STABILITY ANALYSIS WITH THE EFFECTIVE STRESS METHOD FOR EMBANKMENTS CONSTRUCTED ON AN ALLUVIAL MARINE CLAY

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ABSTRACT

Five embankments were rapidly constructed on an alluvial marine clay under a very low factor of safety. One embankment failed and two others must have been on the verge of failure. A stability analysis with the total stress method for each embankment was reported by Hanzawa et al. (1980) and showed quite good agreement with the field behaviour of embankments when the undrained strength of the clay was evaluated from the modified Bjerrum's method in considerations of the effects of anisotropy and strain rate.

In this paper, a stability analysis with the effective stress method as well as comparison of measured and predicted excess pore water pressure is presented. Excess pore water pressure was predicted using pore pressure coefficient, A -value in consideration of the effect of strain rate on pore pressure behaviour and the predicted values showed good agreement with the measured values in spite of the fact that piezometers were installed in considerably different types of clay. Strength parameters, c' and ϕ' of the clay are discussed in detail in the light of fundamental properties of normally consolidated aged clay and a procedure for determining c' and ϕ' for short term stability analysis is proposed. Factors of safety obtained by the effective stress method were 20% to 30% higher than the reasonable values evaluated from the field behaviour of each embankment and strongly suggested the necessity to take consideration of the effects of anisotropy and strain rate in determining c' and ϕ' for practical stability problems.

Key words : anisotropy, clay, consolidated undrained shear, embankment, overconsolidation, pore pressure, shear strength, stability analysis, time effect (IGC : D 6/E 6)

INTRODUCTION

Five embankments for preloading were rapidly constructed on an alluvial marine clay under very low factors of safety. Because of a slight difference in slope and height between the designed embankment and the constructed embankments, one embankment failed immediately after its completion and

two others must have been on the verge of failure judging from their settlement behavior during construction. A detailed stability analysis with the total stress method ($\phi=0$ method) was already reported by Hanzawa et al. (1980) and strongly demonstrated that the undrained strength of the clay at the site should be determined by the modified Bjerrum's method (Hanzawa and Kishida,

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1982) which is explained below: "The in-situ undrained strength is determined from consolidated undrained strength tests in which the specimens from undisturbed samples are reconsolidated at the same stresses they carried in the field and then subjected to shear. Reconsolidation at the field stress is terminated when excess pore water pressure has mostly dissipated (for triaxial tests) or when primary compression has achieved (for direct simple shear tests). The in-situ undrained strength of the clay having been subjected to aging effects should be obtained with this method.

On the other hand, excess pore water pressure was measured beneath an embankment and considerable triaxial compression tests with excess pore water pressure measurement were conducted following the modified Bjerrum's method. Therefore, stability analysis with the effective stress method as well as comparison of predicted and measured excess pore water pressure is then available.

This paper first reports comparison of predicted and measured excess pore water pressure beneath an embankment, in which the effect of strain rate on pore pressure behaviour was considered. Fundamental strength characteristics of normally consolidated aged clay (hereafter called N.C. aged clay) (Bjerrum, 1973; Janbu, 1977; Hanzawa and Kishida, 1981, 1982) are then discussed. Strength parameters, c' and ϕ' in terms of effective stress are considered in the light of fundamental properties of N.C. aged clay and a procedure for determining c' and ϕ' for short term stability analysis is proposed for this type of clay. Results of a stability analysis with the effective stress method which used c' and ϕ' determined by the proposed method are presented as well as the comparison with the results obtained by the $\phi=0$ method.

STRENGTH CHARACTERISTICS OF THE CLAY AT THE PROPOSED SITE

The proposed site is situated on a flood

Fig. 1. Overconsolidation ratio of the clay at the site plotted versus depth

plain developed by the Tigris and the Euphrates rivers at the west end of the Arabian Gulf and consists of an N.C. aged clay which possesses an additional strength developed by aging effects. The properties of the clay were reported in detail by Hanzawa et al. (1980) and Hanzawa and Kishida (1981) as follows: -

1) This clay is in an overconsolidated state ($OCR=1.4$ to 9) through depth as shown in Fig. 1. The causes of overconsolidated state are:

- Desiccation stress for GL. 0m to (-) 5m
- Carbonate cementation (which took place when the sea level was lowered in the past) for GL. (-)5m to (-)6.5m and GL. (-)12m to (-)13m.
- Secondary compression below GL. (-) 8m.

2) Strength ratios among compression, extension and vane strengths are constant irrespective of OCR-values as follows: -

$$\left. \begin{aligned} S_{u(c)} &= 1.6 S_{u(v)} \\ S_{u(e)} &= 0.7 S_{u(c)} = 1.1 S_{u(v)} \end{aligned} \right\} \quad (1)$$

$S_{u(c)}$ = compression strength
 $S_{u(e)}$ = extension strength
 $S_{u(v)}$ = vane strength

3) The value of K_0 (coefficient of earth pressure at rest) is also constant irrespective

of OCR-values and showed a value of 0.5 which is the K_0 value in the normally consolidated state.

4) The in-situ undrained strength, S_{uf} is obtained by Eq. (2).

$$S_{uf} = S_{un}/\bar{\sigma}_v \times \text{OCR} \times \bar{\sigma}_{v0} \quad (2)$$

$S_{un}/\bar{\sigma}_v$ = undrained strength ratio in the normally consolidated state.

