Also for preparing specimens by the wet-tamping method in this investigation, a device was produced in which a tamping rod could smoothly move vertically along a guide which could be displaced smoothly along the circumference of the hollow cylindrical specimen. The diameter of the tamping foot was 16.45 mm which was slightly less than the thickness of the specimen of 20 mm. Initially the density was controlled by adjusting the height of the compaction layer. However, it was found after trying this compaction method several times that the uniformity of density along the circumference of thin hollow cylindrical specimen was not sufficient. After trying several other methods, a method was finally adopted, where the density of each layer was controlled by adjusting the number of tamping with a constant free fall of 3.5 cm. The weight of tamper was 189.5 g and the number of compaction layers was six. And the water content at tamping was 3% for Toyoura Sand and 8% for Sengenyama Sand. To make the vertically uniform specimen, the number of tamping for each layer was increased properly for higher layers and the compacted surface of each layer was scarified as suggested by Ladd (1978).

In the wet vibration method, the prescribed amount of Toyoura Sand with a water content of 3% was placed in layers gently. Then, a hollow cylindrically shaped weight of 1.43 kg was placed on the sand surface. Each layer was compacted by tapping the mold with a wooden hammer uniformly until the height of the layer was reduced to a prescribed value. The number of compaction layers was ten. Also in the wet vibration method, the compacted surface of each layer was scarified.

A high degree of saturation was achieved by circulating CO₂ and de-aired water through a specimen and applying a back pressure of 98 kN/m² or 196 kN/m². The lowest B-value allowed in this test program was 0.96. Most of the measured B-values were larger than 0.98.

A torsional simple shear device (Fig.2) which was recently developed in the Institute of Industrial Science (IIS), the University of Tokyo was used for this investigation. The second author drew the plans under the

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**Fig. 2. Schematic diagram of cyclic torsional simple shear apparatus, (a) general view and (b) plan of cyclic loading device**
supervision of the first author. The stuff members of the workshop in the Institute machined the major parts of the device. A pneumatic cyclic loading system was employed. Cyclic linear motion of the piston of a double-action Bellofram cylinder is converted to cyclic rotational movement of the vertical loading ram through a stretched wire, four pulleys and a wheel. The other single-action Bellofram cylinder is placed on the top plate of the device for providing a vertical load to the specimen.

A hydrostatic confining pressure is provided to confine a hollow cylindrically specimen the outside and inside faces of which is enclosed with conventional rubber membranes. The hollow cylindrical specimen is 100 mm in outer diameter, 60 mm in inner diameter and 100 mm high. Six 1.5 mm high stainless blades are fixed on the surfaces of porous stone of the top cap and the bottom pedestal to prevent slippage of the sand.

One of the features of this apparatus is that the cyclic undrained shear test on saturated specimens in the plane strain condition can be easily performed in a rather simple manner with knowing the effective stress condition throughout consolidation and cyclic loading. With this apparatus, the plane strain condition for undrained saturated specimens is achieved by preventing the axial deformation of specimen by locking vertically the vertical loading ram with allowing only its rotational movement. A usual lucid cell is used as in the conventional triaxial test. The adoption of a low friction air sealing for piston makes it possible to place a strain-gauge type load transducer both for torque and axial load out of the cell without involving testing errors due to piston friction. The torque pickup was calibrated by moment forces produced by dead weights. A slender loading ram with a diameter of 20 mm is used in order that the air sealing works satisfactorily. In the geotechnical laboratory of IIS, the University of Tokyo, same conventional triaxial cells can be used both for triaxial tests and for torsional simple shear tests. For triaxial tests, linear motion bearings are used and these are replaced with stroke bearings for torsional simple shear tests. These arrangements makes the device used in this study much simpler than any of existing other cyclic torsional simple shear devices.

Three different consolidation stress conditions were adopted as follows.

1. Specimens of Toyoura Sand were consolidated isotropically to \( \sigma'_{v_0} = \sigma'_{h_0} = 98 \) kN/m² (\( \sigma'_{v_0} \) and \( \sigma'_{h_0} \) are the effective vertical and horizontal stresses at consolidation, respectively).

