TECHNICAL NOTES

EFFECT OF INITIAL STRESS CONDITIONS
ON LIQUEFACTION OF SANDS:
EXPERIMENTS AND AN INTERPRETATION

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ABSTRACT

Stress controlled cyclic triaxial tests, under undrained conditions, performed on sand samples consolidated under a range of different principal stress ratios (chosen to reflect and simulate the in-situ stress conditions) affect greatly the generation of pore pressures with the increasing number of cycles of repeated loading.

In the following an attempt is made to bring a theoretical interpretation of these experimental observations regarding the tendency of lesser volume changes leading to smaller pore water pressure increases in anisotropically consolidated triaxial sand specimens under dynamic loads.

Key words: anisotropic consolidation, laboratory test, liquefaction, sand, theoretical interpretation (IGC: D 6/D 7)

INTRODUCTION

Numerous research and investigation has been undertaken in order to assess the dynamic behaviour and liquefaction characteristics of sands under different initial stress conditions (Ref.2–12). But, certain contradictions in viewpoints still exist.

Regarding the controversies over the term of “liquefaction”, even when applied to isotropically consolidated samples, whether a saturated sand sample with $D_r=70\%$ “liquefies with limited strain potential” (Prater, 1980) or whether “it is normally considered impossible for cyclic pore pressures to approach or equal the confining pressure” (Casagrande, 1976) is yet another concept to be clarified when applying the term of “initial liquefaction” to anisotropically consolidated triaxial samples (Erguvanli, 1980; Prater, 1980).

In natural or artificial soil deposits, the horizontal effective stresses at rest are about 0.35 to 0.65 of the vertical effective stress, but this value increases above unity, with the degree of overconsolidation, compaction efforts and stabilization measures. Scarcely, in artificial hydraulic fills and in reclaimed areas, the value of coefficient of lateral earth pressure at rest, $K_0=\sigma_d/\sigma_v$, may be around unity (1.0) due to very loose deposition nearly at hydrostatic conditions.

The most important factor occurs to be the assessment of a realistic and valid rela-

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Manuscript was received for review on January 19, 1982.
* Written discussions on these notes should be submitted before January 1, 1984.
rationship between the stress conditions in the field and the conditions created by laboratory experimentation. The development of pore water pressure in cohesionless soils and their liquefaction behaviour is usually investigated in the laboratory with stress-controlled cyclic triaxial tests. If the triaxial test specimen is first consolidated under isotropic confining stress, the cyclic loading completely reverses its direction between triaxial compression and triaxial extension during dynamic testing.

On the other hand, when the triaxial specimen is initially consolidated under anisotropic confining stress, the degree to which cyclic load is reversed depends on the ratio of dynamic stress to initial shear stress. For sufficiently large values of initial shear stress, the load pattern becomes one-way loading and involves no reversal of shear stress. For realistic determination of liquefaction behaviour and the in situ pore water pressure generation of normally consolidated deposits, it may be assumed suitable to carry out dynamic triaxial tests on samples consolidated anisotropically, with principal stress ratios $K_e > 1.5$ (Casagrande, 1976).

It should also be noted that during the process leading to liquefaction, there is an increase or decrease of $(1 - K_e) \sigma_o'$ in the total horizontal stress, where as there is no such stress change in the vertical direction (Ishihara et al., 1977).

**CYCLIC TRIAXIAL TEST RESULTS**

Numerous test results (Casagrande, 1976; Castro and Poulos, 1976; Erguvenli, 1980; Ishihara et al., 1977; Seed, 1976; Silver et al., 1976) have shown that for loose isotropically consolidated samples the pore water pressure rises up to effective confining pressure causing initial liquefaction ($\Delta u_{\text{max}} = \sigma_o' = \sigma_o''$)

For anisotropically consolidated triaxial samples, where $K_e = \sigma_{oc}'/\sigma_{oc}'$, the ultimate pore water pressure increase under cyclic loading has been expressed to be in the form (Erguvenli).

$$\frac{\Delta u_{\text{max}}}{\sigma_o'} = 1 - \frac{3(K_e - 1)}{(1 + 2K_e)} \cdot 1/M$$  \hspace{1cm} (1)

for $K_e > 1.5$ and $M = \frac{6 \sin \phi}{3 - \sin \phi}$

and it is observed not to become equal to initial effective confining stress.

Details of the cyclic testing, the material used and characteristics of the pore-water pressure build up have been given by Erguvenli (1980). A similar build up has also been suggested by Prater (1980). Fig.1 illustrates pore pressure build up in cyclic triaxial tests for loose saturated sand samples under initial isotropic and anisotropic consolidation conditions.

These results are limited to experimental data (Erguvenli, 1980) obtained from the most common and world wide used standard cyclic triaxial tests. Results to be obtained from dynamic simple shear test, under plane strain conditions, may yield different data and may require some other interpretation.