$\bar{\sigma}_{v0}$ = effective overburden stress

The points described in 2), 3) and 4) are the characteristics of N.C. aged clay (Hanzawa and Kishida, 1981) and are very important in applying to practical problems.

PREDICTION OF EXCESS PORE WATER PRESSURE BENEATH AN EMBANKMENT

Determination of Pore Pressure Coefficient "A"

For predicting excess pore water pressure Δu , induced in a clay beneath a strip load like an embankment, the two methods, 1) use of the pore pressure coefficients A and B (Skempton, 1954) and 2) use of the effective stress path (Roscoe and Burland, 1967; Burland, 1971), are generally used. Prediction of Δu in this paper was carried out using the former in which Δu is given by Eq. (3).

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \quad (3)$$

$\Delta\sigma_1$ = increment of major principal stress

$\Delta\sigma_3$ = increment of minor principal stress

Since the embankment was constructed under a very low factor of safety, pore pressure coefficient at failure, A_f was adopted in evaluating Δu . Then Δu is given by Eq. (3)' because B is 1.0 for saturated clay.

$$\Delta u = \Delta\sigma_3 + A_f(\Delta\sigma_1 - \Delta\sigma_3) \quad (3)'$$

The values of A_f obtained from CK_0UC_1 test, in which the specimens from undisturbed samples are K_0 -consolidated at the field stress and then subjected to shear by compression under a strain rate of 0.09%/min, are plotted against OCR in Fig. 2. In determining A -value for predicting Δu , the strain rate effect on pore pressure behaviour should be considered. In order to investigate this problem, CK_0UC_1 test, in which the specimens prepared from a remoulded sample is

Fig. 2. Pore pressure coefficient A at failure plotted versus overconsolidation ratio obtained from K_0 -consolidated triaxial compression tests at a strain rate of 0.09%/min

K_0 -consolidated at OCR=1.0 and then subjected to compression under different strain rates, was conducted. Normalized effective stress paths obtained from the test are presented in Fig. 3 and the followings can be pointed out: -

1) Effective stress paths greatly change with the strain rate and strongly demonstrate the importance to take into account the strain rate effect in evaluating Δu .

2) Angle of internal friction ϕ' slightly changes with the strain rate ($\phi' = 27^\circ$ to 28.1°) but the strain rate effect on ϕ' will be negligible from practical point of view. Berre and Bjerrum (1973) indicated that the effect of strain rate on the undrained strength is not affected by the stress history of the clay. It was assumed, therefore, that the test results on the clay under OCR=1.0 shown in Fig. 3 will hold for the in-situ clay which OCR-values range from 1.4 to 9. From the characteristic that the effect of strain rate on ϕ' is negligible, " A_f -OCR" relationship at a given strain rate can be obtained as shown below.

$$\begin{aligned} \sin \phi' &= (\sigma_1 - \sigma_3)_f / (\sigma_1' + \sigma_3')_f \\ &= \frac{S_{un}/\bar{\sigma}_v \times \text{OCR}}{[S_{un}/\bar{\sigma}_v \times \text{OCR} + K_0 - A_f(2S_{un}/\bar{\sigma}_v \times \text{OCR} - (1 - K_0))]} \\ (\sigma_1 - \sigma_3)_f &= 2S_{uf} = 2S_{un}/\bar{\sigma}_v \times \text{OCR} \times \bar{\sigma}_{v0} \quad (\text{see Eq. (2)}) \end{aligned}$$

When the values of $\sin \phi'$ at the strain rate

Fig. 3. Effective stress paths indicating the effect of strain rate obtained from K_0 -consolidated triaxial compression tests conducted on remoulded clay (OCR=1.0) at different strain rates

of 0.09%/min and at a given strain rate are the same, the pore pressure coefficient at a given strain rate is obtained by Eq. (4).

$$A_f = \frac{(1 - \mu_R)K_0 + \mu_R \cdot A_{f1} \cdot [2(S_{un}/\bar{\sigma}_v)_1 \cdot \text{OCR} - (1 - K_0)]}{2\mu_R \cdot (S_{un}/\bar{\sigma}_v)_1 \cdot \text{OCR} - (1 - K_0)} \quad (4)$$

$(S_{un}/\bar{\sigma}_v)_1 = S_{un}/\bar{\sigma}_v$ -value at the strain rate of 0.09%/min

$A_{f1} = A_f$ -value at the strain rate of 0.09%/min

$$\mu_R = \frac{S_u \text{ at a given strain rate}}{S_u \text{ at the strain rate of 0.09\%/min}}$$

Hanzawa et al. (1980) reported a value of 0.355 for the $(S_{un}/\bar{\sigma}_v)_1$ and adopted a strength that was reduced by 10% of the strength at the strain rate of 0.09%/min for the stability analysis by the $\phi=0$ method, which gives a value of 0.9 as μ_R . The value of A_f corresponding to the undrained strength used for the $\phi=0$ method is then expressed by Eq. (4)' when the K_0 value is 0.5.

$$A_f = \frac{9A_{f1}(1.42 \times \text{OCR} - 1) + 1}{9(1.42 \times \text{OCR} - 1) - 1} \quad (4)'$$

On the other hand, strain condition in clay foundation where piezometers were installed should be in plane strain condition. A relationship shown in Eq. (5) holds for A -value in triaxial and plane strain tests provided that the material is in elastic state (Nakase, 1969). It was assumed that this relationship will hold for when the material is in plastic

Fig. 4. Pore pressure coefficient A at failure in triaxial compression test (at strain rate of 0.09%/min and after correction for strain rate) and plane strain test

state.

$$A_p = 1.5 A_t \quad (5)$$

$A_p = A$ -value in plane strain test

$A_t = A$ -value in triaxial test

The values of A_{ft} (calculated by Eq. (4)') and A_{fp} (obtained using Eq. (5)) are plotted against OCR in Fig. 4.

