ANALYSIS OF WAVE-INDUCED LIQUEFACTION IN SEABED DEPOSITS OF SAND

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ABSTRACT

To explore the nature of cyclic stress alteration in the seabed deposits of sand due to travelling waves, two-dimensional stress analysis was made for homogeneous elastic half-space subjected to a series of harmonic loads moving on its surface. The analysis indicated that changes in shear stress occur in such a way that, while its amplitude is maintained constant, the directions of the principal stresses rotate continuously. With a view to simulating such stress changes in the laboratory test, a series of cyclic triaxial torsion shear tests was conducted on loose specimens of sand. The test results indicated that the conventionally defined cyclic stress ratio is reduced by about 30% if the rotation of the principal stress directions is involved in the cyclic loading. The magnitude of wave-induced pressure at the seabed was determined on the basis of the linear theory of water wave propagation. This water pressure fluctuation was applied over the seabed, which is assumed to be an elastic half-space, to determine the cyclic stress ratio induced within the seabed deposit. The cyclic stress ratio thus determined is compared against the cyclic stress ratio causing liquefaction and cyclic mobility in which the continuous rotation of principal stress directions is taken into account. The procedures as above are integrated into a simple methodology to perform liquefaction analysis under wave loading conditions.

Key words : liquefaction, ocean soil, repeated load (IGC : D7)

INTRODUCTION

The phenomenon of ocean wave-induced liquefaction has arrested increasing attention in recent years in considering the integrity of nearshore and offshore installations such as pipelines, anchors and platform structures. The differential loading on the sea-floor by the pressure wave induces a sequence of cyclic shear stress in the underlying soil

and if the induced shear stress exceeds the strength, significant deformation or liquefaction failure may occur thereby exerting damaging influences on the engineering installations placed nearby. This issue was put in a proper perspective by Henkel (1970, 1982) and several investigators have elaborated to shape up analytical frameworks for the liquefaction analysis under wave loadings. Seed and Rahman (1978) developed

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a methodology in which generation and contemporaneous dissipation of excess pore water pressures during strong wave loadings can be evaluated. Finn et al. (1983) extended this approach for performing stability analyses of the seabed deposit on the basis of the effective stress principle. More practical aspect of the wave-induced liquefaction was dealt with by Nataraja and Gill (1983) who tried to explain several cases of seabed failure on the basis of the simplified analysis of liquefaction. Liquefaction potential of the seafloor deposits at particular sites was studied by Lee and Focht (1975) and Clukey et al. (1980) based on the simplified analysis procedure. Somewhat new method of approach was proposed recently by Ishihara and Yamazaki (1983) and Ishihara (1983) who tried to incorporate two novel features in a framework of the wave-induced liquefaction analysis. The first was to clarify the nature of cyclic alteration of the wave-induced shear stress which is characterized by rotation of the principal stress directions (Ishihara and Towhata, 1983), and the second was to establish a methodology for evaluating the magnitude of cyclic stress on the basis of design storm parameters specified at an offshore location in deep water. The present paper presents a slight modification of the previously proposed methodology with an addition of more elaborate laboratory test results.

CHARACTERISTICS OF WAVE-INDUCED CYCLIC STRESS

Water waves propagating on the ocean may be considered to consist of an infinite number of wave trains having a constant amplitude and wave length. Passage of such an array of waves over the ocean creates harmonic pressure waves on the sea-floor increasing the pressure under the crest and reducing it under the trough. The stresses induced in the seabed are, therefore, analyzed by applying a sinusoidally changing load on the horizontal surface from minus to plus infinity, as illustrated in Fig. 1. If

Fig. 1. Definition of notations

the seabed deposit is assumed to consist of a homogeneous elastic material extending to an infinite depth, the stresses can be readily determined by using the classical solution by Boussinesq for the two-dimensional plane strain problem. Assume that a harmonic load

$$p(x) = p_0 \cos\left(\frac{2\pi}{L}x - \frac{2\pi}{T}t\right) \quad (1)$$

is distributed on the surface of an elastic half-space, where p_0 is the amplitude of the load, L is the wave length and T is the period of waves. The vertical normal stress, σ_v , horizontal normal stress, σ_h , and shear stress, τ_{vh} , induced in the half-space by this load are determined (Yamamoto, 1978; Madsen, 1978) as,

$$\left. \begin{aligned} \sigma_v &= p_0 \left(1 + \frac{2\pi z}{L}\right) e^{-2\pi z/L} \cdot \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) \\ \sigma_h &= p_0 \left(1 - \frac{2\pi z}{L}\right) e^{-2\pi z/L} \cdot \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) \\ \tau_{vh} &= p_0 \frac{2\pi z}{L} \cdot e^{-2\pi z/L} \cdot \sin\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) \end{aligned} \right\} \quad (2)$$

where x and z indicate the spatial coordinates in the horizontal and vertical directions, respectively, as illustrated in Fig. 1. It is well known that the major components of stress associated with the shearing deformation of a solid body are the stress, τ_{vh} , and the stress difference defined as $(\sigma_v - \sigma_h)/2$. For the present problem, the stress difference is calculated from Eq. (2) as follows,

$$\frac{\sigma_v - \sigma_h}{2} = p_0 \frac{2\pi z}{L} e^{-2\pi z/L} \cdot \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) \quad (3)$$

Comparison of the expressions for τ_{vh} and $(\sigma_v - \sigma_h)/2$ indicates that the amplitude of these two components of stress is identical and they differ only in the time phase of cyclic load application. In order to obtain an insight into the nature of the cyclic loading, let the variable, z , be eliminated between τ_{vh} and $(\sigma_v - \sigma_h)/2$. Then, one obtains

$$\tan\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) = \frac{2\tau_{vh}}{\sigma_v - \sigma_h} \quad (4)$$