**THEORETICAL EXPLANATION BASED ON ANISOTROPY**

The above experimental observations point out the need for some theoretical and quantitative considerations. The observation that liquefaction in the sense of excess pore water pressure becoming equal to initial effective confining stress does not occur in the triaxial specimens prepared under anisotropic consolidation stresses, implies that insufficient pore pressures are developed in this case. This in turn, implies that the volumetric deformations of the soil system under dynamic loads are smaller in this case than in the isotropically consolidated case where liquefaction occurs. This may be explained by the following analysis.

For soils the moduli of elasticity are in general stress dependent and are monotonically increasing with the confining pressure in each direction:

$$E = E(\sigma)$$  \hspace{1cm} (2)

with

$$\frac{\partial E}{\partial \sigma} > 0$$  \hspace{1cm} (3)

The inequality above means that the higher the stress is in a direction, the higher will
Fig. 1. Pore water pressure buildup under cyclic loading in isotropically and anisotropically consolidated triaxial sand specimens
be the value of the Young's modulus in that direction. At the end of the initial consolidation state, pore-water has had enough time to drain and hence no excess pore pressures exist. Consequently the initial volumetric deformation is only a reference and it is the difference from this state that causes the pore pressure build up. Therefore the static parts \( \sigma_1^s \) and \( \sigma_2^s \) in the stresses determine the elasticity moduli and strength but do not enter into the dynamics that induce pore pressures. In triaxial testing, under unequal static pressures \( \sigma_1^s = \sigma_3^s \) soil consolidates as an anisotropic sample.

For the isotropic experiments, the static pressures are all equal:
\[
\sigma_1^s = \sigma_2^s = \sigma_3^s = \sigma_0 \quad \text{and} \quad E_0 = E(\sigma_0) \quad \text{(4)}
\]

In the experiments with anisotropic consolidation considered here, we have \( \sigma_1^s + \sigma_2^s + \sigma_3^s = 3 \sigma_0 \) with \( \sigma_2^s = \sigma_3^s < \sigma_1^s \) and \( \sigma_0 \) being equal to the pressures applied in the isotropic samples. In view of the relation and inequality above, we have \( \sigma_1^s > \sigma_0 \) and \( \sigma_2^s < \sigma_0 \).

Moreover for the triaxial test specimens with initial anisotropic confining pressures:
\[
E_1 = E(\sigma_1^s) \quad \text{and} \quad E_3 = E(\sigma_3^s) \quad \text{as} \quad \sigma_3^s = \sigma_2^s \quad \text{in the triaxial set up.} \quad \text{(5)}
\]

More explicitly, with the identities \( \sigma_1^s = \sigma_0 - (\sigma_1^s - \sigma_0) \) and \( \sigma_2^s = \sigma_0 - (\sigma_2^s - \sigma_0) \) one has the Taylor series expansions near the isotropic state as:
\[
E_1 = E_0 + \frac{\partial E}{\partial \sigma} \bigg|_{\sigma_0} (\sigma_1^s - \sigma_0) > E_0 \quad \text{(6)}
\]
\[
E_3 = E_0 + \frac{\partial E}{\partial \sigma} \bigg|_{\sigma_0} (\sigma_2^s - \sigma_0) < E_0 \quad \text{(6)}
\]

The anisotropy induced by consolidation in the triaxial experiments discussed above is of orthotropic type. In this case the constitutive relations in the linear elasticity theory are (Lekhnitskii, 1963).
\[
\varepsilon_1 = -\frac{\sigma_1^s}{E_1} - 2\frac{\nu_{12}}{E_1} \sigma_2 \quad \text{and} \quad \varepsilon_3 = -\frac{\sigma_2^s}{E_3} + \frac{1 - \nu_{23}}{E_3} \sigma_2 \quad \text{(7)}
\]

In obtaining the above expressions use is made of the identities \( E_{1,12} = E_{2,12} \) and the axial symmetry which implies \( E_2 = E_3 \), \( \nu_{23} = \nu_{32} \) and \( \nu_{13} = \nu_{12} \).

The use of the dynamic stresses as
\[
\sigma_1^d = \sigma_d \sin \omega t \quad \sigma_2^d = \sigma_d^2 = 0 \quad \text{(8)}
\]
in (7) gives the strain state as:
\[
\varepsilon_1 = \varepsilon_3 = 0 \quad \text{and} \quad \varepsilon_2 = -\frac{\nu_{12}}{E_1} \sigma_d \sin \omega t \quad \text{(9)}
\]

Hence the volumetric strain is:
\[
\Delta v = \frac{1 - 2
\nu_{12}}{E_1} \sigma_d \sin \omega t \quad \text{(10)}
\]

Similarly for the isotropic case we have:
\[
\Delta v = \frac{1 - 2\nu_{12}}{E_0} \sigma_d \sin \omega t \quad \text{(11)}
\]

where the index "o" indicates the isotropic case. With the substitution of \( E_1 \) as above into the expressions for the anisotropic case, the volumetric deformations from the initial confined state is obtained as:
\[
\Delta v = \frac{1 - 2\nu_{12}}{E_0 (1 + \frac{1}{E_0} \frac{\partial E}{\partial \sigma} \bigg|_{\sigma_0} (\sigma_1^s - \sigma_0))} \sigma_d \sin \omega t \quad \text{(12)}
\]

