EFFECT OF OVERCONSOLIDATION ON DILATANCY OF A COHESIVE SOIL

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ABSTRACT

In this paper, the characteristics of dilatancy, i. e., the volumetric change which occurs when subjected to shear stress under the condition that the effective mean normal stress p is held constant, are examined performing p -constant stress-controlled drained triaxial compression tests on a clayey soil with various OCRs.

For the understanding and interpretation of test results, the state paths on normalized state plane (q/p_e , p/p_e), where p_e is Hvorslev's parameter, and the relations between stress ratio and strain increment ratio are discussed using a few OCR dependent parameters which serve to specify the dilatancy characteristics. The sign of dilatancy volumetric strain is classified according to the degree of overconsolidation and the shear stress level.

Additionally the non-linearity of swelling index κ is shown and the applicability of a relationship between κ and OCR, which has been proposed, is demonstrated for the clay used in this study and other cohesive soils appeared in literatures.

Key words : cohesive soil, dilatancy, drained shear, overconsolidation, stress-strain curve, triaxial compression test (IGC : D 6/D 5)

INTRODUCTION

Many natural deposits of clay have been overconsolidated or apparently overconsolidated during their geological history (Bjerrum, 1967; Soderman and Kim, 1970). According to the engineering purpose, studies on the properties of such clays have been focused mainly on their strengths or some parameters at failure (Henkel, 1956; Henkel, 1959; Ladd and Lambe, 1964; Nakase, 1968). Recently, however, the importance and necessity of stress-strain behaviour of geotechnical materials have been recog-

nized and so there have been some attempts to develop the stress-strain relationships for overconsolidated clays (Roscoe and Burland, 1968; Adachi and Nishi, 1976; Pender, 1978; Ohmaki, 1980).

This paper examines experimentally stress-strain behaviour of a remoulded clay when overconsolidated to various degrees of overconsolidation. A main object is to investigate the influences of the degree of overconsolidation on the dilatancy characteristics and on relations between stress ratio and strain increment ratio, both of which will play an important role for developing adequate stress-

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strain relationships for overconsolidated cohesive soils.

Volumetric strain of an ideal elastic material does not occur unless the isotropic component of stress changes. This is expressed by

$$dv = \frac{1}{K} d\sigma_m \quad (1)$$

where $d\sigma_m$ is a change of mean normal stress σ_m , dv is the corresponding change of volumetric strain v and K is bulk modulus. As is well known, however, particulate materials like soils rarely behave as such ideal elastic materials; volume change of real soils occurs even under the condition that effective mean normal stress is kept constant; and this phenomenon is called 'dilatancy'. In this paper, 'dilatancy' is used in this sense and defined as volume change or volumetric strain which occurs without any change of effective mean normal stress.

Shibata (1963) seems to be the first investigator on quantitative evaluation of the dilatancy for clays. He conducted conventional triaxial compression tests under constant effective mean normal stress for a normally consolidated clay and derived an important conclusion that the dilatancy of normally consolidated clays can be expressed as a linear function of stress ratio. His results have been extended by Karube and Kurihara (1966), resulting in the conclusion that the dilatancy of normally consolidated clays, when subjected to monotonically increasing shear stress, can be expressed by the following equation:

$$dv_D = D \cdot d(\tau_{oct}/\sigma_m') \quad (2)$$

where dv_D is a change in volumetric strain due to dilatancy, D is a material constant called 'dilatancy coefficient' and τ_{oct} is the octahedral shear stress. Using Eq. (2) as the expression for the dilatancy of normally consolidated clays, Ohta (1971) developed a stress-strain relationship based on the theory of plasticity. In addition, the dilatancy predicted by the Cam Clay model (Schofield and Wroth, 1968) for isotropically normally con-

solidated clays is the same as that by Eq. (2).

As for the dilatancy of overconsolidated clays, the type of expression as Eq. (2) should not be employed apriori, because there has been little test data on the dilatancy of overconsolidated clays (Parry and Amerasinghe, 1973; Ohmaki, 1980). In addition to this reason, we should note that the 'dilatancy coefficient' can be expected to be not constant but a function of stress ratio even for normally consolidated clays as hypothesized in the modified Cam Clay model (Roscoe and Burland, 1968) or as shown by test data (Balasubramaniam and Zue-Ming, 1980).