2. Specimens of Toyoura Sand and Sengenjama Sand were consolidated to anisotropic stress conditions. For Toyoura Sand, specimens were consolidated with the ratio of \( \sigma'_{h_0} \) to \( \sigma'_{v_0} \) being kept equal to \( K_0 \)-values measured by another test program up to a stress condition where the value of mean principal stress \( \sigma'_{m_0} = (\sigma'_{v_0} + 2 \sigma'_{h_0})/3 \) equaled 98 kN/m². The \( K_0 \)-values used were measured for normally consolidated triaxial specimens by Oh-Kochi et al. (1981) as

\[
K_0 = 0.52 e_t
\]

where \( e_t \) is the initial void ratio. Since the \( K_0 \) values for Sengenjama Sand were not measured, the following simpler method was adopted. Specimens were first consolidated isotropically to \( \sigma'_{v_0} = \sigma'_{h_0} = 98 \) kN/m². Then, the \( \sigma'_{v_0} \) value was increased to \( \sigma'_{v_0} = 196 \) kN/m². Thus, the ratio of \( \sigma'_{h_0} \) to \( \sigma'_{v_0} \) was 0.5 irrespectively of the density of specimen.

3. Specimens of Sengenjama Sand were initially sheared in the drained condition after being anisotropically consolidated as described above. The ratios of initial shear stress to vertical consolidation stress \( \tau / \sigma'_{v_0} \) were 0.1 and 0.2.

After consolidating the specimens under the stress condition as above for 2 hours, a cyclic undrained test was performed. For an isotropically consolidated specimen, the vertical loading ram was free to move vertically during torsional cyclic loading except ten specimens, for which the vertical loading ram was locked against vertical movement. These tests were performed to know the effects of vertical movement on the strengths.
of the isotropically consolidated specimens. For an anisotropically consolidated specimen, the height of specimen is kept constant under the undrained condition during torsional cyclic loading. During a cyclic test, the total horizontal stress, which was the chamber pressure, was kept constant. The total vertical stress, which was measured with a load cell placed above the chamber, decreased when the specimen began to liquefy. Since during the cyclic test both the vertical and volumetric strains were zero, the change in cross area of the specimen was also zero during cyclic loading. It is not unreasonable to assume that the outside and inside diameters were constant during cyclic loading. In this case, no change in the cross area results in no radial strain in a specimen. This is similar to the in situ plane strain condition in level ground during earthquake shaking. Note that the purpose of maintaining constant height is not for producing the constant volume condition. This condition is achieved by the undrained condition of saturated specimen. Therefore, the degree of effects of system compliance for volume change on test results can be considered similar to that in the conventional cyclic triaxial test. It was considered that errors due to membrane penetration effects was not significant due to the small values of $D_{50}$ of sands tested. Furthermore, to reduce system compliance the length of stiff Synflex tube with an outer diameter of 3.18 mm and an inner diameter of 1.6 mm was reduced to a minimum value and the number of ball valves of no volume change type was also reduced to only four. Test conditions are listed in Table 3.

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<th>Table 3. Testing procedure</th>
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<td>Specimen made on cell</td>
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<td>Time to saturate</td>
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<td>Back pressure in kN/m²</td>
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**TEST RESULTS**

**Tests on Isotropically Consolidated Specimens**

Typical time histories of shear stress, shear strain, and excess pore pressure obtained for an isotropically consolidated Toyoura Sand specimen at a consolidated relative density of 77% are shown in Fig.3. In this test, the loading ram was free to move vertically during cyclic loading. It can be seen from this figure that the amplitude of cyclic shear stress was maintained constant during cyclic loading without being affected by the development of large shear strain. This fact shows that the pneumatic cyclic loading system worked very satisfactorily. It is very important to maintain a constant cyclic shear stress amplitude especially for liquefaction tests on dense specimens in order that the cyclic shear strength for dense sands can be better defined by cyclic shear strain amplitude values. In Fig.3 are also shown the effective stress path and the stress-strain relationship recorded. These are typical of the other test results obtained in this study.