Moreover the Poisson's ratios vary in a small range and can be considered as insensitive to confining pressures. Hence, taking as an approximation \( \nu_{12} = \nu_{23} = \nu_0 \) comparison of (12) with (11) yields:
\[
\frac{\Delta v}{\Delta v_0} = \frac{1}{1 + \frac{1}{E_0} \frac{\partial E}{\partial \sigma} \bigg|_{\sigma_0} (\sigma_1^s - \sigma_0)} < 1 \quad \text{(13)}
\]

The above analysis demonstrates that in triaxial specimens the volumetric deformation for the anisotropically confined case is smaller than the isotropically confined case.

The dependence of moduli of elasticity on confining stress indicated in Eq. (2) is experimentally shown to be defined with the following expression for cohesionless soils
\[
E = kP_a \left( \frac{\sigma_0}{P_a} \right)^n \quad \text{(14)}
\]

where \( \sigma_0 \) is the mean confining pressure, \( P_a \) is the atmospheric pressure, \( k \) and \( n \) are constants. Using this expression (13) can be rewritten as
\[ \Delta v = \frac{1}{1 + n (\sigma_{1}' / \sigma_0 - 1)} < 1 \]  
(15)

For the sand used in this investigation, the exponent \( n \) in (14) is determined to be equal to 0.80, then for the results shown in Fig. 1, taking \( \sigma_{1}' / \sigma_0 = 1.5 \), (15) yields \( \Delta v / \Delta v_0 = 0.81 \) which seems to be consistent with the measured ultimate pore pressure increase under cyclic triaxial loading.

This smaller volumetric change tendency of anisotropically consolidated triaxial samples induces a lesser pore water pressure buildup in comparison with the isotropically consolidated cases. This may be an explanation of the lack of the observation of liquefaction, for samples subjected to dynamic loads under anisotropic initial static stress conditions without shear stress reversal.

CONCLUSIONS

Experimental research has shown that the existence of initial shear stresses due to anisotropic consolidation pressures during specimen preparation, produces a more stable and less mobile soil structure (in comparison with isotropic consolidation under the same mean confining stress, \( \sigma_0 \)) and soils under anisotropic consolidation conditions are more resistant to initial liquefaction, depending on the degree to which shear stresses are reversed.

The proceeding explanation is certainly limited in its prediction of liquefaction. It only states that under cyclic triaxial loading conditions the dynamic volumetric deformations and hence the pore water pressure build up in anisotropically consolidated sand samples are smaller than in the isotropically consolidated samples.

This may be stated, in other words, that the anisotropically consolidated sands are less prone to liquefaction than isotropically consolidated sands under equal static mean confining pressures and dynamic loads.

NOTATION

- \( E_0 \): elastic modulus corresponding to isotropic state of stress
- \( E_i \): elastic moduli in principal stress directions
- \( K_0 \): coefficient of lateral earth pressure at rest
- \( K_c \): consolidation stress ratio
- \( N \): number of cycles
- \( N_c \): number of cycles to reach liquefaction
- \( N_m \): number of cycles to reach maximum pore pressure increase
- \( P_a \): atmospheric pressure
- \( \Delta u_{\text{max}} \): maximum pore pressure increase
- \( \Delta v \): volume change
- \( \Delta v_0 \): volume change under hydrostatic stress conditions
- \( \sigma_0 \): hydrostatic stress
- \( \sigma_1 \): effective principal stresses
- \( \sigma_{m'} \): mean confining stress
- \( \sigma_{v'} \): effective vertical stress
- \( \sigma_{i'} \): static principal stresses
- \( \sigma_d \): dynamic stress
- \( \sigma_f \): dynamic stresses in principal stress directions
- \( \varepsilon_i \): principal strains
- \( \nu \): Poisson's ratio in various principal stress directions
- \( \omega \): frequency of loading

REFERENCES

SIMPLE OPTIMIZATION TECHNIQUES FOR EVALUATING DEFORMATION MODULI FROM FIELD OBSERVATIONS

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ABSTRACT

A numerical procedure consisting of a combination of the finite element method and the mathematical programming, is proposed for estimating material constants of soil deposit based on field measurements. The procedure allows to correct the unknown material constants in a way that the differences between observed and the estimated values decrease sufficiently. Some examples of estimating elastic constants are also presented.

Key words: computer application, elasticity, excavation, field test, finite element method, measurement, natural ground (1GC: E2/H0)

INTRODUCTION

The most difficult aspect in analysing the elastic deformation of a soil deposit is the estimation of the material constants such as Young’s modulus and Poisson’s ratio. The

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Manuscript was received for review on August 4, 1982.