Calculating Method of Stresses Induced by Embankment

Burland presented a simplified procedure for calculating stresses beneath a strip load on elastic-plastic material. This method was based on the studies by Hoeg et al. (1968) and was proved to be reasonable accurate in practice from the results of field embankment tests. In this method stresses induced by a strip load are calculated based on the two assumptions as follows:-

1) Stresses beneath the center of a strip load

The initial effective stresses in the ground are $\bar{\sigma}_{v0}$ and $\bar{\sigma}_{h0}$ ($=K_0\bar{\sigma}_{v0}$) and the undrained strength is S_u . For an elastic material the stress increments due to the strip load are $\Delta\sigma_v^e$ and $\Delta\sigma_h^e$ and the total stresses are $\sigma_v^e = \bar{\sigma}_{v0} + \Delta\sigma_v^e$ and $\sigma_h^e = \bar{\sigma}_{h0} + \Delta\sigma_h^e$. If the total shear stress, $t^e = (\sigma_v^e - \sigma_h^e)/2$ is greater than S_u , plastic flow will occur and the stresses will be redistributed. However, it is assumed

that σ_v^e is not significantly affected by the amounts of plastic flow and hence only σ_h is altered. Since the shear stress, t equals to S_u during local failure, the horizontal stress is given by:

Fig. 5. Idealized plot showing stress calculating method at a plastic failure condition

Fig. 6. Locations and depths of piezometers installed and loading stages adopted in stress calculation

$$\sigma_h^p = \sigma_v^e - 2 S_u$$

2) Stresses at points other than on the center line

A further assumption is required since the shear stress, τ_{vh} as well as σ_h , will be modified by the onset of plasticity. For present purposes the simple assumption will be made that the directions of the final principal stresses are unaffected by the development of the local failure.

From the two assumptions, σ_h^p as well as σ_1^p and σ_3^p under plastic flow conditions are obtained as indicated in Fig. 5. The assumption that the principal stress directions are not affected by the onset of local failure is thought to be reasonable for points beneath the loaded area where rotation are in any case small, but leads to significant errors at points outside the loaded area (Burland, 1971).

Fig. 6 indicates depth and locations of piezometers installed, section of the embankment and the properties of the clay. In determining the undrained strength for the use of stress calculation, anisotropy in undrained strength was considered. It may be said that 1) compression strength, 2) average strength of compression and extension strengths and 3) extension strength will be mobilized at zone 1, zone 2 and zone 3 (shown in Fig. 6) under the final loading conditions. The undrained strength at each zone, which are given as shown below, were used for stress calculation.

$$\text{zone 1 : } S_u = S_{u(c)} = 0.355 \times \mu_R \times \text{OCR} \times \bar{\sigma}_{v0}$$

$$\text{zone 2 : } S_u = [S_{u(c)} + S_{u(e)}] / 2 = 0.355 \times \mu_R \times \text{OCR} \times \bar{\sigma}_{v0} \times (1 + 0.7) / 2$$

$$\text{zone 3 : } S_u = S_{u(e)} = 0.355 \times \mu_R \times \text{OCR} \times \bar{\sigma}_{v0} \times 0.7$$

$$\mu_R = 0.9$$

$$S_{u(e)} = 0.7 S_{u(c)} \quad (\text{see Eq. (1)})$$

Embankment loads were divided into 7 stages as indicated in Fig. 6 and calculations were made assuming that the clay is an isotropic elastic material. Increments of major and minor principal stresses $\Delta\sigma_1$ and $\Delta\sigma_3$ calculated by the method explained in this section are presented beneath the center of the embankment in Fig. 7.

Fig. 7. Increments of the major and minor principal stresses calculated beneath the center of the embankment

Comparison of Predicted and Measured Excess Pore Water Pressure

Using the pore pressure coefficient A_{fp} in plane strain condition shown in Fig. 4 and the values of $\Delta\sigma_1$ and $\Delta\sigma_3$ calculated, excess pore water pressure Δu can be obtained by Eq. (6).

$$\Delta u = \Delta\sigma_3 + A_{fp}(\Delta\sigma_1 - \Delta\sigma_3) \quad (6)$$

Dissipation of Δu during construction of the embankment was calculated as one dimensional consolidation under the condition of a single drainage to the ground surface. Coefficient of consolidation c_v used in the calculation of Δu was determined by trial calculation to fit the actual dissipation rate in the field. The value of c_v thus obtained was $2500\text{cm}^2/\text{day}$ and is about 3 times the maximum c_v -value obtained from oedometer tests on specimens in the overconsolidated state (maximum c_v in oedometer tests = $850\text{cm}^2/\text{day}$).

Comparison of predicted and measured Δu are presented in Fig. 8(a) and (b) with the construction rate of the embankment. The followings will be pointed out from the comparison of predicted and measured Δu shown in the figure.