To obtain a clear relationship between cyclic undrained strength and relative density, several specimens having different density

![Fig. 3. Time histories of shear stress, shear strain, excess pore pressure, effective path and stress-strain relation for isotropically consolidated air-pluviated Toyoura sand](image-url)
Fig. 4. Relationship between relative density and number of loading cycles where 15% double amplitude shear strain was observed for isotropically consolidated air-pluviated Toyoura sand values were tested using an identical cyclic shear stress amplitude. This procedure was repeated for several different cyclic shear stress amplitudes. These relative density values ranged from around 35% to around 90% for Toyoura Sand and from around 45% to around 100% for Seneganyama Sand. For one value of cyclic stress ratio amplitude a relationship was established between the consolidated relative density $D_r$ and the number of cycles $N_e$ at which a certain value of shear strain double amplitude $\gamma(DA)$ was observed (Fig.4). Other figures similar to Fig.4 were prepared for $\gamma(DA)=1.5\%$, 3% and 7.5% and initial liquefaction. For establishing a distinct relationship between relative density and cyclic undrained strength, there are some advantages of the procedure above over the conventional method in which several specimens reconstituted to a predetermined density are tested with several different cyclic shear stress amplitude values. First, it is much easier to reproduce an identical cyclic shear stress amplitude for different specimens having different densities than to prepare several specimens with an identical density, especially for hollow cylindrical specimens which need elaborate procedures to prepare. Secondly, as shown later, even a slight difference in relative density, say ±2%, may induce an error as large as 100% in the cyclic undrained strength for very dense specimens. Such a small variation in relative density seems inevitable even in a careful sample preparation. Such errors can be eliminated in the procedure employed by this investigation.

From such figures as Fig.4, the relationships as shown in Figs.5(a) and (b) were obtained. In these figures, $\tau_{xy}$ means the amplitude of cyclic shear stress and $\sigma'_m$, is the mean principal stress during consolidation which equals $\sigma'_v$ or $\sigma'_h$ for isotropically consolidated specimens. It may be seen from Fig.5(b) that for the specimens of $D_r=80\%$, a large increase in $\tau_{xy}/\sigma'_m$, can be seen with the decrease in $N_e$ less than around 10. Fig.6 shows the relationship between the stress ratio $\tau_{xy}/\sigma'_m$ and the number of loading cycles $N_e$ to a 15% double amplitude shear strain
Fig. 6. Stress ratio versus number of loading cycles to 15% double amplitude shear strain for isotropically consolidated Toyoura sand of $D_r = 50\%, 60\%, 70\%, 80\%$ and $85\%$.

for relative density values of $50\%, 60\%, 70\%, 80\%$ and $85\%$. Inspection of this figure shows that dense Toyoura Sand has significant large resistances against cyclic undrained torsional simple shear loading for smaller numbers of loading cycles. It is also important to note that even for a large number of loading cycles, say larger than 50, dense specimens tested in this study has a resistance much larger than loose specimens. Similar trends, but much less clearly, has been reported for the large simple shear tests by DeAlba et al. (1975). The authors have performed cyclic undrained triaxial tests on Toyoura Sand with regular (unlubricated) and lubricated ends the results of which will be reported in future. It was found that a good separation was observed in the strength curves for different densities even for larger loading cycles in the case of lubricated ends as seen in Fig. 6. However, it was found that the strength curves for different densities collapsed in a single curve for larger loading cycles in the case of regular ends. Thus, it seems to the present authors that when specimens deform uniformly the strength curves for different densities do not collapse in a single curve even for larger loading cycles and vice versa.

The relationships between the stress ratio $\tau_{cy}/\sigma'_{mv}$ and the relative density $D_r$ for failure defined as $3\%, 7.5\%$ and $15\%$ double amplitude shear strains in the tenth and twentieth loading cycles are shown in Figs. 7(a) and (b), respectively. These relationships were determined directly from the $D_r$ versus $N_e$ relationships as shown in Fig. 4. It can be clearly seen from these figures that the cyclic undrained strength of Toyoura Sand obtained by this test program is significantly large for a relative density value larger than around $80\%$ for $N_e=10$ and around $85\%$ for $N_e=20$. The relative density from which cyclic undrained strength increases significantly for the further increase in relative density will be defined as the critical relative
UNDRAINED BEHAVIOR OF SAND

Fig. 8. Effect of relative density on cyclic strength by cyclic torsional simple shear test for failure defined as 15% double amplitude shear strain in the tenth, twentieth, fiftieth and hundredth cycles for isotropically consolidated air-pluviated Toyoura sand as compared to static drained shear strength