Various theories have been developed on stress-strain relationships of soils based on the theory of plasticity. On applying the plasticity theory the existence of the plastic potential is assumed whether the associated flow rule is adopted or not. When obtaining the plastic potential function, either by experiments or by theoretical considerations, it is common to express the strain increment ratio as a function of some parameters concerned with stress: e. g., the energy equations used in the Cambridge theory (Schofield and Wroth, 1968; Roscoe and Burland, 1968) can be regarded as such functions; Pender (1978) has hypothetically proposed an expression for plastic strain increment ratio; Vermeer (1978) has adopted the stress-dilatancy equation proposed by Rowe (1962) to express the flow rule. On the other hand, even without the introduction of the concept in the theory of plasticity, many investigations on strain increment ratio-stress ratio relations have been presented and have played important roles to understand the stress-strain behaviour of granular materials like sands (e. g., Rowe, 1962; Murayama and Matsuoka, 1973).

In this paper, the state of effective stress will be expressed by the effective mean normal stress p and the deviator stress q defined as follows:

$$p = (\sigma_a + 2\sigma_r)/3; \quad (3)$$

$$q = \sigma_a - \sigma_r \quad (4)$$

where σ_a and σ_r are axial and radial effective

stresses in the triaxial test, respectively. The corresponding strain parameters will be the volumetric strain v and the shear strain ϵ defined as

$$v = \epsilon_a + 2\epsilon_r; \quad (5)$$

$$\epsilon = \epsilon_a - v/3 \quad (6)$$

EXPERIMENTAL PROCEDURE

Sample Preparation

The sample used in this study is a silty clay called Fujinomori Clay. After remoulding it with high water content two times its liquid limit for 4 days, it was one dimensionally consolidated in a consolidation cell of 25 cm in diameter under vertical pressure up to 49 kPa, and unloaded to the atmospheric pressure. It took 40 days to complete this preparation process. The physical properties of this soil are listed in Table 1.

Table 1. Physical properties of the sample used

G_s	L. L. (%)	P. L. (%)	P. I.	% finer than $2\mu\text{m}$
2.64	54.0	27.0	26.2	15

Trimming the consolidated sample, cylindrical specimens of 8 cm in height and 10 cm^2 in cross section were made for triaxial tests. The water content when in position in a triaxial cell was about 40%. A filter paper having a series of strips was used around the specimen as a side drain; drainage was not allowed in the vertical direction but in the radial direction only. All the tests were performed at a constant temperature of $20 \pm 0.5^\circ\text{C}$.

Triaxial Tests

All the tests are isotropically consolidated drained triaxial tests, which were performed in standard GEONOR triaxial cells. Fig. 1 represents schematically an effective stress path employed in this study. The triaxial test process can be divided into two processes. First, specimens were isotropically consolidated up to maximum pressure p_c ($=588\text{ kPa}$) and isotropically unloaded to the pressure of p_c/OCR , where OCR is overconsolidation

Fig. 1. Schematic representation of the effective stress path

ratio. The duration for consolidation at each pressure stage was about 1 day. After this consolidation process, shear stress was applied under the condition of constant effective mean normal stress using the method of stress control. The increment of shear stress ratio q/p was held constant at 0.1; this increment was applied every day. Axial deformation and volume change were measured by a dial gauge with the minimum scale of $1/100\text{ mm}$, and by a burette with that of $2/100\text{ mL}$, respectively. All the tests performed are listed in Table 2, where some results which will be discussed later are also given.

Table 2. List of tests performed and some results

Test No.	p_c (kPa)	p (kPa)	OCR	η_{DL}	μ	η_f
F 302	588	588	1	—	—	1.4
F 303	588	588	1	—	—	*
F 304	588	392	1.5	0.4	0.9	1.4
F 305	588	392	1.5	0.5	1.0	1.4
F 306	588	294	2	1.1	1.2	1.5
F 307	588	147	4	1.1	1.6	1.7
F 308	588	147	4	1.1	1.6	1.6
F 309	588	73.5	8	1.4	2.0	2.0
F 310	588	73.5	8	1.4	1.8	1.8
F 311	588	29.4	20	1.3	2.1	2.1
F 312	588	29.4	20	1.7	2.4	2.4

* This test was stopped before failure because of leakage.

