UNDRAINED DEFORMATION CHARACTERISTICS OF
SAND IN MULTI-DIRECTIONAL SHEAR

YASUO YAMADA* and KENJI ISHIHARA**

ABSTRACT

In order to study the deformation and liquefaction characteristics of sand subjected to
cyclic stresses involving changes not only in magnitude but also in direction, several series
of static cyclic tests were performed on cubical saturated sand specimens in undrained conditions
using a true triaxial test apparatus. The octahedral plane in the three-dimensional stress
space was envisaged as representing the horizontal plane in the in-situ sand deposit, and
three principal stresses were cyclically applied to the specimens so that circular, elliptic and
crisscrossing stress paths could be produced in the octahedral stress plane. The analysis of
stress and strain increment vectors on the octahedral plane showed that the sand manifests
the deformation characteristics like an elastic body at the beginning of cyclic loading where
the developed pore water pressure is still small, but at the end of the cyclic loading near the
incidence of liquefaction the sand tends to behave more like a perfectly-plastic body. The
results of the tests also indicated that the resistance to liquefaction became smaller as the
stress paths changed from the straight-line to the ellipse and further to the circle.

Key words: anisotropy, liquefaction, pore pressure, sand, stress path, stress-strain curve
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INTRODUCTION

Basic knowledge on the deformation characteristics of soil under three-dimensional stress conditions is required to obtain in-depth understandings of the mechanism underlying pore water pressure buildup or volume changes taking place during cyclic loading. In view of this, a testing program has been under way at the University of Tokyo using a true triaxial test apparatus (Yamada and Ishihara, 1979, 1981, 1982). In mapping out the test program it was considered appropriate to draw attention to the stress changes that occur on the octahedral plane in the cubic specimen of the true triaxial test device. The application of the three principal stresses was controlled so that any desired pattern of cyclic loads can be induced on the fixed octahedral plane. This plane was envisaged as representing the horizontal plane in the sand deposit and therefore the multi-directional shear stress changes on this plane were considered to simulate the excursions of shear stress during earthquakes. A cyclic test of this type

* Lecturer of Civil Engineering, University of Tokyo, Bunkyo-ku, Tokyo.
** Professor of Civil Engineering, ditto.
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Fig. 1. Representation of stress conditions on the octahedral plane

using a true triaxial test device was performed by Wood (1973) on a kaolin clay employing circular stress paths on the octahedral stress plane. Similar tests were performed by Ishihara and Yamada (1981) on a loose saturated sand employing a wider variety in the pattern of the shear stress path on the octahedral stress plane. The results of the tests showed that the resistance of sand to liquefaction is significantly influenced by the pattern of the multi-directional loading on the octahedral stress plane with the resistance to liquefaction becoming smaller as the stress path changes from a straight-line to an ellipse and further to a circle.

The results of these tests produced some additional data leading to a clarification of the more basic mechanism of sand deformation taking place in a three-dimensional state of stress. This paper is intended to report on the analysis of the test data in this aspect.

REPRESENTATION OF STRESS AND STRAIN

Fig.1(a) shows a rectangular coordinate system in which the \( \sigma_z \) axis is chosen vertically to coincide with the direction in which the specimen was poured. The three-dimensional state of stress can also be expressed in terms of the normal and tangential components of stresses acting on the octahedral plane defined by \( \sigma_x + \sigma_y + \sigma_z = \text{const.} \). The normal component, \( \sigma \), and the tangential component, \( \tau_{oct} \), are usually called "mean principal stress" and "octahedral shear stress", respectively, and are given by

\[
p = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) \tag{1}
\]

\[
\tau_{oct} = \frac{1}{3} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2} \tag{2}
\]

Where stresses imply effective stresses they will be designated with a prime. In order to specify three-dimensional stress conditions it is necessary to introduce another variable, \( \theta \), which determines the direction of the shear stress on the octahedral plane as shown in Fig.1. This variable may also be expressed as a function of the principal stresses in the following manner

\[
\tan \theta = \frac{\sqrt{3} (\sigma_y - \sigma_x)}{2\sigma_z - \sigma_x - \sigma_y} \tag{3}
\]