According to the textbook of the elementary stress analysis, the right-hand side of Eq. (4) is equal to the tangent of twice the angle of the major principal stress direction to the vertical axis, β , as illustrated in Fig. 2(a). Hence, one obtains

$$\beta = \frac{\pi x}{L} - \frac{\pi t}{T} = \frac{1}{2} \tan^{-1}\left(\frac{2\tau_{vh}}{\sigma_v - \sigma_h}\right) \quad (5)$$

This equation implies that, when attention is drawn to stress changes in a soil element at a fixed point x , the direction of the principal stress is rotating continuously through 180 degree during one period, T , of cyclic load application. The same nature of the rotation of the principal stress axes is taking place simultaneously throughout the depth of the half-space. If the variable, $(2\pi x/L - 2\pi t/T)$, is eliminated between the shear stress, τ_{vh} , and $(\sigma_v - \sigma_h)/2$, one obtains

$$\left(\frac{\sigma_v - \sigma_h}{2}\right)^2 + \tau_{vh}^2 = p_0^2 \left(\frac{2\pi z}{L}\right)^2 \cdot e^{-4\pi z/L} \quad (6)$$

Fig. 2. Characteristic variation of wave-induced shear stress

The textbook of the elementary stress analysis shows that the left-hand side of Eq. (6) is equal to the square of the deviator stress which is defined as the difference between the maximum and minimum principal stresses, σ_1 and σ_3 , respectively, divided by two. Hence one obtains,

$$\frac{\sigma_1 - \sigma_3}{2} = \sqrt{\left(\frac{\sigma_v - \sigma_h}{2}\right)^2 + \tau_{vh}^2} = p_0 \frac{2\pi z}{L} e^{-2\pi z/L} \quad (7)$$

This equation indicates that, when attention is drawn to stress changes in a soil element at a fixed depth, the deviator stress is kept unchanged at all times and at all points on the horizontal plane during the cyclic load application. The same nature of cyclic alteration of stress is taking place at other depths in the half-space. As can easily be understood from Eq. (7), the two components of stress, τ_{vh} and $(\sigma_v - \sigma_h)/2$, are increasing or decreasing alternately so as to keep the deviator stress at a constant value during the entire period of cyclic load application. When the two components of stress are represented in a rectangular coordinate system, Eq. (6) is an equation of a circle with its radius equal to the deviator stress, $(\sigma_1 - \sigma_3)/2$. Such a plot is presented in Fig. 2(b). In this type of plot, the angle which a stress vector makes to the horizontal coordinate axis represents twice the angle of the maximum principal stress direction to the vertical, 2β . It can be mentioned in summary that the cyclic change of shear stress induced in an elastic half-space by a harmonic load moving on its surface is characterized by a continuous rotation of the principal stress direction with the deviator stress being always maintained constant. Therefore, if the load on the sea bottom is represented by a propagating harmonic load and if the seabed deposit is assumed to be an elastic half-space, the cyclic stress induced in the deposit has the characteristics such that the cyclic alteration of shear stress is executed with a continuous rotation of the principal stress direction.

Next, consider the distribution of the am-

Fig. 3. Distribution of shear stress versus depth

plitude of the cyclic deviator stress versus depth. Since the deviator stress amplitude is equal to either the amplitude of shear stress, τ_{vh} , or the stress difference, $(\sigma_v - \sigma_h)/2$, it will suffice to consider only the amplitude of shear stress, τ_{vh} , which is given by Eq. (2). Numerical values of the amplitude of τ_{vh} is plotted versus depth in Fig. 3. It can be seen that the shear stress takes a maximum value of $0.368 p_0$ at a depth equal to 15.9% of the wave length. A more significant variable associated with the deformability or strength of soils under cyclic loading conditions is the cyclic stress ratio which is defined as the ratio of amplitude of shear stress to the effective confining stress. In the horizontal deposit of soils, the vertical effective overburden pressure, σ_v' , is generally used as a parameter to represent the confinement of soils in-site. Since the magnitude of σ_v' is expressed by $\sigma_v' = \rho'gz$ the cyclic stress ratio is given by

$$\frac{\tau_{vh}}{\sigma_v'} = \frac{2\pi}{\rho'g} p_0 \frac{1}{L} e^{-2\pi z/L} \quad (8)$$

where z is the depth of the deposit and ρ' is the submerged unit mass of soils in the seabed. The value of the cyclic stress ratio

Fig. 4. Distribution of cyclic shear stress ratio versus depth

at mud line is derived immediately by putting $z=0$ in Eq. (8),

$$\left(\frac{\tau_{vh}}{\sigma_v'}\right)_0 = \frac{2\pi p_0}{\rho'g L} \quad (9)$$

Therefore, the cyclic stress ratio at any depth normalized to the value at mud line is obtained as,

$$\left(\frac{\tau_{vh}}{\sigma_v'}\right) / \left(\frac{\tau_{vh}}{\sigma_v'}\right)_0 = e^{-2\pi z/L} \quad (10)$$

This relationship is numerically presented in Fig. 4, where it can be seen that the cyclic stress ratio decreases sharply with depth near the surface of the seabed.