EXPERIMENTAL RESULTS

Volume Change Behaviour during Isotropic

Fig. 2. Volume change characteristics during isotropic unloading

Consolidation Process

Volume change characteristics during isotropic unloading from $p = p_c / \text{OCR}$ is shown in Fig. 2, where the ratio of the change in water content from virgin consolidated state at $p = p_c$ to the value of natural logarithm of OCR ($\ln \text{OCR}$) is plotted against OCR on the log scale. From this figure it can be assumed that such a linear relation as

$$(w_0 - w_c) / \ln n = a + b \cdot \ln n \quad (7)$$

exists, where n : OCR

w_0 : water content at $p = p_c / n$,

w_c : water content at $p = p_c$, and

a, b : constants independent of OCR.

Differentiating Eq. (7), isotropic swelling index κ , commonly defined as $-dw/d \ln p$, the tangential slope of the swelling line on $(w, \ln p)$ plane, can be given by Eq. (8):

$$\kappa = a + 2b \cdot \ln n. \quad (8)$$

Although it is usually assumed that the isotropic swelling lines are straight and have a constant slope, their non-linearity has been often recognized (Mesri et al., 1978; Shimizu, 1978). According to Mesri et al. (1978), in which the effects of OCR and the duration of swelling on the swelling index

(a) Weald clay (Henkel, 1959)

(b) St. Clair clay till (Soderman and Kim, 1970)

Fig. 3. Swelling characteristics of some soils

are examined for the wide range of OCR, the swelling index generally increases with OCR, however in certain range of OCR, it levels off and even decreases slightly with OCR. It should be noted, hence, that the applicability of Eq. (8) might be limited in a certain range of OCR. It is, however, interesting that the relationship such as Eq. (7) may be applicable for remoulded Weald Clay (Henkel, 1959) and remoulded or undisturbed St. Clair clay till (Soderman

Fig. 4. Change in water content during isotropic consolidation, rebound and at failure

and Kim, 1970) as shown in Fig. 3.

In Fig. 4, the swelling line approximated by Eq. (7) is shown, as well as the virgin consolidation line, where water content is evaluated in terms of the deviation from the water content at the end of consolidation with $p=p_c$ in order to eliminate some experimental scatter in the absolute quantity of water content. It is seen that the approximated swelling line (denoted by S.L. in the figure) is quite close to the experimental results and can well represent the non-linearity. In this figure water content at failure is also plotted. This will be discussed later.

Strain-Time Relations at Shear Stage

In Figs. 5 and 6, volumetric strain and shear strain are shown, respectively, as a function of time elapsed after the moment of the application of an increment of shear stress at each loading stage, for Test F311 as an example. Strain is defined as the natural strain referred to the volume or the

Fig. 5. Volumetric strain v.s. time (Test No. 311)

Fig. 6. Shear strain v.s. time (Test No. 311)

height of specimen at the start of shearing, i.e., at the end of isotropic consolidation process. From these figures it is not likely that deformation ceases completely even after 1 day for each loading stage especially at high values of stress ratio. It will be assumed, however, in the following discussion that the deformation at each loading stage

Fig. 7. Relationships between volumetric strain and stress ratio

Fig. 8. Relationships between shear strain and stress ratio

ceases 1 day after the moment of load application and that strains observed after 1 day of loading are taken as strains in equilibrium corresponding to the state of stress which remains constant during each loading. Thus, time dependent behaviour of shear or volumetric strain will not be considered in this paper.

Stress-Strain Relations at Shear Stage

Volumetric strain and shear strain are plotted against stress ratio in Fig. 7 and Fig. 8, respectively. From Fig. 7 it can be seen that such a dilatancy coefficient as used in Eq. (2) is not constant but depends on stress ratio for each OCR. For specimens with OCR of 4, 8 or 20, negative volumetric strain occurs from the start of shear to failure; that is, volume expansion occurs. For OCR of 1.5 or 2, the sign of volumetric strain changes to positive from negative at a certain value of stress ratio, which seems to depend on OCR. And normally consolidated ones reveal positive volumetric strain once shear stress is applied; that is, volume contraction always occurs. These characteristics of volume change will be discussed later particularly in the light of their dependence on OCR.

