TECHNICAL NOTE

INFLUENCE OF ANISOTROPY OF DEFORMATION MODULUS ON EFFECTIVE STRESS PATH

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ABSTRACT

An influence of anisotropy of deformation modulus on the initial gradient of effective stress paths of cohesive soils is investigated, based on the results of a series of consolidated undrained triaxial compression and extension tests on five kinds of normally consolidated cohesive soils. The correlation of the anisotropy of deformation modulus and the initial gradient of effective stress paths in K_0 consolidated samples is then examined by assuming that the soil is a cross-anisotropic elastic body.

It is found that the initial gradient of effective stress path in K_0 consolidated samples can be predicted qualitatively by introducing the anisotropic effects of deformation modulus of saturated cohesive soils into cross-anisotropic elastic body.

Key words : anisotropy, cohesive soil, consolidated undrained shear, deformation, plasticity, stress path, stress-strain curve, triaxial compression test (IGC : D6)

INTRODUCTION

It has been recognized that the undrained shear strength anisotropy becomes more marked as the plasticity index of soil decreases (Bjerrum, 1973; Ladd et al., 1977; Nakase and Kamei, 1983). This fact suggests that there would exist a similar trend in an anisotropy in the deformation modulus of soils. Deformation characteristics of soils may be most simply indicated by the secant modulus E_{50} from the stress-strain curve of shear tests.

Then the values of the secant modulus

E_{50} are examined from results of a series of triaxial tests on five kinds of cohesive soils having different values of the plasticity index I_p . An attempt is made to correlate the deformation modulus anisotropy with the initial gradient of effective stress paths.

EXPERIMENTS AND SOIL SAMPLES

A series of consolidated undrained triaxial compression and extension tests are performed on five different soil samples. In the compression tests, specimens are consolidated isotropically or under K_0 condition

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and then subjected to undrained compression by increasing axial pressure, lateral pressure being kept constant during compression process. In the extension tests, specimens are consolidated isotropically or under K_0 condition and then subjected to undrained extension by decreasing axial pressure, lateral pressure being kept constant during extension process.

Two of the soil samples are Kawasaki clays, and other three are reconstituted by mixing the Kawasaki clay ($I_p=30$), Toyoura sand and partly crushed Toyoura sand. These samples are named M-10, M-15, M-20, M-30 and M-51 respectively after their values of the plasticity index I_p . M-51 and M-30 are the Kawasaki clays. Details of soil samples and testing procedures have been reported previously (Nakase and Kamei, 1983).

TEST RESULTS AND DISCUSSIONS

E_{50}/c_u Value

Figs. 1 and 2 show relationship between the ratio of the secant modulus to the undrained shear strength, E_{50}/c_u , and the plasticity index I_p in the $\overline{\text{CIU}}$ and $\overline{\text{CK}}_0\text{U}$ tests respectively. It has been reported that the value of E_{50}/c_u obtained by compression tests has the same order of magnitude as for any clays (Skempton and Bishop, 1954). In the figures, it will be seen that the E_{50}/c_u values by the extension tests are always

larger than those by the compression tests, and the difference between the two values increases with a decrease in I_p . It can also be seen that the E_{50}/c_u value increases with a decrease in I_p , except for the case of $\overline{\text{CIU}}$ tests, where the value is almost constant irrespective of I_p . This trend is consistent with that reported by Ladd et al. (1977). In addition, the E_{50}/c_u values of the K_0 consolidated samples are always larger than those of the isotropically consolidated samples. This result agrees with that reported by Nakase et al. (1972).

Fig. 1. Relationship between E_{50}/c_u and I_p in $\overline{\text{CIU}}$ tests

Fig. 2. Relationship between E_{50}/c_u and I_p in $\overline{\text{CK}}_0\text{U}$ tests

Fig. 3. Relationships between $(E_{50})_{\max}$, n and I_p in $\overline{\text{CK}}_0\text{U}$ tests (vertical effective consolidation pressure : 392 kPa)

Anisotropy of Undrained Deformation Modulus

The E_{50} values are plotted against I_p for both $\overline{CK}_0UC$ and $\overline{CK}_0UE$ tests in Fig. 3. In the figure, the maximum values of E_{50} obtained for each kind of the samples are shown, since the maximum value of E_{50} is supposed to be nearest to the ideal value.

It is noted that each plot in the figure represents the maximum value of E_{50} obtained from, in general, three test results. It is seen that the values of E_{50} by the compression tests, $(E_{50})_C$, are always larger than those by the extension tests, $(E_{50})_E$, and the difference between the two increases with a decrease in I_p . The ratio n of the moduli, $n = (E_{50})_E / (E_{50})_C$, increases as I_p increases as shown in the figure. It is, therefore, said that the anisotropy in the modulus of deformation does exist and the degree of the anisotropy increases as the plasticity index decreases.

Engineering implication of the result is that soils with low plasticity, in particular, may be treated as a cross-anisotropic elastic body when stress distribution and displacement are estimated. Some indication of the effect of anisotropy on the distribution of vertical stress and vertical displacement beneath a uniformly loaded circle has been given. In his report, Koning (1957) states that for a given value of Young's modulus of vertical direction, the vertical stress spreads deeper and wider and vertical displacement increases as the value of the ratio n decreases. This means that the displacement could be underestimated if the isotropic elastic body is assumed by adopting the Young's modulus of vertical direction.

Effect of Anisotropy on Effective Stress Path

Another evidence of the dependence of degree of anisotropy on I_p may be found by the detailed study of effective stress paths obtained by the standard triaxial tests. In this section, the correlation of the anisotropy of deformation modulus and the initial gradient of effective stress path shall be examined by assuming that the soil is a cross-

anisotropic elastic body. The gradient of effective stress path in terms of $\Delta q / \Delta p'$ in the p - q plane is considered.