WAVE-INDUCED PRESSURE AT SEA BOTTOM

The height of waves in the ocean is generally very small as compared to the length of the waves. The basic equation of water waves solved on this assumption leads to the wellknown theory of small amplitude wave (Horikawa, 1978). According to this theory, the amplitude of the pressure fluctuation exerting on the sea

bottom, p_0 , due to the travelling wave is given by

$$p_0 = \rho_w \cdot g \frac{H}{2} \frac{1}{\cosh\left(2\pi \frac{h}{L}\right)} \quad (11)$$

where H denotes the height and L is the length of the wave at the place where the water depth is h , as illustrated in Fig. 1. The term, ρ_w , is the unit mass of the water and g is the gravity acceleration. The values of H and L in Eq. (11) can not be arbitrarily chosen, but are derived, as follows, from the basic solution of the small amplitude wave theory,

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi h}{L}\right) \quad (12)$$

and from the energy continuity equation,

$$H = H_0 \left[\left(1 + \frac{4\pi \frac{h}{L}}{\sinh\left(4\pi \frac{h}{L}\right)} \right) \tanh\left(2\pi \frac{h}{L}\right) \right]^{-1/2} \quad (13)$$

where H_0 denotes the height of waves in deep water condition where the depth of water is large compared to the half of wave length. Eq. (13) is derived on the assumption that no energy loss is involved during the wave propagation over the sloping bottom towards the shore whereby the water depth decreases gradually. The wave profiles at two cross sections over the sloping bottom are shown in Fig. 5. Section I is assumed to be located in the nearshore region, off the surf zone, where the waves are still stable. Section II is assumed to be sufficiently offshore so that the length and height of the waves are determined by

fully developed wind waves irrespective of the depth of water. The wave profile at a location such as Section II is referred to as deep water condition. The energy continuity equation of Eq. (13) is established by equating the energy being transmitted by the wave train at the Section I to the energy being transmitted by the wave system in deep water. In deriving Eq. (13), it is assumed that the wave period remains unaltered in water of any depth, whereas the wave length, velocity and height vary as the wave proceeds towards the shore. For the waves travelling in deep water, Eq. (12) is reduced to,

$$L_0 = \frac{gT^2}{2\pi} \quad (14)$$

where L_0 denotes the wave length in deep water. Combining Eqs. (12) and (14), one obtains

$$L = L_0 \tanh\left(\frac{2\pi h}{L}\right) \quad (15)$$

This equation indicates how the wave length changes as the wave propagates in water of decreasing depth towards the shore.

A most commonly used procedure to evaluate the pressure, p_0 , exerting on the sea bottom at a location of given water depth, h , is to envisage an overall profile of waves propagating in water of decreasing depth from the deep water towards the shore, as illustrated in Fig. 5, and to determine the wave height, H , and length, L from Eqs. (13) and (15) on the basis of the key wave parameters, T , L_0 and H_0 specified at a location in deep water. The values of L and H thus determined at water depth, h , are then introduced in Eq. (11) to determine the amplitude of pressure, p_0 .

CYCLIC STRESS RATIO AT MUD LINE

With the amplitude of pressure, p_0 , known by the procedure as described above, it is now possible to evaluate the cyclic stress ratio at mud line. By introducing Eq. (11) into Eq. (9), a more explicit expression can be obtained as follows,

Fig. 5. Deformation of wave profile during shoaling

$$\left(\frac{\tau_{vh}}{\sigma_v'}\right)_0 = \pi \frac{\rho_w}{\rho'} \frac{H}{L} \frac{1}{\cosh\left(2\pi \frac{h}{L}\right)} \quad (16)$$

Introducing the expressions for H and L from Eqs. (13) and (15), one can rewrite Eq. (16) as,

$$\left(\frac{\tau_{vh}}{\sigma_v'}\right)_0 = \frac{\pi \frac{\rho_w}{\rho'} \cdot \frac{H_0}{L_0}}{\sinh\left(\frac{2\pi h}{L}\right) \sqrt{\tanh\left(\frac{2\pi h}{L}\right) \left(1 + \frac{4\pi h/L}{\sinh(4\pi h/L)}\right)}} \quad (17)$$

Eq. (17) is numerically presented in Fig. 6 for different values of H_0/L_0 ranging between 0.01 and 0.13. As mentioned in the foregoing section, values of L_0 and H_0 in deep water are first specified. Then, using Eq. (15), a value of wave length, L , at the location of a given depth, h , is determined.

Fig. 6. Cyclic stress ratio at mud line

Fig. 7. Wave length versus water depth

By inserting all these values into Eq. (17), the cyclic stress ratio at mud line can be determined. To facilitate the computation, Eq. (15) is expressed in a form of a chart in Fig. 7 by plotting the value of L versus h . In some cases, the wave conditions in deep water are more conveniently specified in terms of the period, T , and the wave steepness, H_0/L_0 . In such cases, it is expedient first to compute the wave length, L_0 , through Eq. (14) and then proceed in the same way as mentioned above.

Eq. (17) indicates that the cyclic stress ratio at mud line can increase infinitely when the water depth becomes small relative to wave length. In such a situation, the wave height is large as compared to wave length, as easily seen from Eq. (16). However, it has been shown that the wave steepness defined by H/L can not increase beyond a certain critical value at which the wave breaks. According to some studies made in the expertise of coastal engineering (Hori-kawa, 1978), the critical condition for progressive waves to break is given by,

$$\frac{H}{L} \leq \frac{1}{7} \tanh\left(\frac{2\pi h}{L}\right) \quad (18)$$

Eq. (18), setting an upper limit for the wave steepness, also put an upper boundary for the cyclic stress ratio to exist. By merging the condition of Eq. (18) with Eq. (16), the limiting condition for the cyclic stress ratio can be obtained as,

$$\left(\frac{\tau_{vh}}{\sigma_v'}\right)_0 \leq \frac{\pi}{7} \frac{\rho_w}{\rho'} \frac{\sinh(2\pi h/L)}{\cosh^2(2\pi h/L)} \quad (19)$$