DISCUSSION

Critical State for Overconsolidated Soils

Water content at failure evaluated in terms of the change in water content from the end of consolidation under $p = p_c$ is plotted against the normalized effective mean normal stress p/p_c in Fig. 4 presented above. For OCRs of 1, 1.5 and 2, states failure can be represented by a straight line being parallel to the virgin consolidation line (denoted by V.C.L.). Assuming that the failure state for normally consolidated or lightly overconsolidated specimens corresponds to the critical state as defined in the Cambridge theory, the critical state line (C.S.L.) can be expressed by the following equation:

$$w_{crit} = \Gamma - \lambda \cdot \ln (p/p_u) \quad (9)$$

where subscript 'crit' denotes the critical state, Γ is the water content at p of unit stress p_u ($=100$ kPa), and λ compression index for normally consolidated states ($= -dw/d \ln p$).

As is well known, modes of failure of clays depend on the degree of overconsolidation. Normally consolidated or lightly overconsolidated clays usually fail with the shape like a cask; heavily overconsolidated clays fail with one or two slip surfaces. It might be supposed, hence, that water content would not be uniform throughout the specimen, especially for heavily overconsolidated clays. To investigate this locality of water content, the distribution of water content at failure was measured for specimens of OCR=4, 8 or 20.

The specimens were divided to nine or ten small masses and the water content of each small mass was measured. The distribution of water content within the specimen for Test F311 is illustrated in Fig. 9 as an example. The directions for dividing the specimen were taken so that at least one small mass could include the slip surface. As is clear in the example given in Fig. 9, water content was relatively high for masses

Fig. 9. An example of the distribution of local water content

in the bulged portion, and the highest for one which includes the slip surface.

Maximum and minimum local water contents thus obtained and mean water content of the whole specimen are plotted in Fig. 4. It can be seen that the maximum local water content gets near to the C. S. L. determined from the results for normally consolidated and lightly overconsolidated specimens.

The measurement of local water content was done at the moment of rupture to avoid the complete rupture of the specimen, resulting from the testing method of stress control. Since further redistribution will occur if the deformation after the rupture could be controlled to continue more slowly, local water content may be expected to be higher than measured quantities shown in Fig. 4. Taking into account this expected tendency of further increase in local maximum water content, it may be assumed that the critical state line for heavily overconsolidated states is the same as that for normally consolidated and lightly overconsolidated states. This is expressed by Eq. (9).

Dilatancy of Overconsolidated Clay

The results shown in Fig. 7 suggest that the volume change behaviour of overconsolidated specimens strongly depends on the degree of overconsolidation even if the normalized shear stress q/p is used. Several parameters which would serve to describe the characteristics of the volume change behaviour will be used and the variation of such parameters with the degree of overconsolidation will be investigated by using experimental results.

Fig. 10 shows state paths on $(q/p_e, p/p_e)$ plane, where p_e is the equivalent mean normal stress defined by Hvorslev (1960) as

$$p_e = p_u \cdot \exp\left(\frac{w_a - w}{\lambda}\right) \quad (10)$$

where w_a is water content at the point on the V.C.L. for unit stress p_u . From this figure it appears that any stress path except those for normally consolidated specimens is fairly straight in some portion from the start of shearing. Beyond this straight portion, it begins to be curved as positive dilatant behaviour becomes more and more

Fig. 10. State paths on $(q/p_e, p/p_e)$

marked for heavily overconsolidated specimens ($\text{OCR}=4, 8$ or 20) and as negative dilatant behaviour for lightly overconsolidated ones ($\text{OCR}=1.5$ or 2). State paths for normally consolidated specimens are curved just from the beginning of shearing because of the negative dilatant behaviour. It should be noted here that the positive dilatant behaviour results in the decrease of p_e and the increase of p/p_e since p was held constant during the shearing.