The general theory of strain energy is applied to the anisotropically modified Hooke's law and the equation obtained is expanded under axisymmetrical and undrained condition. By adopting the assumption made by Henkel (1971), the following equation is obtained (Parry and Wroth, 1981):

$$\frac{\Delta q}{\Delta p'} = \frac{3\{n'(5\nu_{vH}' - 1) + \nu_{vH}' - 2\}}{(1 - n')(\nu_{vH}' - 2)}$$

where $n' = E_H' / E_V'$, E_H' is equal to Young's modulus of horizontal direction in terms of effective stress, E_V' is equal to Young's modulus of vertical direction in terms of effective stress and ν_{vH}' is equal to Poisson's ratio in terms of effective stress.

The above expression shows that the $\Delta q / \Delta p'$ value can be determined if n' and ν_{vH}' are obtained. If ν_{vH}' is constant, then the Young's modulus in terms of total stress is proportional to that in terms of effective stress, hence n' value is equal to n value. The value of ν_{vH}' is determined by using the figure which shows experimental values of ν' plotted against plasticity index for a number of different soils (Wroth, 1975). As a result, the above equation can be expressed in terms of only n . Thus obtained values of $\Delta q / \Delta p'$, based on the relationship between n and I_p in Fig. 3 are shown in Table 1 and compared with the measured values in the $\overline{CK}_0UC$ tests.

In the case of the samples M-10 and M-15, where the anisotropy is marked the calculated values of $\Delta q / \Delta p'$ compare favourably with the measured values. Typical effective stress paths for the M-10 and M-30 (Kawasaki clay) in the $\overline{CK}_0U$ tests are shown in Figs. 4 and 5, where the principal stress difference q and the mean effective stress p' are normalized by dividing by the effective vertical consolidation pressure σ'_{vc} . As shown in Fig. 4, the calculated value of the initial gradient is very close to the actual one in the $\overline{CK}_0UC$ test of the M-10 which has marked anisotropy.

Fig. 4. Comparison between observed and calculated value in $\overline{CK}_0UC$ test

Fig. 5. Comparison between observed and calculated value in $\overline{CK}_0UC$ test

It seems from the Table 1 that there is a considerable difference between the calculated and observed values of $\Delta q/\Delta p'$. When these values are compared in the plotted stress path, however, they may be considered as within the scatter band of experimental data, as seen in Fig. 5. In the case of the M-51 (Kawasaki clay), the difference is large and the sign of the value is reversed. This might be due to the selection of E_{50} value. In this case of M-51 (Kawasaki clay), the average value of E_{50} obtained from extension tests is greater than that from compression tests, so n is larger than unity and the

Table 1. Comparison between observed and calculated value

Materials	$\Delta q/\Delta p'$	
	Obs.	Cal.
M-51	-16.0	7.74
M-30	22.0	7.56
M-20	13.4	7.41
M-15	7.13	6.84
M-10	5.27	5.25

sign of $\Delta q/\Delta p'$ value becomes minus. Note that the values shown in Fig. 3 are the maximum and not the average of the tests. Therefore, for soil with large I_p , attention should be paid to the selection of E_{50} value or n value. In engineering aspect, however, no inconvenience will be experienced by assuming that the effective stress path rises vertically or n value equal to unity. It is generally considered that the state of normally K_0 consolidated cohesive soil can be expressed as a point on the yield surface and the effective stress path during undrained shear progresses along this surface. But, the actual effective stress path progresses approximately vertical upwards from the point of consolidation. Regarding this, Burland (1971) explained this by the effect of secondary compression, i.e., as the void ratio decreases while mean effective stress is constant, the yield surface extends and results in the rising of the effective stress path from the point of consolidation to the expanded yield surface both vertically and elastically during undrained shear. After the effective stress path has reached the yield surface, plastic strain is produced and it progress along the yield surface.

As a result, by employing the anisotropic effects of deformation modulus of saturated cohesive soils into cross-anisotropic elastic body, the initial gradient of effective stress path can be predicted qualitatively.

CONCLUSION

Undrained deformation modulus of cohesive samples of various I_p values, M-51 (Kawasaki clay), M-30 (Kawasaki clay),

M-20, M-15 and M-10, have been studied by a complete set of consolidated undrained triaxial tests, i. e., $\overline{CIUC}$, $\overline{CIUE}$, $\overline{CK_0UC}$ and $\overline{CK_0UE}$ tests.

The conclusion from this study is that by employing the anisotropic effects of deformation modulus of normally consolidated cohesive soils into cross-anisotropic elastic body, the initial gradient of effective stress path can be predicted qualitatively.

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NOTATION

M = letter M is named after their plasticity index

$\overline{CIUC}$ = isotropically consolidated undrained compression tests

$\overline{CK_0UC}$ = K_0 consolidated undrained compression tests

$\overline{CIUE}$ = isotropically consolidated undrained extension tests

$\overline{CK_0UE}$ = K_0 consolidated undrained extension tests

p' = mean effective stress : $(\sigma_a' + 2\sigma_r')/3$

q = deviator stress : $\sigma_a - \sigma_r$

σ_{vc}' = vertical effective consolidation pressure

E_{50} = secant modulus E_{50} passing through the origin and the point on the stress-strain curve at a stress equal to 50% of the maximum strength

c_u = undrained shear strength

$(E_{50})_C$ = the value of E_{50} by the compression tests

$(E_{50})_E$ = the value of E_{50} by the extension tests

n = the ratio of the $(E_{50})_E$ value to the $(E_{50})_C$ value

ν_{VH}' = poisson's ratio in terms of effective stress which is the ratio of strain in horizontal

direction to an applied strain in vertical direction

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