This equation is numerically presented in Fig. 6, setting an upper boundary for the cyclic stress ratio. It is to be remembered that the cyclic stress ratio lying above this boundary curve in Fig. 6 can not exist in reality in the seabed deposit because of the breaking occurring at the crest of propagating waves. It is also to be noted that the cyclic stress ratio at the wave breaking is a function of only h/L and can be determined without knowing a value of the wave steepness in deep-water condition. However, if the wave steepness, H_0/L_0 , corresponding to a

given value of h/L needs to be known, it can easily be determined through the following formula which has been obtained by equating Eqs. (17) and (19).

$$\frac{H_0}{L_0} = \frac{1}{7} \left[\tanh\left(\frac{2\pi h}{L}\right) \right]^{5/2} \sqrt{1 + \frac{4\pi h/L}{\sinh(4\pi h/L)}} \quad (20)$$

It is of interest to note that the limiting condition of Eq. (19) specifies an absolute maximum for the cyclic stress ratio that can exist under the condition of wave breaking. Some simple analytical treatment of Eq. (19) shows that the maximum value of the cyclic stress ratio is equal to,

$$\left(\frac{\tau_{vh}}{\sigma_v'} \right)_0 = 0.2379 \quad (21)$$

and this occurs when $h/L=0.140$. The value of the wave steepness, H_0/L_0 , in deep water condition corresponding to the above maximum cyclic stress ratio can be determined by introducing $h/L=0.140$ into Eq. (20). The value thus obtained was $H_0/L_0=0.0765$ and indicated accordingly in Fig. 6.

WAVE PROFILE IN DEEP WATER

As pointed out in the foregoing, a knowledge of the wave length, L_0 , and wave height, H_0 , in deep water is necessitated for evaluating the cyclic stress ratio at mud line, $(\tau_{vh}/\sigma_v')_0$, at any water depth on a shoaling shore.

One of the most commonly used procedures to estimate the wave parameters in deep water is to make use of the empirical formulae proposed by Wilson (1965) who tried to express the significant wave height, $H_{1/3}$ (m), and period, $T_{1/3}$ (sec), in terms of fetch length, F (km), and wind velocity, U (m/sec),

$$\frac{g}{U^2} H_{1/3} = 0.30 \left[1 - \frac{1}{(1 + 0.004(gF/U^2)^{1/2})^2} \right] \quad (22)$$

$$\frac{g}{2\pi U} T_{1/3} = 1.37 \left[1 - \frac{1}{(1 + 0.008(gF/U^2)^{1/3})^5} \right] \quad (23)$$

The significant wave height is defined as

an average taken for all waves belonging to the highest one-third in a group of waves under consideration. The fetch length implies the distance through which wind blows to generate a sequence of sea waves. The value of wind velocity, U , is generally determined by the wind speed at an elevation 10 m above the sea level. The above formulae have been known to be applicable with a reasonable degree of accuracy for fairly wide range of variation in the fetch length and wind velocity.

Having known the value of the period, $T_{1/3}$, from Eq. (23), the value of the length of the significant wave, $L_{1/3}$ can be readily obtained by using the relation of Eq. (14). In order to facilitate computation, the empirical formulae given by Eqs. (22) and

Fig. 8. Plots of Wilson's empirical formulae for the wave profile parameters in deep water as functions of fetch length and wind velocity

Fig. 9. Plot of different observed data on the mean period versus significant wave height (Wiegel, 1964)

(23) are presented in Fig. 8 in a form of charts. Also shown in this chart is the wave steepness, $H_{1/3}/L_{1/3}$, which are calculated from Eqs. (22) and (23) together with Eq. (14).

Apart from the procedures as described above, an attempt was made by Wiegel (1964) to assemble numerous sets of wave profile data from in-situ and laboratory observations and to plot them in terms of the mean wave period, $\bar{T}$, versus the significant wave height, $H_{1/3}$. The result of such a data arrangement is presented in Fig. 9. Considering an almost linear relationship between the square of mean period and significant wave height in the plot of logarithmic scale, Wiegel (1964) drew a straight line near the lower boundary as indicated in Fig. 9, which is expressed as

$$H_{1/3} = 0.137 \bar{T}^2 \quad (24)$$

Since most data points in Fig. 9 lie in the zone above this straight line, the empirical relation of Eq. (24) would have to be construed as a condition setting an upper limit for the height of the significant wave to exist in reality for a given wave period. Therefore, Eq. (24) should rather be rewritten as an inequality condition,

$$H_{1/3} \leq 0.137 \bar{T}^2 \quad (25)$$

In the above discussions, the wave profile has been specified in terms of the significant wave height, $H_{1/3}$, wave length, $L_{1/3}$, and period $T_{1/3}$, or mean period $\bar{T}$. Since the analytical treatment presented in the early part of this paper is based on the use of the parameters, H_0 , L_0 and T appearing in the theory of harmonic wave, it is now necessary to discuss the correspondence between these two sets of wave parameters.