1) When the height of embankment is relatively low (less than 3 m), predicted values showed considerably higher values than measured values. This phenomenon will be attributed to the facts that 1) clay was assumed to be an isotropic elastic material in stress calculation and 2) pore pressure coefficient at failure was used in predicting

Fig. 8(a). Comparison of predicted and measured excess pore water pressure at piezometers P-1, P-2 and P-3 located beneath the center of the embankment

Fig. 8(b). Comparison of predicted and measured excess pore water pressure at piezometers P-4, P-5 and P-6

Table 1. Comparison of predicted and measured maximum Δu and properties of clays

Piezometer	Depth	$(\Delta u_p)_{\max}$	$(\Delta u_m)_{\max}$	$\frac{(\Delta u_p)_{\max}}{(\Delta u_m)_{\max}}$	properties of clay
P-1	-2.5m	96 kN/m ²	92.5 kN/m ²	1.04	Drying crust, OCR=4.5
P-2	-9.5	140	122	1.15	Lightly O.C. clay, OCR=1.55
P-3	-13.0	126 85	101	1.25 0.84	Boundary between lightly O.C. clay (OCR=1.5) and cemented clay (OCR=2.6)
P-4	-7.0	140	116	1.20	Lightly O.C. clay, OCR=1.65
P-5	-10.0	107	103	1.04	Lightly O.C. clay, OCR=1.55
P-6	-5.0	74 65	77	0.96 0.84	Boundary between drying crust (OCR=2.2) and cemented clay (OCR=5.3)

Δu .

2) The increasing rate of predicted Δu from the 15 days after commencement of embankment at which construction rate was rapidly increased showed good agreement with the increasing rate of measured Δu .

3) In spite of the fact that piezometers were installed considerably different clay layers, 1) drying crust, 2) lightly cemented clay and 3) lightly overconsolidated clay, predicted Δu indicated good agreement with measured Δu . The comparison of maximum measured and predicted Δu as well as the properties of the clay are summarized in Table 1.

4) Predicted Δu beneath the center or near the center of the embankment (piezometer P-1, P-2, P-3 and P-4) indicated somewhat larger values than measured Δu . However, predicted Δu at P-6 which is located near the toe was smaller than measured Δu . Burland (1971) also reported the same tendency that the predicted pore pressures were underestimated for points lying outside the loaded area. This may be due to the assumption made about stress calculation during contained plastic flow. It may also be due to the influence of rotation of principal stress which Ladd (1965) showed to give significantly larger pore pressures than the values if no rotation occur.

A CONSIDERATION ON c' AND ϕ' OF NORMALLY CONSOLIDATED AGED CLAY

There are two methods in evaluating stability of clay foundations, 1) the effective stress method and 2) the total stress method. So far as stability of saturated clay is con-

Fig. 9. Idealized plot showing the relation of the shear strength used for the effective stress analysis and the $\phi=0$ analysis

cerned, the $\phi=0$ method is taken as the total stress method. It should be first noted that the total shear strength as a function of effective vertical stress is used in the $\phi=0$ method while it is divided into the two components, c' and ϕ' and the resultant shear strength is used in the effective stress method. Since there should be no difference in the total and the resultant shear strengths as shown in Fig. 9, the factor of safety from the both methods should coincide with each other provided that the drainage conditions are consistent. Akai (1967), Parry (1968) and Janbu et al. (1977) conducted stability analysis of clay foundations (which actually failed) with both the effective and the total stress method ($\phi=0$ method). They reported that the factor of safety with the total stress analysis was much higher than a reasonable value evaluated from the field behaviour of the structures. The shear strength that they used, however, was either unconfined compression strength or in-situ vane strength. It

has recently been demonstrated that the stability of clay foundations evaluated with these two strengths is usually empirical and of indeterminate accuracy (Ladd and Foott, 1974; Schmertman, 1975; Hanzawa et al. 1979, 1980; Hanzawa and Kishida, 1982). That is, their criticism to the total stress analysis is in fact the direct use of these two strengths to the practical stability problems. The engineer who supports the total stress method has no objection to their criticism regarding this point. On the other hand, Foott (1977) pointed out that the factor of safety with the effective stress method for embankments at and near failure were some 25% to 50% larger than the more correct factors of safety given by the total stress method.

In this section of the paper, a consideration on c' and ϕ' of N.C. aged clay which is the most common type of soft clay deposits will be given in the light of fundamental properties of this type of clay. Before the discussion on this subject is progressed, it will be desirable to summarize the fundamental undrained strength characteristics of N.C. aged clay.

The in-situ undrained strength of N.C. aged clay is composed of the two components, 1) S_{un} developed by primary compression under the effective overburden stress $\bar{\sigma}_{vo}$ and 2) S_{ua} developed by aging effects (Hanzawa and Kishida, 1982). When it is defined that the shear strength is a function of the effective stress, the effective stress that a N.C. aged clay possesses may be divided into the two components, 1) external effective stress, $\bar{\sigma}_{ext}$ and 2) internal effective stress $\bar{\sigma}_{int}$ reserved in clay with development of aging. However, the mechanism of aging effects is quite different from each other as demonstrated by Bjerrum (1967). For example, secondary compression is a phenomenon that water content w decreases under a constant $\bar{\sigma}_{ext}$, whereas in chemical bonding it is a usual case that bonding structure is developed under a constant w . Desiccation is accompanied by the increase of $\bar{\sigma}_{int}$ and the decrease of w . Therefore, the state of N.C. aged