In conformity with the aforementioned definition for the stress state, the volumetric strain, \( \nu \), the octahedral shear strain, \( \tau_{oct} \), and an angular parameter for strain, \( \omega \), are defined as follows:

\[
\nu = \varepsilon_x + \varepsilon_y + \varepsilon_z \tag{4}
\]

\[
\tau_{oct} = \frac{2}{3} \sqrt{\varepsilon_x - \varepsilon_y}^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 \tag{5}
\]

\[
\tan \omega = \frac{\sqrt{3} (\varepsilon_y - \varepsilon_x)}{2\varepsilon_z - \varepsilon_x - \varepsilon_y} \tag{6}
\]

where \( \varepsilon_x, \varepsilon_y \) and \( \varepsilon_z \) are the strain components in the \( x, y \) and \( z \) directions, respectively, and are positive in the case of compressive strains.

In representing states of stress at several stages of loading, stress paths plotting the octahedral stress, \( \tau_{oct} \) versus the mean principal stress, \( \sigma \), will be used. In some cases, an oblique presentation of the coordinate system will be used to visualize overall features of stress changes (see Fig.3). The behavior of strains will be represented by a polar coordinate system consisting of strain vectors projected on the octahedral plane in the rectangular \( \varepsilon_x - \varepsilon_y - \varepsilon_z \) strain space. A
strain vector on the octahedral plane can easily be constructed by laying off numerical values of $\gamma_{\text{oct}}$ and $\omega$ calculated by Eqs. (5) and (6) in the radial and circumferential directions, respectively. In the case of undrained tests, there occurs no volume change in the specimen. Therefore, when a point is plotted on the octahedral plane in terms of $\gamma_{\text{oct}}$ and $\omega$, it means that all three components $\varepsilon_x$, $\varepsilon_y$ and $\varepsilon_z$ are known because $\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$. Consequently, it is possible to put numerical scale on each of the strain component axes as indicated in Figs.3 through 8 in the following pages. Scaling relationship between each strain component and octahedral strain can be obtained by putting, for example, $\varepsilon_y = \varepsilon_x$ in Eq. (5). Using the relationship $\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$, one obtains $\gamma_{\text{oct}} = \sqrt{2} \varepsilon_x$. Similar relationships can be obtained for other strain components, $\varepsilon_x$ and $\varepsilon_y$. The above relationship means that the scale in $\varepsilon_x$-axis should be 1.414 times larger than the scale of $\gamma_{\text{oct}}$ taken in the radial direction. The scaling in Figs. 3 to 8 is made in this proportion.

APPARATUS AND MATERIAL

The test apparatus used in this study was a true triaxial apparatus which is capable of applying three principal stresses independently to a specimen of 100 mm x 100 mm x 100 mm by means of three pairs of rubber pressure bags. Details of the equipment and its operation are described elsewhere (Yamada and Ishihara, 1979).

The sand taken from Fuji river bed near Tokyo was used in this investigation. It consisted of subrounded to subangular particles with a specific gravity of 2.73. The mean diameter of the sand was 0.40 mm and uniformity coefficient was 2.14. The maximum and minimum void ratios were 1.03 and 0.48, respectively. The saturated sand was poured vertically into the cubical cell filled with deaired water to form a specimen. The specimen was compacted by tapping the side of the container so that the specimen would have a desired void ratio between 0.82 to 0.86 after isotropic consolidation of 98 kN/m².

TEST SCHEME

In this investigation the specimens were first consolidated to an effective confining pressure of 98 kN/m². After consolidation a back pressure of 49 kN/m² was applied to the specimens under undrained conditions to ensure full saturation. The specimens were then subjected to cyclic stresses under undrained conditions keeping the total mean principal stress at a constant value of 147 kN/m². By measuring the pore water pressure, the effective mean principal stress, $p'$, was determined at each stage of the loading.