With respect to the period it has been observed (Horikawa, 1978) that an average period taken for those wave trains belonging to the longest one-third is longer by approximately 10% than an average period taken for all wave trains in a group of waves under consideration. Therefore, the period, $T_{1/3}$, may be taken equal to 1.1 times the mean period, $\bar{T}$. With respect to the wave

height, it has been shown (Horikawa, 1978) empirically that the significant wave height is approximately equal to 1.6 times the mean wave height, $\bar{H}$. Therefore,

$$T_{1/3} \doteq 1.1 \bar{T} \quad H_{1/3} \doteq 1.6 \bar{H} \quad (26)$$

In order to represent a series of actual wave trains with different heights and periods in a form of sinusoidal waves, it is necessary to make some assumption on the correspondence between the wave parameters pertaining to the significant wave and to the sinusoidal wave. There appear to be two possibilities for the choice of the correspondence. One is to assume that actual waves are represented by a harmonic wave with the period and the wave length equal to the mean values of the actual waves. The other assumption is that the actual waves are represented by a harmonic wave having the period and wave length equal to the respective values of the significant wave. The correspondences as above are written as,

$$T = \bar{T} \left(= \frac{1}{1.1} T_{1/3} \right), \quad H_0 = \bar{H} \left(= \frac{1}{1.6} H_{1/3} \right) \quad (27)$$

$$T = T_{1/3} (= 1.1 \bar{T}), \quad H_0 = H_{1/3} (= 1.6 \bar{H}) \quad (28)$$

When adopting these correspondences, the inequality relation of Eq. (25) is rewritten, respectively, as,

$$H_0 \leq 0.0856 T^2 \quad (29)$$

$$H_0 \leq 0.113 T^2 \quad (30)$$

Using the relation of Eq. (14), Eqs. (29) and (30) are respectively transformed into the inequalities specifying the range of variation in the wave steepness in deep water.

$$\frac{H_0}{L_0} \leq 0.055 \quad (31)$$

$$\frac{H_0}{L_0} \leq 0.0725 \quad (32)$$

In a previous paper by Ishihara and Yamazaki (1983), the assumption similar to Eq. (27) with $\bar{T} = T_{1/3}$ was adopted for establishing a possible range of variation for the wave steepness in deep water. How-

ever, in view of the fact that the strength of sand in cyclic loading conditions is influenced primarily by the repetition of large-amplitude components in a series of irregular pattern of loads, it seems logical to adopt the assumption of Eq. (28) in which the wave height, H_0 , and length, L_0 , are chosen as being equal to those pertaining to significant wave. Therefore, Eq. (32) will be adopted in the following. The range of variation for H_0/L_0 specified by Eq. (32) is also indicated in Fig. 8 for comparison with the value of H_0/L_0 which is determined by the empirical formulae of Eqs. (22) and (23). It may be seen that the wave steepness value evaluated by means of Eqs. (22) and (23) falls well within the range of variation established from Wiegel's data arrangements, if the nondimensional parameter, gF/U^2 , takes a value greater than approximately 55 which is encountered in most cases.

UPPER LIMIT IN THE CYCLIC STRESS RATIO

It has been shown that the cyclic stress ratio induced by waves at mud line is determined by Eq. (17) as a function of h/L , if the wave steepness in deep water, H_0/L_0 , is known. It has also been pointed out that the cyclic stress ratio thus determined does not become infinitely large, but is bounded by the condition of wave breaking as represented by Eq. (18). Since the variation of the wave steepness, H_0/L_0 , is practically limited within a certain range as indicated above, the value of the cyclic stress ratio is accordingly limited to vary within a corresponding range. This range of variation is indicated in Fig. 6 by a dotted area. It is important to notice that there exists practically a maximum value in the cyclic stress ratio which is equal to 0.2375, and this maximum occurs with $H_0/L_0=0.0725$ when $h/L=0.1355$, as indicated in Fig. 6. The maximum cyclic stress ratio at mud line specified by the observed upper limit of the wave steepness in deep water happens to be

practically the same as the maximum possible cyclic stress ratio specified by the condition of wave breaking.

CYCLIC TRIAXIAL TORSIONAL SHEAR TEST

In order to study the resistance of sands under the cyclic loading conditions simulating the characteristic variation of stresses induced in the seabed by waves, a series of laboratory tests was conducted using a triaxial torsional shear test apparatus in which the horizontal shear stress component, τ_{vh} , and the component of the stress difference, $(\sigma_v - \sigma_h)/2$, can be varied independently. In this apparatus, a hollow cylindrical test specimen 10 cm in outer diameter, 6 cm in inner diameter and 10.1 cm in height was encased in rubber membranes and put in place in the triaxial chamber.

The sand used was a type of Japanese standard sand called Toyoura sand. The mean particle size, D_{50} , is 0.17 mm and the uniformity coefficient of this sand is 2.0. Subrounded to subangular in grain shape, the sand has the maximum and minimum void ratios of 0.98 and 0.60, respectively. The specific gravity is 2.65. In the present study, test specimens were prepared by the method of pluviation to different states of packing having a relative density of around 50%, 80% and 95%. Specimens with these densities will be referred, respectively, to loose, dense and very dense states of packing. Air-dried sand was spread in the mold from a height of fall of 12, 35 and 180 cm to form specimens of loose, dense and very dense states, respectively. Saturation of the specimen was aided by circulating carbon dioxide gas and by applying a back pressure of 150 kN/m². The specimens were consolidated isotropically under a confining stress of 98 kN/m² and then subjected to cyclic loads under undrained conditions.

Three types of cyclic loading tests were conducted by changing the two components of shear stress either singly or in combination. The loading scheme of these tests are

Fig. 10. Stress paths in cyclic loading tests

illustrated in the stress space shown in Fig. 10. In the cyclic test with circular rotation of the principal stress direction, two components of shear stress were increased or decreased alternately so that the deviator stress, $(\sigma_1 - \sigma_3)/2$, defined by Eq. (7) was maintained at a constant value. In other words, the torsional component, τ_{vh} , and the stress difference component, $(\sigma_v - \sigma_h)/2$, were cyclically applied to the specimen 90 degree out of phase with each other. Other two types of tests were the conventional cyclic triaxial test and the cyclic torsion shear test, as illustrated respectively in the stress paths in Figs. 10(a) and 10(b). The frequency of the cyclic load was 0.08 Hz for each of the triaxial mode and torsional mode of cyclic loading.