The static shear strength ratio $\tau_r/\sigma'_{m_a}$ is plotted against relative density in Fig. 8 for the purpose of comparison. The values of $\tau_r/\sigma'_{m_a}$ were obtained by Fukushima and Tatsuoka (1981) by means of static drained torsional simple shear tests on isotropically consolidated air-pluviated Toyoura Sand ($\sigma'_{v_r} = \sigma'_{v_r} = 98 \text{kN/m}^2$) with the axial and radial stresses being kept constant during shear test. The height of specimen for static test was 20 cm and the other details in test procedures were identical both for static and cyclic tests. It may be seen from Fig. 8 that the cyclic undrained strength as defined as in the figure becomes as large as the static shear strength for a relative density value above $(D_r)_{critical}$, while the cyclic strength is considerably less than the static drained strength for a relative density less than $(D_r)_{critical}$. This fact above indicates that the effect of cyclic undrained loading on the stress strain behavior of saturated sand when the density is larger than the critical value is rather different from its effect when the density is less than the critical value. Another comparison of the cyclic stress-strain behavior with the static one is shown in Fig. 9. In Fig. 9 are shown the relationships between stress ratio and double amplitude shear strain at $N_e=10$ for several relative density values for Toyoura Sand. It may be seen from Fig. 9 that the resistance of dense specimens against cyclic undrained loading is strongly affected by the shear strain amplitude value used for defining the resistance. It is also to be noted in Fig. 9 that the shapes of the relationships between stress ratio and double amplitude shear strain by cyclic undrained loading for relative density values of 85% and 90% are very similar to the shape of the stress-strain relationship by monotoneous (static) undrained loading for a relative density of 84%. The static undrained tests were performed by Fukushima et al. (1980) with use of specimens similar to ones used for static drained tests (see Fig. 8). It is known that the strain hardening behaviors of dense specimens under monotonous undrained loading as shown in Fig. 9 are due to the dilative nature of the specimens which are observed as a development of large negative excess pore pressure (see Fig. 10). Therefore, it is likely that the larger resistance against cyclic undrained loading for an increased double amplitude shear strain of dense specimens are also due to the dilative nature of the specimens.

Some tests were performed on isotropically
consolidated specimens in which the vertical movement of the loading ram was prevented by clamping during cyclic loading (see Fig. 14). The test results showed that the effects of clamping the loading ram on the test results are not significant. It was observed that the variation of vertical stress was very small until shear strain became large, say 5% in double amplitude. When shear strain became larger than around 5%, a slight variation of vertical stress was observed, which was around 10% of the initial vertical stress in the largest. It is likely that this small variation did not affect the cyclic stress-strain behavior of isotropically consolidated specimens as shown in Fig. 14.

Tests on $K_0$ Consolidated Specimens

Fig. 11 shows typical time histories of shear stress, shear strain, excess pore pressure and total vertical stress decrease obtained for a dense Sengenyama Sand specimen by a cyclic torsional simple shear test under the plane strain condition. The values of effective vertical stress $\sigma'_{ve}$ and effective horizontal stress $\sigma'_{h}$ taken when the cyclic horizontal stresses were zero and the maximum values were calculated from measured values shown in Fig. 11 and plotted in Fig. 12. A smooth effective stress path seen in Fig. 12 may indicates that the arrangements provided for cyclic plane strain tests worked very satisfactorily. Similar tests were performed both for Toyoura Sand and for Sengenyama Sand.

Test results for Sengenyama Sand are summarized in Fig. 13. It may be seen from

**Fig. 9.** Relationship between stress ratio and double amplitude shear strain in the tenth loading cycle for relative densities of 60%, 70%, 80%, 85% and 90% as compared to static undrained stress-strain behaviors for relative densities of 40% and 80% (static undrained tests are by Fukushima et al., 1980).