In the test scheme adopted in this study, the shear stress changes in the level ground during earthquakes were simulated by stress changes in the octahedral plane within a cubic specimen. The stress paths employed are shown in Fig. 2. They consisted of three groups, uni-directional paths, alternate criss-crossing paths and rotational stress paths.

Two kinds of stress paths were adopted for the uni-directional loading tests as illustrated in Fig.2(a) and 2(b). The stress path shown in Fig. 2(a) corresponds to the conventional type cyclic loading scheme executed alternately in compression and extension using a triaxial test apparatus. In this type of cyclic loading, the deformation characteristics of the specimen are significantly different between compression (ZC) and extension (ZE) sides due to the anisotropic struc-
ture of the specimen and the stress state represented by \( \delta = (\sigma'_x - \sigma'_y)/(\sigma'_x - \sigma'_y) \) (Yamada and Ishihara, 1979 and 1981). In order to perform tests free from this anisotropic behavior, another straight-line stress path shown in Fig. 2(b) being perpendicular to that of conventional cyclic triaxial test, was also adopted. In this stress path the stress conditions on both sides of the origin, designated by RS 90° and RS 270°, are identical having a \( b \)-value of 0.5.

The tests with the crisscrossing stress path employed two uni-directional cyclic loadings which were executed alternately in two mutually perpendicular directions as shown in Figs. 2(c) and 2(d). In one of the crisscrossing shear tests, the ratio of the cyclic shear stress amplitude between the two directions was set at one to one (Fig. 2(c)), while in the other tests the ratio was taken as two to one (Fig. 2(d)).

Three kinds of rotational shear stress path were adopted in this study as illustrated in Figs. 2(e) to 2(g). The stress path shown in Fig. 2(e) has a circular shape and the other stress paths shown in Figs. 2(f) and 2(g) have an elliptic shape with the ratio of the shear stress amplitude between the longest axis and the shortest axis of two to one. These two elliptic stress paths were adopted so as to compare the effects of the anisotropy formed in the specimen, when it was prepared by the method of deposition in water.

For each of the above stress paths, a set of cyclic loading tests, each having different shear stress amplitude, was carried out and the effects of stress path on the deformation and liquefaction characteristics of the sand were examined.

Fig. 3. Measured stress path and strain paths for a circular loading (\( \tau_{\text{act}} = 12.5 \text{ kN/m}^2, e_0 = 0.822 \))

Fig. 4. Measured strain paths for a uni-directional loading
Fig. 5. Measured strain paths for a crisscrossing loading

Fig. 6. Measured strain paths for a crisscrossing loading

Fig. 7. Measured strain paths for an elliptic loading

Fig. 8. Measured strain paths for an elliptic loading
RESULTS OF CYCLIC TESTS

Typical cyclic tests results for a specimen with a relative density of 38% are shown in Fig. 3 for the case of the circular stress path. Fig. 3(a) shows the effective stress path plotted in three-dimensional stress space. After isotropic consolidation the specimen was subjected to a radial shear stress along the $\sigma_r$-axis shown in Fig. 2(e) tracing the effective stress path from point 0 to point 1 as illustrated in Fig. 3(a). The cyclic load application along the circular stress path started at point 1 and stopped at point 7 which led to the decrease in effective confining stress with increasing cycles by following a spiral stress path in three-dimensional stress space. The shear stress was then
unloaded from point 7 to point B, whereupon the effective confining stress became nearly equal to zero. Based on the above test result, the strain path was also constructed in the diagram of octahedral plane as shown in Fig. 3(b). The strain path beyond point A is shown in Fig. 3(c) with a reduced scale.

In these figures the scale of the octahedral shear strain, \( \tau_{oct} \), defined by Eq. (5) is also shown in a radial direction as well as the scale for each component of strain, \( \epsilon_x, \epsilon_y \), and \( \epsilon_z \).