From this figure it appears also that all the state paths have a tendency to arrive at a small area within which the critical state point will be located. The critical state point can be easily determined from two assumptions that the unique critical state line expressed by Eq. (9) exists on (p, w) plane and, additionally, that the value of stress ratio at the critical state M would be constant regardless of the degree of overconsolidation. For a point at the critical state, from the definition of p_e ,

$$p_{e \text{ crit}} = p_u \cdot \exp\left(\frac{w_a - w_{\text{crit}}}{\lambda}\right) \quad (11)$$

Substituting Eq. (9) into the above, we obtain

$$p_{e \text{ crit}} = p_{\text{crit}} \cdot \exp\left(\frac{w_a - \Gamma}{\lambda}\right) \quad (12)$$

and, hence, $(p/p_e)_{\text{crit}} = \exp\{(\Gamma - w_a)/\lambda\}$ and $(q/p_e)_{\text{crit}} = M \cdot (p/p_e)_{\text{crit}}$. By using the values of w_a , Γ , λ and M the critical state point can be determined as $(p/p_e)_{\text{crit}} = 0.569$ and $(q/p_e)_{\text{crit}} = 0.797$. The values of constants mentioned above are listed in Table 3.

Table 3. List of constants for the determination of the critical state point

w_a (%)	Γ (%)	λ ($\times 10^{-2}$)	M
34.5	32.0	4.34	1.4

Any state path for overconsolidated specimens may be divided into two parts: straight and curved. Stress ratio corresponding to the point at which the linearity of a state path disappears and at which the state path begins to be curved will be referred to as

Fig. 11. Variation of the stress ratio at the 'threshold of dilatancy' (η_{DL}) with OCR

the 'threshold of dilatancy' hereafter. A subscript 'DL' will be used to specify the state of the threshold of dilatancy. Stress ratio at the threshold of dilatancy η_{DL} and the gradient of the linear portion of the state path as shown in Fig. 10, D^* , will be employed as parameters which characterize the volume change behaviour in the linear portion of a state path.

The variation of η_{DL} with OCR can be seen in Fig. 11, where η_{DL} is plotted against the value of $\ln \text{OCR}$ in the log-log scale. The following relationship may be obtained between η_{DL} and OCR:

$$\eta_{DL} = M_{DL} \cdot (\ln n)^{m_{DL}} \quad (13)$$

where M_{DL} and m_{DL} are constants: the for-

Fig. 12. Variation of the gradient of the linear portion of state path (D^*) with OCR

mer means the value of η_{DL} when $\ln n=1$ or $n=e$ (e : the base of the natural logarithm); the latter the gradient of the straight line represented in Fig. 11. The best fitting by means of the least square method gives 0.935 and 0.567 to M_{DL} and m_{DL} , respectively.

Fig. 12 shows the variation of D^* with the degree of overconsolidation. In this figure plots denoted by 'corrected' were obtained by taking into account the expansion or compression of the filter paper used as a side drain. The quantity of water expelled from or absorbed by the filter paper due to change in cell pressure was experimentally determined (Shimizu, 1981). 'Non-corrected' plots were obtained directly from the results shown in Fig. 7. From Fig. 12 it can be seen that the inverse of D^* and that of OCR

can be linearly related whether or not the correction was made, resulting in the expression as follows:

$$1/D^* = m^*(1/n - 1/n^*) \quad (14)$$

The best fitting for corrected plots gives -0.065 and 1.7 to m^* and n^* , respectively. This equation indicates that D^* is negative when OCR is smaller than n^* . Hence, we assume that for a specimen with OCR smaller than n^* its state path will have a linear portion with a negative value of D^* in the beginning of shearing. This assumption implies that very lightly overconsolidated clay will contract just from the start of shearing as well as normally consolidated clays.

Now, we note that the dilatancy which

appears at stress ratio smaller than η_{DL} is negligibly small, comparing that at stress ratio higher than η_{DL} .

Stress Ratio-Strain Increment Ratio Relations

The purpose of this section is to understand, experimentally, the relative influence of the degree of overconsolidation to the relationships between strain increment ratio and stress ratio. Discussions will be given being associated with the volume change behaviour shown above.

Fig. 13 illustrates the relations between stress ratio $\eta (=q/p)$ and strain increment ratio $-dv/d\varepsilon$, obtained from the p -constant tests of which results were presented above. dv and $d\varepsilon$ are volumetric and shear strain increments, respectively, corresponding to an applied stress increment.

Stress ratio-strain increment ratio relations are interesting in applying the theory of plasticity. Instantaneous strains at the application of load increment, which seem to be attributed to the elastic deformation (Casagrande and Wilson, 1951), were subtracted from the measured strains. Therefore, the strain increments used in Fig. 13 may be regarded to be plastic.

This figure shows that the strain increment ratio-stress ratio relations vary with OCR. This OCR dependency will be discussed in detail.