**Fig. 10.** Effective stress paths by static undrained torsional simple shear test (Fukushima et al., 1980).

**Fig. 11.** Time histories of shear stress, shear strain, total vertical stress decrease and excess pore pressure by plane strain cyclic undrained torsional simple shear tests for dense air-pluviated Sengenyama sand ($\sigma'_{ve} = 196 \text{ kN/m}^2$ and $\sigma'_{h} = 98 \text{ kN/m}^2$).
Fig. 12. Effective stress path for dense air-pluviated Sengenyama sand by plane strain cyclic undrained torsional simple shear test

Fig. 13. Relationship between stress ratio and number of loading cycles where 15% double amplitude shear strain was observed for anisotropically consolidated air-pluviated Sengenyama sand for $D_r=40\%$, 60\%, 80\%, 90\% and 95\%

Fig. 14. Effect of relative density on cyclic strength for failure defined as 15% double amplitude shear strain in the tenth cyclic loading for isotropically consolidated and $K_o$-consolidated Toyoura sand and anisotropically consolidated Sengenyama sand

that for Toyoura Sand the strengths by plane strain tests on $K_o$ consolidated specimens are similar to those of isotropically consolidated specimens for relative density values less than around 65\%. This fact coincides with the results reported by Ishihara and Takatsu (1979) for loose Fuji River Sand of $D_r=55\%$. However, it was found from this investigation that the plane strain strength of anisotropically consolidated specimens were much larger than those of isotropically consolidated ones for relative density values larger than around 70\%. This fact above means that the value of $(D_r)_{crit, test}$ is smaller for the plane strain strength of anisotropically consolidated specimens than for the strength of isotropically consolidated ones. The reason for the fact described above is not known to the present authors. However, it is likely that to simulate the behavior of a dense sand element in the level ground under cyclic simple shear deformation by the laboratory test, it is necessary to perform the plane strain simple shear tests on $K_o$ consolidated specimens.

It is also seen from Fig. 14 that for Sengen-
yama Sand, the value of the critical relative density is much larger than for Toyoura Sand. This difference in strength between Sengenyama Sand and Toyoura Sand is
probably due to their differences in fine content. Sengenyama Sand contains fine particles to some extent which can somewhat prevent larger particles to be located in more stable positions when pluviated through air. On the other hand, Toyoura Sand is a clean uniform sand which does not involve any fine particle. Therefore, particles of Toyoura Sand can be located in stable positions more easily than Sengenyama Sand when pluviated through air. The results shown in Fig.14 also show that cyclic undrained stress-strain relationship depends on not only the relative density but also other factors. Further researches are necessary to clarify these unknown factors.

Effect of Initial Shear Stress $\tau_i$

Test results for anisotropically consolidated Sengenyama Sand specimens initially sheared at consolidation are shown in Figs.15 and 17 as compared to those for the specimens without being sheared at consolidation. The strength for specimens initially sheared were defined as 15% double amplitude shear strain $\gamma(DA)$ and as 7.5% maximum shear strain $\gamma_{\text{max}}$ (see Fig.16). The values of $\gamma(DA)$ may be used to estimate values of cyclic shear strain amplitude in slopes or earthstructures during earthquakes. On the other hand, the residual strains $\gamma_R$ and $\gamma'_R$

![Fig. 15. Effect of initial shear stress on relationship between relative density and cyclic strength for failures defined as 15% double amplitude shear strain or 7.5% maximum shear strain in the tenth cyclic loading for anisotropically consolidated Sengenyama sand](image)

![Fig. 16. Definitions of maximum shear strain $\gamma_{\text{max}}$ and residual shear strains $\gamma_R$ and $\gamma'_R$](image)

![Fig. 17. Effect of initial shear stress on cyclic strength defined for $\gamma_{\text{max}}$ =7.5% and $\gamma(DA)$ =15% at $N_c=5$, 10 and 20 for $D_r=60\%$, 80% and 90% for anisotropically consolidated Sengenyama sand](image)

are defined as strains at zero cyclic shear stress from which the residual deformations of slopes or earthstructures just after earth-
quakes may be estimated. It was found from this study that the value of \( \gamma_{\text{max}} \) could be defined much more clearly than the values of \( \gamma_R \) or \( \gamma'_R \) and was only a slight overestimate of \( \gamma'_R \). Therefore, it was considered that the values of \( \gamma_{\text{max}} \) could be used as alternatives of \( \gamma'_R \). Thus, the values of \( \gamma_{\text{max}} \) will be used in the following. It may be seen from Figs. 15 and 17 that when \( \tau_0/\sigma'_v = 0.1 \), the effect of initial shear stress on the cyclic shear strength as defined above is not significant and the value of \( \langle D_c \rangle_{\text{critical}} \) can be determined as similar to the case when the initial shear stress is zero. When \( \tau_0/\sigma'_v = 0.2 \), the value of \( \langle D_c \rangle_{\text{critical}} \) cannot be well defined. However, it can be seen that the strength for the failure defined as above of dense Sengenyama Sand when \( \tau_0/\sigma'_v = 0.2 \) is much larger than the strength for the case when the initial shear stress is zero. In summary, the cyclic strength defined as sufficiently large strain values was not reduced by the presence of initial shear stress but increased in some cases especially for dense specimens. These test results are well in accordance with those reported by Vaid and Finn (1979). It seems, therefore, that the torsional simple shear apparatus used in this study is a very useful tool for evaluating cyclic undrained stress strain behavior of sands.