Typical strain paths in the octahedral plane for the other loading paths are plotted in
Fig. 11. Effective mean principal stress versus octahedral shear stress for crisscrossing loadings

Figs. 4 to 8. In this set of figures, figure (a) refers to the plots of strain path up to about 0.5% of octahedral shear strain, and figure (b) shows with a contracted scale the strain path for the larger strain range up to about a few percent close to the state of liquefaction. Fig. 4 shows the strain path obtained in a uni-directional cyclic loading test carried out along the RS 90°-RS 270° stress path shown in Fig. 2(b). The strain path obtained from another type of uni-directional cyclic test along the ZC-ZE stress path (Fig. 2(a)) is not shown here, because the strain path simply goes back and forth along the ε_ε-axis. Figs. 5 and 6 show the strain paths for the cases of two types of crisscrossing loading tests of which the stress paths are shown in Figs. 2(c) and 2(d),
Fig. 12. Effective mean principal stress versus octahedral shear stress for crisscrossing loadings

respectively. Typical strain paths of the elliptic loading tests are plotted in Figs. 7 and 8 for the tests with loading pattern shown in Figs. 2(f) and 2(g), respectively. In Figs. 7 and 8 the strain points corresponding to the longest axes of elliptic stress path are indicated by solid circles.

Effective stress paths observed in the above cyclic tests are shown in Figs. 9 to 15. When plotting the octahedral shear stress, the value of \( \tau_{\text{oct}} \) was modified by the following formulas.

\[
\tau_{\text{oct}}^* = \tau_{\text{oct}} \times \cos \theta \quad (7)
\]

\[
\tau_{\text{oct}}^{**} = \tau_{\text{oct}} \times \sin \theta \quad (8)
\]

where the angle, \( \theta \), indicates the direction of the shear stress in the octahedral plane as illustrated in Fig. 1. The value of \( \tau_{\text{oct}}^* \) means the projection of \( \tau_{\text{oct}} \) in the vertical
direction in the octahedral plane, while the value \( \tau_{oct}^{**} \) means the projection of \( \tau_{oct} \) in the horizontal direction in the octahedral plane. In establishing Figs. 9, 13 and 14, the modified value, \( \tau_{oct}^{*} \) was plotted in the ordinate so that the shear stress corresponding to the longest axis of the stress path coincides with the vertical axis in these figures. For the results of tests with the longest axis of the shear stress path directed in the horizontal direction, the modified value \( \tau_{oct}^{**} \) was employed as shown in Figs. 10 and 15. In Figs. 11 and 12 showing the results of the crisscrossing loading tests, values of \( \tau_{oct}^{*} \) were plotted with solid lines for the vertically directed portion of the stress path and for the portion of the stress path going horizontally in the octahedral
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Fig. 14. Effective mean principal stress versus octahedral shear stress for elliptic loadings

plane of Fig. 2 values of \( \tau_{oct} \) were plotted with dashed lines. In the plot of the test results employing rotational loading paths, (Figs. 13, 14 and 15), the stress paths corresponding to the portion of monotonic loading at the beginning and monotonic unloading at the end of the rotational cyclic loading are indicated with broken lines.

DEFORMATION CHARACTERISTICS DURING CYCLIC LOADING

In the cases of cyclic loading tests with straight-line and crisscrossing stress paths, when the amplitude of the applied shear stress was relatively small, the amount of increase in pore water pressure in the first cycle was large compared to the pore water
Fig. 15. Effective mean principal stress versus octahedral shear stress for elliptic loadings

pressure developing in the subsequent cycles as may be seen in Figs. 9(a)(b), 10(a)(b), 11(a)(b) and 12(a)(b). In the meanwhile the strain path traced a similar pattern cyclically although the amplitude of strain increased gradually as shown in Figs. 4, 5 and 6. This is because of the increase in effective stress ratio, \( \tau_{\text{oct}}/\sigma' \), due to the increase in pore water pressure as illustrated in Figs. 9 to 12. As the load is cycled several times the pore water pressure increases gradually and the effective stress ratio, \( \tau_{\text{oct}}/\sigma' \), eventually reaches a value corresponding to the angle of phase transformation (Ishihara, Tatsuoka and Yasuda, 1975), whereupon a significant increase in pore water pressure occurs during the subsequent unloading as shown in Figs. 9 to 12. At this point the specimens reached the
state of initial liquefaction where the effective mean principal stress was equal to zero. As the stress ratio approached the value corresponding to the angle of phase transformation, the shear strain also increased significantly as shown in Figs. 4, 5 and 6.