Plots for normally consolidated specimens appear to be aligned on a straight line expressed by Eq. (15) although for small values of η there seems to be some scatter.

$$\eta = -dv/d\varepsilon + M \quad (15)$$

where M is the value of stress ratio at $-dv/d\varepsilon=0$ or at failure. This parameter can be considered to correspond to the critical state stress ratio. Eq. (15) is the very energy equation in the original Cam Clay model.

Lightly overconsolidated specimens ($OCR = 1.5$ or 2) start from states of $-dv/d\varepsilon$ positive, pass through the region within which $-dv/d\varepsilon$ is negative and reach the straight line expressed by Eq. (15) at a certain value of η , which will be denoted by μ . Once reaching

Fig. 14. Variation of the boundary stress ratio (μ) with OCR

the line, they move along it and fail at $\eta=M$ with $-dv/d\varepsilon=0$.

Plots for heavily overconsolidated specimens ($OCR=4, 8$ or 20) are always in the region of $-dv/d\varepsilon$ positive. At failure they are nearly on the straight line expressed by Eq. (15). The stress ratio μ , defined above, hence, corresponds to stress ratio at failure for heavily overconsolidated specimens.

In Fig. 14, the variation of μ with OCR is shown. From this figure μ can be related to OCR by the expression:

$$\mu = M_\mu (\ln n)^{m_\mu} \quad (16)$$

where M_μ and m_μ are constants which can be determined from Fig. 14. M_μ represents the value of μ when $\ln n=1$, i.e., when $n=e$ (e : the base of the natural logarithm). The best fitting by means of the least square method gives $M_\mu=1.39$ and $m_\mu=0.428$. It should be noted that the value of M_μ is almost the same as the value of M , the critical state stress ratio. From the coincidence between the values of M_μ and M , we may suppose that for OCR greater than e the sign of dilatancy due to shear is always positive.

There has been no data with which the coincidence of M_μ and M would be examined for clays other than the sample used here. However, recalling that OCR of the value of e is the boundary between 'dry' and 'wet' in the Cam Clay model, the coincidence between M and M_μ would not be strange. Accordingly, we adopt the notation M instead of M_μ . Eq. (16) leads to

$$\mu = M \cdot (\ln n)^{m_\mu} \quad (17)$$

State Boundary Surface

It is very interesting that, both for heavily overconsolidated specimens at stress ratio of μ and for lightly overconsolidated specimens at stress ratio greater than μ , stress ratio-strain increment ratio relations can be represented by the same equation as the energy equation in the original Cam Clay model.

According to the original Cam Clay model, Eq. (15) is assumed to be valid for any state on the State Boundary Surface which can be derived from Eq. (15) with a few hypotheses concerned with the concept of theory of plasticity. It will not be very difficult to suppose that in (p, q, w) state space a unique boundary surface like the State Boundary Surface in the Cam Clay model exists and that on the boundary surface stress ratio-strain increment ratio relation as expressed by Eq. (15) is valid in spite of the degree of overconsolidation although the validity is limited to a certain range of stress ratio.

In order to visualize the form of the state boundary surface, Fig. 15 is prepared, where dilatancy behaviours are compared between specimens of various OCR but of the same

value of effective mean normal stress p . Plots for a certain value of OCR were calculated by temporarily assuming that the dilatancy behaviour of a specimen consolidated to $p = p_{c0} = \text{OCR} \cdot p_c$ and subsequently unloaded to $p = p_{c0}/\text{OCR} = p_c$ can be analogized by the behaviour of a specimen consolidated to $p = p_c$ and unloaded to $p = p_c/\text{OCR}$. The abscissa represents the change in water content referring to the water content at the point of $p = p_c$ on V.C.L.

In this figure, the envelope is considered to be a portion of the state boundary surface. It can be seen that for lightly overconsolidated specimens beyond the points corresponding to stress ratio μ defined already, the curves get near to those for $\text{OCR} = 1$. On the other hand, plots for heavily overconsolidated specimens tend to lie on the envelope at failure, i.e., $\eta = \mu$. This means that for the states on the state boundary surface there exists a unique relation between stress ratio and strain increment ratio such as Eq. (15), i.e., the energy equation proposed in the original Cam Clay model may be valid even for overconsolidated specimens if their states reach the state boundary surface.