Fig. 10. Idealized plot showing a method to evaluate the aging effects as a function of effective vertical stress

clay can not be presented using the " $e-\bar{\sigma}_v$ " relationship and this makes it difficult to draw a clear figure on the shear strength of N.C. aged clay. But when "structure-stress" relation is used instead of " $e-\bar{\sigma}_v$ " relation as presented in Fig. 10, it would be possible to evaluate the aging effects as a function of stress. When the equivalent structure to that at point A (in the field) is obtained on the line showing yielding of the structure (which is equivalent to the virgin compression curve), it will be given by the structure at point B as shown in Fig. 10. Since consolidation yielding stress $\bar{\sigma}_{vy}$ proposed by Mikasa (1962), which is a more generalized expression of preconsolidation pressure p_c , can be defined as the stress where the structure of clay starts to yield, $\bar{\sigma}_{vy}$ of a N.C. aged clay presented by point A will be given by the effective vertical stress at point B. Therefore, $\bar{\sigma}_{int}$ of a N.C. aged clay will be given by " $\bar{\sigma}_{vy} - \bar{\sigma}_{vo}$ ".

$$\bar{\sigma}_{int} = \bar{\sigma}_{vy} - \bar{\sigma}_{ext} = \bar{\sigma}_{vy} - \bar{\sigma}_{vo} \quad (7)$$

As previously described, mechanism of aging effects is quite different from each other. It will be very difficult, therefore, to evaluate each effect on soil properties quantitatively. But their total effect on soil properties will be reflected in the "structure". Therefore, the "structure" described in this section can be evaluated by consolidation yielding stress $\bar{\sigma}_{vy}$.

When the undrained strength is used instead of the structure, the interrelations

Fig. 11. Idealized plot showing relations between the undrained strength (S_{uf} , S_{un} and S_{ua}) and the effective vertical stress ($\bar{\sigma}_{v0}$, $\bar{\sigma}_{vy}$, $\bar{\sigma}_{ext}$ and $\bar{\sigma}_{int}$) for a normally consolidated aged clay

among $\bar{\sigma}_{ext}$, $\bar{\sigma}_{int}$, S_{un} and S_{ua} become more clear as presented in Fig. 10. The paths $O \rightarrow A$ and $A \rightarrow B$ in the figure indicate 1) development of S_{un} with the increase of $\bar{\sigma}_{int}$ (sedimentation process) and 2) development of S_{ua} with increase of $\bar{\sigma}_{int}$ under constant $\bar{\sigma}_{ext}$ (aging process). As indicated in the figure, the total undrained strength (or the in-situ undrained strength) S_{uf} and strength components S_{un} and S_{ua} can be expressed as follows: -

$$\left. \begin{aligned} S_{uf} &= S_{un} + S_{ua} = S_{un}/\bar{\sigma}_v \times \bar{\sigma}_{vy} \\ S_{un} &= S_{un}/\bar{\sigma}_v \times \bar{\sigma}_{ext} = S_{un}/\bar{\sigma}_v \times \bar{\sigma}_{v0} \\ S_{ua} &= S_{un}/\bar{\sigma}_v \times \bar{\sigma}_{int} = S_{un}/\bar{\sigma}_v \times (\bar{\sigma}_{vy} - \bar{\sigma}_{v0}) \end{aligned} \right\} \quad (8-1)$$

Therefore, OCR of N.C. aged clay can be given by the strength ratio between S_{uf} and S_{un} .

$$OCR = S_{uf}/S_{un} \quad (8-2)$$

Hanzawa (1977, 1979), Hanzawa et al. (1980) and Hanzawa and Kishida (1981) reported that OCR-values from Eq. (8-2) and from oedometer test showed quite good agreement for alluvial marine clays which are classified to N.C. aged clay.

When a N.C. aged clay is subjected to a consolidation stress $\Delta\bar{\sigma}$, the structure of the clay should change little provided that $\Delta\bar{\sigma}$ is less than $\bar{\sigma}_{int}$ and consequently its undrained strength is not changed. The efficiency of this suggestion will be supported by the fact

Fig. 12. Idealized plot showing the change of the undrained strengths, S_{uf} , S_{un} and S_{ua} with the change of external effective stress for a normally consolidated aged clay

that OCR of N.C. aged clay can be given by the S_{uf}/S_{un} ratio as expressed by Eq. (8-2). Parry (1970) also supports this suggestion. However, $\bar{\sigma}_{ext}$ and $\bar{\sigma}_{int}$ at this stage should be different from the initial condition as follows: -

$$\left. \begin{aligned} \bar{\sigma}_{ext} &= (\bar{\sigma}_{ext})_0 + \Delta\bar{\sigma} \\ \bar{\sigma}_{int} &= (\bar{\sigma}_{int})_0 - \Delta\bar{\sigma} \end{aligned} \right\} \quad (8-3)$$

$(\bar{\sigma}_{ext})_0 = \bar{\sigma}_{ext}$ at the initial stage
 $(\bar{\sigma}_{int})_0 = \bar{\sigma}_{int}$ at the initial stage

That is, a certain portion of the structure developed by $\bar{\sigma}_{int}$ is destroyed and replaced by the structure developed by $\bar{\sigma}_{ext}$. The undrained strengths S_{un} and S_{ua} at this stage, therefore, can be expressed by Eq. (8-4) as shown in Fig. 12.

$$\left. \begin{aligned} S_{un} &= S_{un}/\bar{\sigma}_v \times [(\bar{\sigma}_{ext})_0 + \Delta\bar{\sigma}] \\ S_{ua} &= S_{un}/\bar{\sigma}_v \times [(\bar{\sigma}_{int})_0 - \Delta\bar{\sigma}] \end{aligned} \right\} \quad (8-4)$$

As expressed in Eq. (8-4), S_{ua} is partly replaced by S_{un} while the total strength remains unchanged. S_{ua} decreases as $\Delta\bar{\sigma}$ increases and when $\Delta\bar{\sigma}$ reaches $(\bar{\sigma}_{vy} - \bar{\sigma}_{v0})$ where the clay is in the normally consolidated state, S_{ua} is completely replaced by S_{un} as presented in Fig. 12.