In the tests with straight-line and criss-crossing stress paths of relatively large amplitude of shear stress, the pore water pressure increased significantly from the first cycle and the effective stress ratio, \( \tau_{\text{ost}}/\rho' \), reached a value corresponding to the angle of phase transformation in a few cycles. During the subsequent unloading process the pore water pressure increased a great deal to make the specimen reach the state of initial liquefaction.

It is noteworthy in the tests with straight-line stress path directed vertically in the octahedral plane (Fig. 2(a)) that the pore water pressure built up more pronouncedly when the cyclic stress application was executed on the side of triaxial extension than when it was applied on the side of triaxial compression as illustrated in Fig. 9. This is mainly due to the anisotropic structure of the specimen formed by deposition of sand, although the stress conditions designated by \( b \)-value are also different between ZC and ZE directions. It was shown that, in the range of shear stresses not sufficient to cause maximum volume contraction, the amount of pore water pressure developed for some given octahedral shear stress during monotonic loading tests employing a radial stress path increased as the angle of shearing direction, \( \theta \), changed from zero to 180 degrees irrespective of the \( b \)-value (Yamada and Ishihara, 1981). This type of anisotropic behavior did not show up in the plots of the stress paths as seen in Fig. 10, when the cyclic stress application was executed along the straight-line stress path directed horizontally in the octahedral plane (Fig. 2(b)). However, the effects of the anisotropy appeared in the plot of strain path shown in Fig. 4, where it may be seen that the strain path shifts downward as a whole in the octahedral plane.

In the cyclic loading tests with the rotational stress paths the pore water pressure was generated in an almost similar manner to that of the tests with a straight-line stress path until the effective stress ratio arrived at a value corresponding to the angle of phase transformation as shown in Figs. 13 to 15. As can be seen in Figs. 13 and 14, the pore water pressure built up more remarkably when the cyclic stress application was executed in the stress conditions corresponding to the lower half of the octahedral plane than when the cyclic stress application was executed on the upper half. This characteristic behavior is again due to the effect of the inherent anisotropy built in to the sand specimens during the sample preparation process. Once the stress ratio arrived at a value corresponding to the phase transformation, the observed effective stress paths with rotational stress paths were entirely different from those for the straight-line stress paths. As can be seen in Figs. 13 to 15 the pore water pressure never becomes equal to the initial confining pressure, and the corresponding stress path moves back and forth along almost the same loop. It should be noticed in Fig. 13 that, in the loops observed in the tests with a circular stress path, the pore water pressure decreased as the stress changed from the state of triaxial compression defined by \( b=0 \) (\( \theta=0 \), 120 and 240 degrees) to the state of triaxial extension defined by \( b=1 \) (\( \theta=60 \), 180 and 300 degrees), while it increased as the stress changed from the state of triaxial extension to compression. This is associated with the fact that the effective stress ratio, \( \tau_{\text{ost}}/\rho' \), corresponding to the angles of phase transformation decreases as the \( b \)-value increases from 0 to 1 (Yamada and Ishihara, 1981). In the tests with a rotational stress path it was not until the shear stress was brought back to zero that the pore water pressure became equal to the confining pressure, the state of initial liquefaction. This phenomenon was also recognized in the rotational simple shear tests performed by Ishihara and Yamazaki (1980).

It may be seen in Figs. 3(b)(c), 7 and 8,
Fig. 16. Schematic illustration of stress and strain increment vectors

Fig. 17. Strain increment vectors along a circular stress path

that, in the range where the pore water pressure does not sufficiently build up the strain paths for tests with a rotational stress path trace almost similar loops to those of the applied stress paths, but that the diameters of the strain paths become gradually larger with the increase in pore water pressure. It is also to be noted in these figures that the strain paths tend to drift downward in the octahedral plane because of the inherent anisotropy of the specimens. After arriving at the stress state corresponding to the angle of phase transformation the specimens produce large strains as shown in Figs. 3, 7 and 8.