Fig. 15. Dilatancy behaviour on the state plane with the same value of mean normal stress

CONCLUSION

The characteristics of dilatancy, i.e., the volume change which reveals when subjected to shear stress under the condition that the effective mean normal stress is held constant, have been examined experimentally for a saturated clay soil of various OCRs.

In order to facilitate the understanding and interpretation of the results, the state paths on the normalized state plane (q/p_e , p/p_e) and the relations between stress ratio and strain increment ratio were utilized. Several parameters which serve to specify the dilatancy characteristics were chosen and used.

From the behaviour of state paths on (q/p_e , p/p_e) plane (Fig. 10) it was shown that:

i) When stress ratio is smaller than an OCR dependent value, denoted by η_{DL} , state paths are straight, and in the straight portion the variation of p/p_e , accordingly the change in water content, is negligibly small comparing to that beyond this portion.

ii) The gradient of the straight portion of a state path, $D^* = d(q/p_e)/d(p/p_e)$, is positive for OCR greater than n^* (n^* is about 1.7 for the sample used) and it is negative for OCR smaller than n^* ; that is, overconsolidated specimens of OCR greater than n^* dilate just from the start of shearing while those of OCR smaller than n^* contract.

As to the relations between stress ratio and strain increment ratio, findings are summarized as follows:

i) For any degree of overconsolidation, when stress ratio reaches an OCR dependent value, denoted by μ , the relation is represented by the expression of Eq. (15), which is the same as the energy equation assumed in the original Cam Clay model.

ii) State paths in (p , q , w) state space reach the state boundary surface at the stress ratio μ , and there seems to exist a unique relation between stress ratio and strain increment ratio on the state boundary surface as assumed in the original Cam Clay model.

iii) e (the base of the natural logarithm)

Fig. 16. Variation of the sign of dilatancy with OCR and shear stress

is a boundary value of OCR, because for OCR greater than e the sign of strain increment ratio $-dv/d\varepsilon$ might be always positive while for OCR smaller than e it changes from positive to negative; that is, overconsolidated specimens of OCR greater than e dilate all the way from the start of shearing to failure.

Considering both the behaviours of state paths on (q/p_e , p/p_e) and the relations between stress ratio and strain increment ratio, the sign of dilatancy can be classified according to the degree of overconsolidation and the shear stress level as illustrated in Fig. 16, in which shear stress q and effective mean normal stress p are both normalized by maximum normal consolidation stress p_c .

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NOTATION

- ε =shear strain ($=\varepsilon_a-v/3$)
 v =volumetric strain
 ε_a =axial strain in triaxial test
 p =effective mean normal stress ($=(\sigma_a+2\sigma_r)/3$)
 q =deviator stress
 σ_a and σ_r =axial and radial stresses in triaxial test
 η =stress ratio ($=q/p$)
 p_e =equivalent effective mean normal stress
 $p_{e\text{crit}}=p_e$ at the critical state
 p_c =maximum isotropic consolidation stress
 w =water content
 w_0 =water content at the end of isotropic consolidation
 w_c =water content at $p=p_c$
 w_{crit} =water content at the critical state
 w_a and Γ =water contents at the virgin consolidation and at the critical state, respectively, corresponding to $p=p_u$ ($=100$ kPa)
 M =the value of η at the critical state
 η_{DL} =stress ratio at the 'threshold of dilatancy'
 μ =stress ratio at which stress ratio-strain increment ratio relation reaches the relation for normally consolidated state
 D^* =gradient of the linear portion of state path on (q/p_e , p/p_e) plane
 M_{DL} , m_{DL} =constants to describe η_{DL} -OCR relation
 m^* , n^* =constants to describe D^* -OCR relation
 M_μ , m_μ =constants to describe μ -OCR relation