The processes described above are then discussed using c' and ϕ' in terms of effective stress. The shear strength τ_f is expressed by Eq. (9-1).

$$\tau_f = c' + \sigma' \tan \phi' \quad (9-1)$$

Since τ_f expressed by Eq. (9-1) corresponds to the undrained strength expressed by Eq. (8-1), it is suggested that the effective normal stress acting on the failure plane may be divided into the two parts, 1) an external normal stress σ'_{ext} and 2) an external normal stress σ'_{int} . The value of $\tan \phi'$ will be constant and equals to the value in the normally consolidated state because $\tan \phi'$ is considered to be equivalent to the undrained strength ratio in the normally consolidated state. The shear strength τ_f is then given by:

$$\left. \begin{aligned} \tau_f &= (\sigma'_{ext} + \sigma'_{int}) \tan \phi'_n \\ c' &= \sigma'_{int} \tan \phi'_n \end{aligned} \right\} \quad (9-2)$$

$\phi'_n = \phi'$ in the normally consolidated state

When the clay is subjected to a consolidation stress $\Delta \bar{\sigma}$, τ_f at this stage can be expressed by Eq. (9-3) as shown in Fig. 13.

$$\tau_f = [\{ (\sigma'_{ext})_0 + \Delta \sigma' \} + \{ (\sigma'_{int})_0 - \Delta \sigma' \}] \tan \phi'_n \quad (9-3)$$

$\Delta \sigma'$ = increment of external effective normal stress

As expressed in the above equation c' ($= \sigma'_{int} \tan \phi'_n$) decreases with increasing $\Delta \sigma'$ while τ_f remains unchanged. When the clay has reached the normally consolidated, c' completely disappears as presented in Fig. 13.

DETERMINATION OF c' AND ϕ' OF THE CLAY FOR STABILITY ANALYSIS

Determination of c' and ϕ' of the clay for the stability analysis will be given in this section based on the discussion made in the previous section. The value of c' at the initial stage can be obtained from Eq. (10) as follows: -

$$c' = [(\sigma_1 - \sigma_3)_f / 2 - (\sigma'_1 + \sigma'_3)_f \sin \phi' / 2] / \cos \phi' \quad (10)$$

$$(\sigma_1 - \sigma_3)_f / 2 = S_{un} / \bar{\sigma}_v \times OCR \times \bar{\sigma}_{v0}$$

$$(\sigma'_1 + \sigma'_3)_f / 2 = \bar{\sigma}_{v0} [S_{un} / \bar{\sigma}_v \times OCR + K_0 - A_f \{ 2 S_{un} / \bar{\sigma}_v \times OCR - (1 - K_0) \}]$$

In calculating c' -values from Eq. (10), the following values were used: -

A_f : A_{ft} -values shown in Fig. 4 after correction for strain rate

Fig. 13. Idealized plot showing c' and ϕ' in terms of effective stress for a normally consolidated aged clay and their change with the change of the effective normal stress

OCR : OCR-values presented by the solid line shown in Fig. 1

$S_{un} / \bar{\sigma}_v$: 0.32 (0.355×0.9) = undrained strength ratio in the normally consolidated state at the strain rate of 0.01%/min

K_0 : 0.5

The value of ϕ' in the normally consolidated state was obtained from CK_0UC_2 test in which the specimens prepared from undisturbed samples were K_0 consolidated at OCR = 1.0 and then subjected to compression at the strain rate of 0.09%/min. Effective stress paths obtained from the test are presented in Fig. 14 and a value of 30° was obtained as ϕ'_n which will be little dependent on the strain rate as previously described. The

Fig. 14. Effective stress paths of the clay at OCR=1.0 obtained from K_0 -consolidated triaxial compression test conducted on undisturbed samples at a strain rate of 0.09%/min

Fig. 15. Normalized cohesion in terms of effective stress, $c'/\bar{\sigma}_{v0}$ of the clay at the site plotted versus overconsolidation ratio

values of c' of the clay at the initial stage obtained from Eq. (10) are indicated in Fig. 15 by " c' -OCR" relationship.