The results of the tests are presented in Fig. 11 by plotting the cyclic stress ratio versus the number of load cycles required to cause 5% double amplitude deviator strain. The cyclic stress ratio is defined as the ratio between the amplitude of the cyclically applied deviator stress, $(\sigma_1 - \sigma_3)/2$, and the initial confining pressure, σ'_0 . The definition in a broader sense as above embodies, as a special case, the cyclic stress ratio that has been conventionally used for the triaxial or torsional mode alone of cyclic loading. The amplitude of the deviator strain, $\varepsilon_1 - \varepsilon_3$, was calculated from the recorded amplitudes of the triaxial mode of strain, $\varepsilon_v - \varepsilon_h$, and of the torsional mode of strain, γ_{vh} , on the basis of the following formula

$$\varepsilon_1 - \varepsilon_3 = \sqrt{(\varepsilon_v - \varepsilon_h)^2 + \gamma_{vh}^2} \quad (33)$$

The double-amplitude, 5%, was chosen, without any specific reasons, to define a state

of failure as represented by cyclic mobility or liquefaction of saturated sand.

Among the results of the three types of test, it appears appropriate to compare the result of the cyclic torsion shear test with that of the cyclic test with rotation of principal stress axes, in order to single out the effects of the principal stress axis rotation. It can be seen in Fig. 11(a) that the

Fig. 11. Cyclic stress ratio versus number of cycles for three types of tests with or without principal stress axis rotation

cyclic stress ratio causing the failure strain in a given number of cycles, say, 100 cycles is about 30% smaller in the cyclic test with the rotation of the principal stress direction as compared to the cyclic test without the rotation. Fig.11(a) also shows that the cyclic stress ratio causing failure in the specimen in the cyclic triaxial test is smaller than the cyclic stress ratio obtained in the cyclic torsion shear test for all the number of cycles employed in the tests, but that the difference in the cyclic stress ratio between these two types of test becomes smaller as the number of cycles concerned is increased. This appears to result from the fact that in the case of the cyclic triaxial test, a larger amount of pore water pressure and hence shear strain is produced when the sample is deformed towards the triaxial extension side than when it is deformed in the triaxial compression side. This tendency becomes increasingly pronounced as the amplitude of cyclic load is increased. Such an unsymmetrical nature of sand response in the triaxial loading condition makes the sample fail at a smaller cyclic stress ratio in a given number of cycles as compared to the corresponding cyclic stress ratio executed in the torsional mode of loading. Therefore, at a large number of cycles, say, 100 cycles, where the amplitude of cyclic stress required to cause failure is small, the difference in the cyclic strength between the cyclic triaxial and cyclic torsional tests practically disappears as seen in Fig. 11(a). The results of similar cyclic loading tests on dense specimens with a relative density of approximately 78% are presented in Fig.11(b). Approximately the same proportion of decrease in cyclic stress ratio is observed in the result of the test with rotation of the principal stress direction as compared to the result of the cyclic torsion test. It is to be noted that the difference in the failure-inducing cyclic stress ratio between the cyclic triaxial test and cyclic torsion test becomes negligibly small, as the required number of load cycles increases. The results of the three types of cyclic

Fig. 12. Relationship between the cyclic strength and relative density

loading test on very dense specimens with a relative density of approximately 92% are presented in Fig.11(c). It may as well be noted that the continuous rotation of the principal stress direction reduces the cyclic stress ratio by about 30% below the corresponding cyclic stress ratio required to cause failure in the cyclic torsional loading condition.

Values of cyclic stress ratio producing 5% double amplitude of strain in 100 load cycles were read off from each of the test results shown in Fig.11 and plotted in Fig.12 versus the average value of the relative density of the test specimens. A straight line was drawn through average data points for each result of the test types to express the cyclic strength as a function the relative density. The straight-line relationships thus establish are written as

$$\left(\frac{\tau_{vh,l}}{\sigma_0'} \right)_{100} = 0.0027 D_r \quad (34)$$

for the average data obtained under cyclic triaxial and cyclic torsional loading conditions without rotation of principal stress, and

$$\left(\frac{\tau_{vh,l}}{\sigma'_0}\right)_{100} = 0.0019 D_r \quad (35)$$

for the cyclic loading conditions involving rotation of the principal stress direction, where $\tau_{vh,l}$ denotes the amplitude of cyclic shear stress causing liquefaction or cyclic mobility and σ'_0 is the effective confining pressure. The number of load cycles, 100, would probably represent the number of the significant waves occurring in the ocean during the storm period. The choice of this number of load cycles is also warranted by the fact that no significant variation occurs in the cyclic stress ratio for the number of cycles exceeding 100 as indicated in Fig. 11.

Another important fact that must be considered in evaluating the cyclic strength is the effect of anisotropic consolidation which is evaluated in terms of the earth pressure coefficient at rest, K_0 . In the case of cyclic loading test without rotation of principal stress direction, the effects of K_0 -consolidation were shown to be evaluated simply by taking the mean principal stress as the effective confining stress. (Ishihara et al., 1977). A series of more recent laboratory tests conducted by Yamazaki (1984) indicated the same concept to be applicable also for the case of cyclic loading involving the rotation of principal stress direction. On the basis of this consideration, the empirical formula of Eq. (35) is modified as

$$\left(\frac{\tau_{vh,l}}{\sigma'_v}\right)_{100} = 0.0019 \cdot D_r \cdot \frac{1+2K_0}{3} \quad (36)$$

The cyclic strength value estimated by this formula will be used for the analysis of liquefaction presented in the following pages.