**Effects of Sample Preparation Methods**

The effects of sample preparation methods were investigated for isotropically consolidated Toyoura Sand and for anisotropically consolidated Sengenyama Sand (see Table 2). In Fig.18 are shown the relationships between stress ratio and number of loading cycles to a 15% double amplitude shear strain for isotropically consolidated Toyoura Sand of a relative density value of 60% which were reconstituted by the air pluviation, wet tamping and wet vibration methods. It is seen from Fig.18 that the strengths of the specimens prepared by the air pluviation and wet tamping methods are very similar for the number of loading cycles larger than around 5, while the specimen prepared by

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**Fig. 18. Stress ratio versus number of loading cycles to 15% double amplitude shear strain for isotropically consolidated Toyoura sand of \( D_r = 60\% \) prepared by air-pluviation, wet tamping and wet vibration**

**Fig. 19. Effect of sample preparation method on cyclic strength defined as 15% double amplitude shear strain in the tenth loading cycle for isotropically consolidated Toyoura sand**
critical relative density \( D_r \) is smaller for wet tamped specimens than for air-pluviated ones. In Fig. 20 are shown the relationships between stress ratio and double amplitude shear strain in the tenth loading cycle for \( D_r = 60\% \) and 75\% for air-pluviated and wet tamped specimens. While the relationships are very similar for both kinds of specimen for \( D_r = 60\% \), the relationships are quite different for \( D_r = 75\% \), with wet tamped specimens having a larger strength for a larger shear strain. It is interesting that the effect of sample preparation on the cyclic strength increases with the increase in shear strain amplitude used for defining the strength. It is also shown in Fig. 19 that the wet vibrated specimens have a larger strength than those prepared by the other methods for the relative density range examined. A comparison for anisotropically consolidated Sengenyama Sand similar to those shown in Fig. 19 is shown in Fig. 21. Also in this case, the wet tamped dense specimens showed a larger strength than the air-pluviated dense specimens.

The aforementioned comparisons of strength clearly indicate that sample preparation methods may have significant effects on cyclic undrained strength for torsional simple shear test as for triaxial test with the effects being larger for denser specimens. A more detailed investigation will be necessary in this respect.

CONCLUSIONS

On the basis of the limited number of tests reported in this paper on the cyclic undrained strength of sands by cyclic torsional simple shear tests, the followings were found:

(1) Cyclic undrained torsional simple shear tests under the plane strain condition on anisotropically consolidated specimen can be easily performed with knowing effective stress conditions throughout tests using the newly developed device in which a pneumatic cyclic loading system, an air-sealing for piston and a locking device for the vertical loading ram are provided.

(2) The concept that the cyclic undrained strength of sands is proportional to relative density may give an underestimated value for dense clean sands. For a relative density larger than the critical value, cyclic undrained strengths defined as 7.5\% or 15\% double amplitude shear strains of sands tested become extremely high. When the specimen is dense enough, the stress ratio to induce 7.5\% or 15\% double amplitude shear strains can become as large as static drained shear strengths.

(3) For dense specimens, cyclic undrained strength of Toyoura Sand, which is a clean, uniform sand, is much larger than Sengenyama Sand, which is medium fine sand including some fine particles, for an identical relative density value.
Generally, the cyclic undrained strength is affected significantly by many factors when specimens become denser, such as the value of shear strain amplitude used for defining the failure, the stress condition at consolidation, the initial shear stress, the sample preparation method, the kind of sand or so, while these factors above have only a slight effect on the cyclic undrained strength when specimens are loosely packed. Therefore, the cyclic undrained strength of dense sands should be defined and evaluated much carefully than those of loose sands.

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