Effective stresses of the clay at the completion of embankment should be different from the initial stage and then c' -values obtained using Fig. 15 should be modified. In determining c' -values for the stability analysis, vertical and horizontal effective stresses $\bar{\sigma}_v$ and $\bar{\sigma}_h$ at the completion of embankment were first obtained as follows: -

$$\begin{aligned}\bar{\sigma}_v &= \bar{\sigma}_{v0} + \Delta\sigma_v - \Delta u \\ \bar{\sigma}_h &= \bar{\sigma}_{h0} + \Delta\sigma_h - \Delta u\end{aligned}$$

The mean effective stress at this stage is given by:

$$\bar{\sigma}_m = (\bar{\sigma}_v + \bar{\sigma}_h)/2 \quad (11)$$

On the other hand, the mean effective stress at the initial stage and at the stage of OCR = 1.0 are:

$$\left. \begin{aligned}\bar{\sigma}_{m0} &= (1 + K_0)\bar{\sigma}_{v0}/2 \\ \bar{\sigma}_{mn} &= (1 + K_0)\bar{\sigma}_{vn}/2\end{aligned} \right\} \quad (12)$$

From the comparison of $\bar{\sigma}_m$ -values from Eqs. (11) and (12), OCR of the clay at the completion of embankment can be obtained. The values of c' for the stability analysis can be easily obtained from Fig. 15 using newly obtained OCR. In Fig. 16, c' -values at the initial stage and at the completion of embankment are shown for embankment No. 1.

Fig. 16. Comparison of c' -values at the initial stage and at the completion of embankment No. 1-beneath the center and the toe-

As shown in Eq. (10), c' is a function of $\bar{\sigma}_{v0}$, OCR, S_u , A_f and ϕ' . Among these parameters, S_u is affected by both anisotropy and strain rate and A_f by strain rate. In addition, as pointed out by Ladd (1965; Ladd et al., 1977), there could be a high probability that A_f and ϕ' are significantly affected by anisotropy though it was not confirmed in this investigation. When these matters are considered, it may be concluded that strength parameters c' and ϕ' in terms of effective stress can not be uniquely determined depending on anisotropy and strain rate. It can be also concluded that the effective stress analysis requires exactly the same testing program as the total stress analysis and that it has the added complication of requiring precise pore pressure measurement plus- and this is very important-absolutely requiring good practical technique for extrapolating this pore pressure behaviour to field situations as pointed out by Foott (1977).

RESULTS OF STABILITY ANALYSIS FOR FIVE EMBANKMENTS

A stability analysis with the effective stress method which used c' and ϕ' determined from the procedure explained in the previous section was carried out for 5 embankments. Conditions adopted in the stability analysis are first explained below: -

1) The values of OCR of the clay at the completion of embankment were obtained from the comparison of $\bar{\sigma}_m$ shown in Eqs. (11) and (12).

2) The values of c' for the stability analysis were determined by " $c'/\bar{\sigma}_{vo}$ -OCR" relationship shown in Fig.15 using OCR-values obtained in 1) and $\bar{\sigma}_{vo}$ -values at the initial stage. The values of c' thus obtained change with the locations of the clay beneath embankment as shown in Fig.16.

3) The values of ϕ' of the clay and the embankment were evaluated to be 30° and 35° , respectively.

4) It was assumed that Δu is zero in embankment.

5) A stability analysis was conducted using circular arc method and σ' acting on the sliding plane was calculated as shown in Fig.17.

Field behaviour of each embankment was reported in detail by Hanzawa et al. (1980) and is summarized in Table 2 including their heights, lengths and construction durations and others.

Circular arc giving F.S. $_{min}$ obtained from the effective stress method and the total stress method ($\phi=0$ method) are indicated in Fig.18 for the failed embankment No.4. As shown in the figure, the sliding planes obtained from the two methods showed good agreement with the actual one which was

Fig.17. Diagrammatic representation showing a method to calculate the effective normal stress on the sliding plane for the effective stress analysis

Fig.18. Comparison of circular arc giving F.S. $_{min}$ obtained from the effective stress analysis and the $\phi=0$ analysis and the typical cross section of embankment No.4 after failure

predicted from the tension crack and the heave at the toe observed in the field. Particularly, the sliding plane obtained from the effective stress analysis showed quite good agreement with the actual one.

Factors of safety with the effective stress analysis are presented in Fig.19 together with those obtained from the $\phi=0$ method which were obtained using the strength values by the modified Bjerrum's method explained at the beginning of this paper. The values of F.S. $_{min}$ with the effective stress method are 20% to 30% greater than the values obtained

Table 2. Summary of embankments

Embak. No.	Remarks	Height	Length	Construction duration	Volume
No.1	stable	11.0m	100m	20 days	47 000m ³
No.2	very critical for failure	11.5	300	35	145 000
No.3	critical for failure	11.5	300	50	145 000
No.4	failed	11.5	250	45	120 000
No.5	stable	11.5	250	45	120 000

Fig. 19. Values of $F.S._{min}$ obtained from the effective stress analysis for 5 embankments and their comparison with $F.S._{min}$ values obtained from the $\phi=0$ analysis using the strength from the modified Bjerrum's method

with the $\phi=0$ method and are too high when compared with the field behaviour of embankments shown in Table 2, which is consistent with Foott's indication as previously described. The values of $F.S._{min}$ when c' -values are reduced to zero are also presented in Fig. 19 and showed good agreement with the values by the $\phi=0$ method. Parry (1968) also reported the same case. Errors in calculating stresses induced by embankment and in evaluating excess pore water pressures may be one of the causes that $F.S._{min}$ with the effective stress analysis was greater than reasonable values. However, the true reason for this would be attributed to 1) c' is affected by both anisotropy and strain rate, 2) A_f is greatly affected by strain rate and 3) ϕ' and A_f could be significantly dependent on anisotropy.