In order to visualize the relationship between the applied shear stresses and resulting shear strains during the tests of the rotational shear, the angle on the octahedral plane between the directions of strain increment vector and stress increment vector was computed. This angle, denoted by $\beta$, is illustrated schematically in Fig.16. For the case of the circular loading test, the vectors of octahedral strain increment were computed from the test results shown in Fig.3 at an angle interval of 15 degrees and laid off at the corresponding stress point on the octahedral plane as shown in Fig.17. For example, an increment of octahedral strains obtained from the start of the test (ZC-state) to the state 15 degrees away clockwise is laid off in a vector form at a stress point midway between zero and 15 degrees, i.e., at 7.5 degrees stress point. Likewise, the octahedral strain increment vectors are successively plotted round the circle. In Fig.17, the strain increment vectors obtained in the first circular loading are plotted with dashed-line arrows with a scale indicated in the inset. The octahedral strain increment vectors in the third cycle are indicated by solid-line arrows with a scale reduced to one-tenth. The stress vectors radiating from the origin on the octahedral plane are also indicated in Fig.17, while the stress increment vector can be established simply by connecting the ends of the two neighboring stress vectors. Turning attention to the behavior in the first cycle, one can see in Fig.17 that the strain increment vector deviates only slightly from the stress increment vector within the range of angle zero to 90°, but the deviation between the two vectors become pronounced as the angle increases up to 180 degrees where a state of triaxial extension is encountered. After passing through this state of stress, the direction of the octahedral strain increment vector becomes closer to that of the stress increment vector as the angle returns to the original state. It is to be noted further that the magnitude of the octahedral strain vector takes its greatest value at the state of triaxial
extension. The specific behavior as above observed in the vicinity of the triaxial extension is considered due to the anisotropic nature of the cubic specimen which is more compressible on the triaxial extension side (ZE-test) than on the triaxial compression side (ZC-test). The direction of the octahedral strain increment vector in the third circular loading is diverted farther from the stress increment vector as compared to that observed in the first loading. At the same time, the magnitude of the strain increment is significantly increased. This behavior can be explained by the low values of effective confining pressure remaining in the specimen in the third load cycle. In fact, the pore water pressure behavior of this test demonstrated in Fig. 13(c) shows that, at the beginning of the last load cycle, the effective confining pressure was 38% of the initial consolidation stress and decreased to only 15% at the end of the third cycle. It appears that the effective stress ratio, $\tau_{cel}/p'$, reached a value corresponding to the angle of phase transformation when approximately a quarter cycle had proceeded from the beginning of the third cycle producing a pore water pressure about 70% of the initial confining pressure. The deviation of the directions of the strain and stress increment vectors becomes significantly large, and the effective stress ratio increases beyond a value corresponding to the angle of phase transformation. At this stress state, the magnitude of the octahedral strain increment also increases drastically. For the purpose of visualizing a feature of change in the angle of $\beta$ defined above in a more quantitative manner, the value of $\beta$ is plotted in Fig. 18 for each cycle of shear stress application versus the angle of stress rotation, $\theta$, as measured from the positive $\sigma_z$-axis (see Fig. 1(b)). It may be seen in Fig. 18 that, during the first and second cycles where the buildup of pore water pressure is still small accompanied by small strains, the angle, $\beta$, takes values between 10 and 50 degrees reaching a peak at $\theta=180$ degrees (ZE-state). It may also be noted that the value of $\beta$ becomes significantly larger in the third cycle. If the direction of the strain increment vector coincides with that of the stress increment vector ($\beta=0$), this implies that the strain produced at each stage of load increment is determined solely by changes in stress irrespective of the current state of stress. As is well known, this situation is encountered when material behavior
is elastic. On the other hand, when changes in strain occur depending only on the current state of stress irrespective of the stress increment ($\beta=90^\circ$), the material behavior is referred to as perfectly-plastic. Consequently, it can be concluded that, at an early stage of loading where pore water pressure build-up is still small, the behavior of sand resembles that of an elastic material, but as the pore water pressure increases in the course of rotational shear, the sand tends to behave more like a perfectly-plastic material.