In making the analysis of liquefaction, it will be assumed that a train of harmonic waves is travelling over the sloping sea floor in water of decreasing depth from the deep water location towards the shore, as illustrated in Fig. 5. It will also be assumed that a sand is uniformly deposited in the seabed throughout the depth and through the distance of wave travel. The analysis of liquefaction involves the determination of water depth at a location where liquefac-

Fig. 13. Flow chart for performing wave-induced liquefaction analysis for the seabed deposit

tion occurs and the determination of depth in the deposit to which liquefaction advances below the sea bottom. The analysis will be made by following the steps as illustrated in a flow chart of Fig. 13.

Step 1

It is necessary first of all to specify conditions of a design storm. The design storm conditions need to be specified at a location far offshore where the assumption of the deep water is satisfied. It would be convenient to specify the wave steepness, H_0/L_0 and the wave length, L_0 . In case the wave period, T , is specified, the value of L_0 can readily be determined by Eq. (14).

Step 2

It is necessary to evaluate the cyclic strength of the sand composing the seabed deposit. The cyclic strength is expressed in terms of the cyclic stress ratio causing liquefaction or 5% double-amplitude shear strain in the sand after a sufficient number of cyclic load has been applied. For all

practical purposes, 100 load cycles are considered appropriate. In what follows, the cyclic strength thus defined will be referred to as cyclic stress ratio at failure. In evaluating the cyclic strength, the effects of K_0 -condition and the rotation of the principal stress direction must be taken into account.

Step 3

If the cyclic stress ratio at failure is greater than 0.238, nowhere in the seabed deposit liquefaction type failure is expected to occur. Otherwise, liquefaction takes place down to some depth of the seabed deposit through some distance with varying water depth in the path of wave shoaling. For given values of the wave steepness, H_0/L_0 , and the cyclic stress ratio at failure estimated from Eq. (36), a value of h/L can be read off from the chart in Fig. 6. It is also convenient to read off values of h/L corresponding to several arbitrarily chosen values of the cyclic stress ratios which are in excess of the cyclic stress ratio at failure.

Step 4

For each value of h/L determined in Step 3, the water depth, h , and the wave length, L , can be determined using the chart of Fig. 7 whereby referring to the prescribed value of the wave length, L_0 .

Step 5

The value of the water depth determined from the cyclic stress ratio at failure indicates the water depth where liquefaction occurs just at the fluid-bed interface. In other words, this water depth was determined in such a way that the wave-induced cyclic stress ratio becomes equal, at the mud line, to the cyclic stress ratio of sand at failure. In the areas with shallower depth, the cyclic stress ratio induced at the mud line becomes greater than the cyclic stress ratio at failure. In such cases, it may well be assumed that the induced cyclic stress ratio at the mud line decays with increasing depth of the seabed deposit in accordance with the rela-

tionship of Eq. (10). Then, using the chart in Fig. 4, it is possible to determine a depth factor, z_c/L , at which the depthwise decreasing cyclic stress ratio is reduced to a value at failure, where z_c denotes the depth to which failure due to liquefaction penetrates.

Step 6

Using the wave length, L , determined in Step 4 and the depth factor, z_c/L , calculated in Step 5, it becomes possible to assess the depth, z_c , to which liquefaction type failure penetrates.

EXAMPLES OF ANALYSIS

Using the analysis procedure as described above, hypothetical cases will be considered in this section for illustration purposes. Assume a severe storm conditions in deep water where, $H_0/L_0=0.0725$ and $L_0=150$ m. The liquefaction resistance of the sand in the seabed sediment is assumed to be given by a representative formula in Eq. (36). The sand deposit in the seabed is assumed to be uniform and to have a relative density of $D_r=60\%$ and a K_0 -value of 0.5. Introducing these soil parameters in Eq. (36), the cyclic stress ratio at failure is determined as

$$\left(\frac{\tau_{vh,l}}{\sigma_v'}\right)_{100} = 0.076 \quad (37)$$

The depth of water at which the cyclic stress ratio at mud line becomes equal to 0.076 is determined by locating a point in Fig. 6 where the curve of $H_0/L_0=0.0725$ intersects with a horizontal line of $(\tau_{vh}/\sigma_v')_0=0.076$. Reading off a value of h/L in the abscissa corresponding to the point of intersection as determined above, one obtains

$$\frac{h}{L} = 0.288$$

Then, entering in the chart of Fig. 7, a straight line corresponding to $h/L=0.288$ is drawn through the origin and a point is located where the line of $h/L=0.288$ intersects with a curve of $L_0=150$ m. By reading off the values in the abscissa and in the ordi-

nate, the water depth and wave length are determined respectively as,

$$h=41\text{ m}, L=142\text{ m}$$

Therefore, under the conditions of wave profile and sand deposit as specified above, the cyclic stress ratio at mud line induced by waves becomes just equal to the cyclic stress ratio at failure when the waves reach to a location with the water depth of 41 m over the seabed with decreasing depth towards the shore. At this location, the wave length is 142 m. This location is indicated in Fig. 14(a).

As the waves travel towards the shore over the seabed with decreasing depth, the wave profile changes thereby increasing the wave steepness, H/L , and hence increasing the cyclic stress ratio induced at mud line. Therefore, when the waves come in to a zone with a water depth shallower than 41 m, the induced cyclic stress ratio at mud line becomes always greater than the cyclic stress ratio causing liquefaction or a state

of cyclic mobility in the sand. In such a case, it may well be assumed that the liquefaction can penetrate into the ground down to a depth at which the induced cyclic stress ratio is reduced to the cyclic stress ratio at failure. Therefore, the next problem of concern is to determine the depth of liquefaction penetration.