SUMMARY AND CONCLUSIONS

A stability analysis with the effective stress method for 5 embankments constructed on an alluvial marine clay at and near failure was reported as well as the comparison of predicted and measured excess pore water pressure beneath an embankment. From the study the following conclusions and suggestions were obtained.

1) Prediction of excess pore water pressure beneath an embankment was made using the test results obtained from the

modified Bjerrum's method in considerations of the effect of strain rate on pore pressure behaviour. Predicted excess pore water pressure showed good agreement with the measured values in spite of the fact that 1) piezometers were installed in considerably different clay layers such as drying crust, lightly cemented clay and lightly overconsolidated clay and 2) so simplified stress calculation method was used.

2) It may be possible to evaluate the aging effects (which are different in mechanism) as a function of stress when the concept of structure described in this paper is adopted. From this point of view, undrained strength and effective stress of a normally consolidated aged clay can be expressed as follows:-

$$S_{uf} = S_{un} + S_{ua}$$

$$S_{un} = S_{un}/\bar{\sigma}_v \times \bar{\sigma}_{v0} = S_{un}/\bar{\sigma}_v \times \bar{\sigma}_{ext}$$

$$S_{ua} = S_{un}/\bar{\sigma}_v \times (\bar{\sigma}_{vy} - \bar{\sigma}_{v0}) = S_{un}/\bar{\sigma}_v \times \bar{\sigma}_{int}$$

$\bar{\sigma}_{v0}$ = effective overburden stress

$\bar{\sigma}_{vy}$ = consolidation yielding stress

$\bar{\sigma}_{ext}$ = external effective stress = $\bar{\sigma}_{v0}$

$\bar{\sigma}_{int}$ = internal effective stress developed by aging effects = $\bar{\sigma}_{vy} - \bar{\sigma}_{v0}$

S_{uf} = in-situ (or total) undrained strength

S_{un} = undrained strength developed by primary compression under $\bar{\sigma}_{v0}$ (= $\bar{\sigma}_{ext}$)

S_{ua} = undrained strength developed by aging effects

$S_{un}/\bar{\sigma}_v$ = undrained strength ratio in the normally consolidated state

3) The concept described in 2) enables to express the shear strength of normally consolidated aged clay in terms of effective stress as follows:-

$$\tau_f = (\sigma'_{int} + \sigma'_{ext}) \tan \phi'_n$$

$$c' = \sigma'_{int} \tan \phi'_n$$

τ_f = in-situ (or total) shear strength in terms of effective stress

σ'_{int} = internal effective normal stress corresponding to $\bar{\sigma}_{int}$

σ'_{ext} = external effective normal stress corresponding to $\bar{\sigma}_{ext}$

$\phi'_n = \phi'$ in the normally consolidated state

4) When the clay is subjected to an additional stress, S_{ua} (or c') decreases and

S_{un} (or $\sigma'_{ext} \tan \phi'$) increases while S_{uf} (or τ_f) remains unchanged before the clay reaches the normally consolidated state. That is, S_{ua} (or c') is partially replaced by S_{un} (or $\sigma'_{ext} \tan \phi'$). When the clay has reached the normally consolidated state, S_{ua} (or c') is completely replaced by S_{un} (or σ'_{ext}).

5) A stability analysis with the effective stress method which used c' and ϕ' determined by the method proposed in this paper, was conducted for 5 embankments. Factors of safety obtained by the effective stress analysis were 20% and 30% greater than the values obtained by the $\phi=0$ method and were too high when compared with the field behaviour of embankment, which strongly suggested that the values of c' , ϕ' and A_f could be significantly affected by both anisotropy and strain rate.

NOTATION

A = pore pressure coefficient, A
 A_f = A value at failure
 A_p = A value in plane strain test
 A_t = A value in triaxial test
 c' = cohesion in terms of effective stress
 K_0 = coefficient of earth pressure at rest
OCR = overconsolidation ratio
 S_u = undrained strength
 S_{ua} = undrained strength developed by aging effects
 S_{uf} = in-situ undrained strength
 S_{un} = undrained strength in normally consolidated state
 $S_{u(c)}$ = undrained compression strength
 $S_{u(e)}$ = undrained extension strength
 $S_{u(v)}$ = vane strength
 $\bar{\sigma}_{ext}$ = external effective stress
 σ'_{ext} = external effective normal stress on failure plane
 $\bar{\sigma}_{int}$ = internal effective stress
 σ'_{int} = internal effective normal stress on failure plane
 $\bar{\sigma}_m$ = mean effective stress
 $\bar{\sigma}_{mn}$ = mean effective stress at OCR=1.0
 $\bar{\sigma}_{mo}$ = mean effective stress at the initial stage
 $\bar{\sigma}_h$ = effective horizontal stress
 $\bar{\sigma}_v$ = effective vertical stress
 $\bar{\sigma}_{v0}$ = in-situ effective overburden stress
 $\bar{\sigma}_{vy}$ = consolidation yielding stress

$\Delta \bar{\sigma}$ = consolidation stress

$\Delta \sigma'$ = incremental effective normal stress on failure plane

$\Delta \sigma_h$ = incremental horizontal stress

$\Delta \sigma_v$ = incremental vertical stress

$\Delta \sigma_1$ = incremental major principal stress

$\Delta \sigma_3$ = incremental minor principal stress

μ_R = correction coefficient for strain rate on undrained strength

ϕ' = internal friction angle in terms of effective stress

τ = shear stress

τ_f = shear strength

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