In Fig.18 it is also recognized that during the last cycle the angle, $\beta$, exhibits three peaks at $\theta$-values of 60, 180 and 300 degrees where $b=1.0$. This may be explained by the fact that the stress ratio, $\tau_{oct}/p'$, corresponding to the angle of phase transformation takes the smallest value in the shearing directions having a $b$-value of 1.0, i.e. triaxial extension states.

Shown in Fig.19 are octahedral shear strain increment vectors obtained from a test of rotational shear employing an elliptic stress path in the octahedral plane. Details of the test results are already shown in Fig.7 and 14(b). In Fig.19, the strain increment vectors are plotted at an angle interval of 30 degrees at a midpoint between two successive stress points where the strain increment was calculated. The stress vectors at the midpoint are also indicated in the figure. It may be seen that in the first half cycle, the strain increments are directed nearly parallel to the stress increments at the beginning, but as the stress application goes around, these two increment vectors tend to deviate. This is indicative of the inherent anisotropy of the test specimen. As the stress changes from the point of 180 degrees back to the original state, the strain increment vector becomes again nearly parallel to the stress increment vector. As for the data in the last cycle, only those portions beginning from the 180 degree point are plotted in Fig.19. It may generally be seen that the octahedral strain increment vectors are more outwardly directed, indicating that the angle, $\beta$, defined in Fig.16 is larger than that in the first rotational load cycle. This indicates that the behavior of the sand in the first cycle is close to that of an elastic material, whereas in the last cycle the sand characteristic is more like a perfectly-plastic body. Fig.14(b) shows that at the beginning of the last cycle, the pore water pressure had developed about 70% of the initial confining pressure. Therefore, the specimen had been brought to a state of the effective stress ratio, $\tau_{oct}/p'$, exceeding a value corresponding to the angle of phase transformation. Consequently, it may be concluded that if the mobilized angle of shear stress exceeds the angle of phase transfor-

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**Fig. 120.** Cyclic stress ratio versus number of cycles for loadings predominantly oriented in the ZC-ZE direction

**Fig. 121.** Cyclic stress ratio versus number of cycles for loadings predominantly oriented in the RS $90^\circ$-RS $270^\circ$ direction
mation the sand tends to behave more like a perfectly-plastic body.

LIQUEFACTION IN THREE-DIMENSIONAL STRESSES

In the cyclic tests with straight-line stress paths in the octahedral plane (Figs. 2(a) to 2(d)), it is possible to read off the numbers of cycles to cause initial liquefaction from the effective stress paths such as shown in Figs. 9 to 12. In the cyclic tests with rotational stress paths (Figs. 2(e) to 2(g)), however, the pore water pressure did not become equal to the confining pressure even if the cycle of rotational loading was executed a number of times as seen in Figs. 13 to 15. Therefore the numbers of cycles at which the pore water pressure began to increase or decrease repetitively were taken as those required to cause liquefaction. If the applied shear stress was removed at this stage the pore water pressure immediately increased to induce initial liquefaction in the specimen as described in the preceding section.

The numbers of cycles required to cause liquefaction thus determined for each loading path are plotted versus the cyclic stress ratio, \( \tau_{ec}/\rho' \), in Figs. 20 and 21. Fig. 20 shows the results of the tests with the stress paths whose longest axis coincides with the vertical direction of the octahedral plane (Figs. 2(a), (c), (e) and (f)), whereas Fig. 21 shows the results of the tests with the stress paths whose longest axis coincides with the horizontal direction of the octahedral plane (Figs. 2(b), (c), (d), (e) and (g)). The results of the tests having the same amplitude of shear stress in the vertical and horizontal directions in the octahedral plane (Figs. 2(c) and (e)) are plotted in both figures. It is to be noted that in the tests with crisscrossing stress paths (Figs. 2(c) and (d)) a set of loadings carried out successively along the two perpendicular directions is counted as one cycle.