Consider an arbitrary chosen value of cyclic stress ratio in excess of the cyclic strength evaluated in Eq. (37), say, $(\tau_{vh}/\sigma_v')=0.12$. Let us seek for the water depth at which the wave-induced cyclic stress ratio becomes equal to the pre-assumed value of 0.12 at the mud line. By locating a point of intersection in Fig. 6 between a horizontal line of $(\tau_{vh}/\sigma_v')=0.12$ and the curve of $H_0/L_0=0.0725$, the ratio of h/L can be read off as being $h/L=0.218$. By drawing a straight line of $h/L=0.218$ in Fig. 7 through the origin, and by locating a point of intersection between the straight line of $h/L=0.218$ and the curve of $L_0=150\text{ m}$, the water depth and wave length can be read off as

$$h=29\text{ m}, L=132\text{ m}$$

Consequently, it can be mentioned that at the location where water depth is 29 m, the wave length is 132 m and the induced cyclic stress ratio at mud line becomes equal to 0.12. The depth of liquefaction penetration can be determined first by computing the ratio between the cyclic stress ratio at failure and the induced cyclic stress ratio at mud line, i.e.

$$\frac{(\tau_{vh,l}/\sigma_v')}{(\tau_{vh}/\sigma_v')_0} = \frac{0.076}{0.12} = 0.633 \quad (38)$$

Then, entering in the chart in Fig. 4, a point on the indicated curve is located corresponding to a value of 0.633 in the abscissa. Reading off the depth factor, z/L , on the ordinate, one obtains

$$z/L=0.07$$

Since the wave length is already known, the depth of liquefaction penetration, z_c , is determined as

$$z_c=0.07 \times 132=9.2\text{ m}$$

Fig. 14. Examples of wave-induced liquefaction analysis for shoaling seabed deposit

The result of the above analysis is indicated in Fig. 14(a).

Similar calculation can be made for other assumed values of cyclic stress ratio induced at mud line and corresponding water depth and depth of liquefaction penetration can be determined. By repeating the above calculation as many times as necessary, the zone of liquefaction can be established as demonstrated in Fig. 14(a). The result of analysis shown in Fig. 14(a) indicates that, after liquefaction begins to occur when the waves come to the location with a water depth of 41 m, the liquefaction continues to penetrate deeper in the seabed deposit until the waves come to the location of 14 m water depth where the waves break. The sediment is liquefied to a depth of 19 m. The waves propagating further are continuously subjected to breaking and consequently, with decreasing magnitude of induced cyclic stress ratio, the depth of liquefaction penetration becomes again equal to zero at the location of 0.6 m water depth.

For comparison sake, the other cases will be considered in which the density of sand deposit is 80% and 90% and respective K_0 -value is assumed to be 0.7 and 1.0. The assumption on the wave profile in deep water is the same as in the case of the aforementioned example. The result of the analysis conducted in the same manner as above is also shown in Fig. 14(a). It can be seen that liquefaction develops shallower in the deposit and narrower in the lateral extent. It is to be noted that the location of the deepest liquefaction penetration is the same for all cases considered above. This is due to the fact that the location of the maximum induced cyclic stress is determined by the condition of wave breaking which is in turn is related only with the wave profile factors. The other cases with different conditions, $L_0=100$ m, in deep water are similarly analyzed as shown in Fig. 14(b).

The method of liquefaction analysis described above does not take into account the effects of pore water pressure dissipation that could take place during a storm period.

Therefore, if the liquefaction were to develop only in the shallow depth of deposits, the analysis result will grossly be on a safe side. However, if the liquefaction penetrates deep in the seabed deposit, the result of analysis is conceived to be closer to a real picture of liquefaction occurrence.

CONCLUSIONS

By assuming the seabed deposit to be a homogeneous elastic half-space and also by assuming the wave-induced load on the seabed to be a train of harmonic waves, two-dimensional stress analysis was made to examine the nature of cyclic stresses induced in the seabed deposit. The analysis showed that the wave-induced stresses are characterized by the continuous rotation of the principal stress directions. In an effort to simulate such conditions of cyclic loading, a series of the cyclic triaxial torsion shear tests was conducted on sand specimens prepared in the laboratory to a relative density of 45%, 78% and 92%. On the basis of the results of these tests, an empirical formula was established to evaluate the cyclic strength allowing for the effect of rotation of the principal stress directions.

In order to evaluate the magnitude of cyclic stresses induced by waves in the seabed deposit, the linear theory of wave propagation was used and the changing feature of wave profile as the wave propagates shorewards on the sloping seabed was clarified. It was shown that the wave steepness, i.e. the height to length ratio increases as the waves travel over the sloping seabed with decreasing water depth and that the wave starts breaking when the wave steepness reaches a certain limit which is specified by the water depth and wave length. It was further shown that the condition imposed by the wave breaking leads to a maximum value of the cyclic stress ratio of 0.2379 that can be induced in the seabed deposit. A methodology was proposed to evaluate the wave-induced cyclic stress ratio at locations of given water depth, based on

the wave conditions given offshore in deep water.

The methodology to evaluate susceptibility of the seabed sediment to liquefaction type failure was then established by incorporating the above mentioned empirical formula of the cyclic strength into the analysis procedure to evaluate the cyclic stress ratio induced by waves. The above methodology for liquefaction analysis was applied to some typical cases in which the most severe storm conditions are considered. The results of analysis showed that, for a medium dense deposit of sand with a 60% relative density, liquefaction could extend down to a depth of about 19 m at a location of 14 m water depth where wave breaks for the first time in the course of the wave propagation.

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