REFERENCES

- Adachi, T. and Nishi, K. (1976): "Constitutive equations for overconsolidated clays," *Annals, Disaster Prevention Research Institute, Kyoto Univ.*, No. 19 B-2, pp. 1-13 (in Japanese).
- Balasubramainam, A. S. and Zue-Ming, H. (1980): "Yielding of weathered Bangkok Clay," *Soils and Foundations*, Vol. 20, No. 2, pp. 1-15.
- Bjerrum, L. (1967): "Engineering geology of Norwegian normally consolidated marine clays as related to settlements of buildings," *Seventh Rankine Lecture, Géotechnique*, Vol. 17, pp. 83-117.
- Casagrande, A. and Wilson, S. D. (1951): "Effect of rate of loading on the strength of clays and shales at constant water content," *Géotechnique*, Vol. 2, pp. 251-263.
- Henkel, D. J. (1956): "The effect of overconsolidation on the behaviour of clays during shear," *Géotechnique*, Vol. 6, pp. 139-150.
- Henkel, D. J. (1959): "The relationships between the strength, porewater pressure, and volume change characteristics of saturated clays," *Géotechnique*, Vol. 9, pp. 119-135.
- Hvorslev, M. J. (1960): "Physical components of the shear strength of saturated clays," *Proc. Research Conf. on Shear Strength of Cohesive Soils*, ASCE, pp. 163-273.
- Karube, D. and Kurihara, N. (1966): "Dilatancy and shear strength of saturated remoulded clay," *Proc. JSCE*, No. 135, pp. 16-24 (in Japanese).
- Krishna Murthy, M., Sridharan, A. and Nagaraj, T. S. (1981): "Shear strength behaviour of overconsolidated clays," *Soils and Foundations*, Vol. 21, No. 2, pp. 73-82.
- Ladd, C. C. and Lambe, T. W. (1964): "The strength of 'undisturbed' clay determined from undrained tests," *ASTM Special Technical Publication*, No. 361, pp. 342-371.
- Mesri, G., Ullrich, C. R. and Choi, Y. K. (1978): "The rate of swelling of overconsolidated clays subjected to unloading," *Géotechnique*, Vol. 28, No. 3, pp. 281-307.
- Murayama, S. and Matsuoka, H. (1973): "A microscopic study on shearing mechanism of soils," *Proc. 8th ICSMFE*, Vol. 1, Part 2, pp. 293-298.
- Nakase, A. (1968): "Effect of overconsolidation on undisturbed strength of clays," *Report of the Port and Harbour Research Inst., Ministry of Transport, Japan*, Vol. 7, No. 1, pp. 4-23.
- Ohmaki, S. (1980): "Strength and deformation characteristics of an O.C. clay," *Proc. 3rd Int. Conf. on Numerical Methods in Geomechanics*, pp. 465-474.
- Ohta, H. (1971): "Analysis of deformations of soils based on the theory of plasticity and its application to settlement of embankments," *Dr. Thesis, Kyoto Univ.*
- Parry, R. H. G. and Amerasinghe, S. F. (1973): "Components of deformation in clays," *Proc. Symposium on Plasticity and Soil Mechanics*, Cambridge, pp. 108-128.
- Pender, M. J. (1978): "A model for the behaviour of overconsolidated soil," *Géotechnique*, Vol. 28, No. 1, pp. 1-25.
- Roscoe, K. H. and Burland, B. J. (1968): "On

- the generalized stress-strain behaviour of 'wet' clay," Engineering Plasticity, Cambridge Univ. Press, pp.535-609.
- 19) Rowe, P. W. (1962): "The stress-dilatancy relation for static equilibrium of an assembly of particles in contact," Proc. Roy. Soc. London. Ser. A. Vol.269, pp.500-527.
- 20) Schofield, A. N. and Wroth, C. P. (1968): Critical State Soil Mechanics, McGraw-Hill.
- 21) Shibata, T. (1963): "On the volume change of normally consolidated clays," Annals, Disaster Prevention Research Inst., No.6, pp.128-134 (in Japanese).
- 22) Shimizu, M. (1978): "Characteristics of dilatancy and strength of remoulded overconsolidated clay under low effective stress level," Annals, Disaster Prevention Research Inst., No.21 B-2, pp.1-17 (in Japanese).
- 23) Shimizu, M. (1981): "Factors affecting the measurement of volume change of cohesive soils in drained triaxial tests," Soils and Foundations, Vol.21, No.2, pp.121-128.
- 24) Soderman, L. G. and Kim, Y. D. (1970): "Effect of ground water levels on stress history of the St. Clair clay till deposit," Can. Geotech. J., Vol.7, pp.173-187.
- 25) Vermeer, P. A. (1978): "A double hardening model for sand," Géotechnique, Vol.28, No.4, pp.413-433.