It is apparent in Figs. 20 and 21 that the resistance of sand to liquefaction decreases as the stress path changes its configuration from uni-directional, elliptic, crisscrossing to circular shape, i.e. the resistance to liquefaction decreases as the magnitude of shear stress in the direction perpendicular to the axis of the largest shear stress increases. Similar observations can be made when comparing the resistance to liquefaction between the tests with uni-directional, elliptic and circular stress paths.

It is interesting to compare the resistance to liquefaction obtained from tests with the same shape of stress path but with a different direction for the longest axis of shear stress application. In both uni-directional and elliptic loadings the resistance of

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**Fig. 22.** Cyclic stress ratio versus number of cycles averaged for different loading paths

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**Fig. 23.** Cyclic strength in the rotational and alternate shears normalized to the cyclic strength in the uni-directional shear
sand to liquefaction is greater for the tests with the longest axis of shear stress directed horizontally in the octahedral plane (Fig. 21) than for the tests with that directed vertically in the octahedral plane (Fig. 20). This characteristic comes from the anisotropic behavior of the sand specimen formed in the laboratory via the method of deposition under water. (Yamada and Ishihara, 1981). This anisotropic behavior is manifested in the pore water pressure buildup shown in Figs. 9 to 15. The average values of the cyclic resistance obtained in the two classes of tests which may eliminate the effect of inherent anisotropy of specimens were calculated based on the results shown in Figs. 20 and 21, and are plotted in Fig. 22. It is apparent in Fig. 22 that the resistance of sand to liquefaction decreases as the stress path changes its configuration from uni-directional, elliptic, crisscrossing to circular shape, which is a similar tendency to that observed in Figs. 20 and 21.

In order to examine the effect of cyclic stress application in the direction perpendicular to the major loading direction, the cyclic stress ratios in the direction of larger load amplitude required to induce initial liquefaction in 2, 5, 10 and 20 cycles for the crisscrossing, elliptic and circular loading conditions were normalized to the cyclic stress ratio similarly defined for the unidirectional loading conditions using the curves plotted in Fig. 22. These ratio are plotted in Fig. 23 versus the ratio of the cyclic stress amplitude between the two directions. Also shown in this figure are the results of the multi-directional simple shear test reported previously (Ishihara and Yamazaki, 1980). It can be seen in the results of both types of tests that the resistance of sand to liquefaction decreases as the cyclic shear stress amplitude in the direction perpendicular to the direction of major loading increases. However, this tendency is more marked in the multi-directional simple shear tests than in the true triaxial test as indicated in Fig. 23.

CONCLUSIONS

By means of a true triaxial test equipment several series of cyclic loading tests were conducted on cubic specimens of loose sand. The application of the three principal stresses was regulated so that rotational and crisscrossing stress paths were induced in the octahedral stress plane in the cubic specimens. Detailed analysis of the rotational test results indicated that the octahedral strain increment vector is directed almost in the same direction as the corresponding stress increment vector on the octahedral plane at an early phase of the cyclic loading when the pore water pressure buildup is still small, but that the strain increment vector tends to lag behind the direction of the stress increment towards the end of the cyclic loading and the incidence of liquefaction. This fact appears to indicate that, when the effective shear stress ratio stays below a value corresponding to the angle of phase transformation at an early stage of cyclic loading, the sand behaves nearly as if it were an elastic material, whereas it behaves as though it were a perfectly-plastic material when the effective shear stress ratio is increased, near the end of the cyclic loading, beyond a value of the angle of phase transformation.

The test results also showed that the cyclic stress ratio causing initial liquefaction in the specimens in a given number of cycles is largest for the loading with uniaxial stress path and smallest for the loading with circular stress path.

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DEFORMATION IN MULTI-DIRECTIONAL SHEAR


