ESTIMATION OF THE FUNDAMENTAL PERIOD OF LARGE EARTHFILL AND ROCKFILL DAMS

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ABSTRACT

A semi-empirical expression has been derived for the fundamental period of large earthfill and rockfill dams. Derivation is based on analytical solutions and has been calibrated using the empirical data available. Analytical solution involves a heterogeneous shear wedge with a realistic distribution of small-strain shear modulus. The effect of the "third dimension" has also been taken into account by making use of two and three dimensional finite element and shear wedge solutions, and a simple expression has been derived by the aid of a single degree of freedom idealization. Empirical data come from forced vibration tests and response observations during small earthquakes. Recently the fundamental period of a large rockfill dam became available which had not been used in the development of the period expression presented. The accurate prediction of the period of this dam has shown that the equation will be useful in practice for estimation of the small strain fundamental periods of large dams.

Key words : dam, earthfill, earthquake, field test, rockfill, (soil dynamics), vibration (IGC : E 8/F 7)

INTRODUCTION

In the design of large embankment dams a critical loading case that often dominates the entire design process is the earthquake condition. Therefore it is necessary to consider this condition starting in the preliminary phases in design. Significant advances have been made in realistic dynamic response analyses of embankments in the past few decades, and powerful design/analysis procedures have been developed using shear wedge idealizations (Ambraseys

and Sarma, 1967; Gazetas, 1980), plane strain finite elements (Chopra, 1967; Lysmer et al, 1974), and recently by three dimensional finite elements (Hayashi, Kitahara and Komada, 1973; Mejia, Seed and Lysmer, 1982). These procedures have been extremely useful in practice and improved our understanding of the dynamic behavior of dams during earthquakes. However, these methods are not suitable for hand calculations for preliminary design of important dams and for simplified analyses of less important dams.

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A simplified procedure has been proposed by Makdisi and Seed (1977) to compute maximum crest accelerations and natural periods for embankments with known dynamic properties. This method is essentially a modal analysis using a design response spectrum in which mode shapes and frequencies of the conventional homogeneous shear wedge theory are utilized. This method allows the consideration of nonlinearity using strain dependent material properties, and has been shown to agree with the results of more detailed finite element analyses. The information derived by using this method may in turn be used in estimating the permanent displacements induced by a design earthquake.

Makdisi-Seed simplified procedure for estimating peak crest accelerations and fundamental periods can be useful for many practical purposes. A successful application, however, requires (a) an estimation of strain dependent shear modulus and damping values, and (b) that the embankment geometry allows a plane strain idealization. Determination of cyclic shear modulus of fill materials in the laboratory is a difficult task due to such factors as high confining pressures necessary to simulate the stress level involved and large particle sizes especially in the case of rockfill. In addition, the ratio of crest length to dam height, l/h , in large dams lies mostly between 2 and 10, and therefore, plane strain approximation may not be valid for a significant percentage of large dams where l/h ratio is less than about 5. If these factors are taken into account, the usefulness of the simplified procedures will improve significantly.

In this study a simple formula has been derived for the fundamental period of embankments. Derivation is based on approximate analytical solutions of free vibrations of a heterogeneous shear wedge with a realistic distribution of stiffness. The effect of the "third dimension" has also been taken into account by making use of two and three dimensional finite element and

shear wedge solutions, and a simple expression has been derived by the aid of a single degree of freedom idealization. Next, the equation has been calibrated using the empirical data available.

The semi-empirical formula derived in this study has been applied to predict the small-strain fundamental period of a large rockfill dam for which data has become available recently.

HETEROGENEOUS SHEAR WEDGE

Representation of Modulus Distribution

It is assumed that the shear modulus, G , of embankment materials vary, as in soils (Seed and Idriss, 1970; Hardin and Drnevich, 1972), with the square root of effective confining pressure:

$$G = AF\sqrt{\sigma'_0 p_a} \quad (1)$$

where AF is a dimensionless modulus factor, σ'_0 is the mean effective stress at a point, and p_a is the atmospheric pressure in the same units as that used for modulus and stress. Strain dependency of modulus and the presence of zones in the embankment are not considered at this point. As a result of the confinement dependency of

Fig. 1. Heterogeneous modulus distribution within a "homogeneous" embankment

small-strain shear modulus defined by Eq. (1), G varies vertically and horizontally within the embankment as shown in Fig. 1. This distribution is obtained if it is assumed that the material has a constant unit weight γ' and a coefficient of lateral pressure K (somewhere between K_0 and K_a), and that the vertical stress at any depth z from the surface to be approximated as $\gamma'z$, such that the mean effective stress is given by

$$\sigma'_0 = \frac{1+2K}{3} \gamma' z \quad (2)$$

The maximum value of the shear modulus, G_1 , for a horizontal section of the embankment, at a depth y from crest, is then

$$G_1 = AF \sqrt{\frac{1+2K}{3} \gamma' y p_a} \quad (3)$$

and the average modulus for the slice is

$$\bar{G} = G_0 \sqrt{y} \quad (4)$$

where

$$G_0 = \frac{2}{3} AF \sqrt{\frac{1+2K}{3} \gamma' p_a} \quad (5)$$

Zoned Dams

The above representation of the modulus distribution simplifies the analytical treatment of the problem. Comparisons of shear wedge and finite element solutions (e.g., Chopra, 1967; Makdisi and Seed, 1977) show that the shear wedge idealization yields a sufficiently accurate fundamental period value. Since the primary aim of the present study is the derivation of a simple period formula, the approximation above (Eqs. (4) and (5)) should serve the purpose. In the case of a zoned dam weighted averages of the parameters may be used since the relative horizontal thicknesses of various zones of the embankment do not change significantly in the vertical direction.

Mode Shapes and Frequencies

In order to determine the natural frequencies and mode shapes of the heterogeneous shear wedge, a differential equation of motion is formulated and solved for free

vibrations. For the sake of completeness the solution is briefly described in Appendix A. From this solution, the natural circular frequencies are

$$\omega_i = \alpha_i \sqrt{\frac{G_0}{\rho h^{3/2}}} \quad (6)$$

where ρ is the mass density, h is height of the embankment, G_0 is defined in Eq. (5), i is the mode number, and

$$\alpha_1 = 2.177, \quad \alpha_2 = 4.525, \quad \alpha_3 = 6.88, \dots$$

corresponding mode shapes and modal participation factors are given in Appendix A.

An expression for the fundamental period, T_1^* , of the heterogeneous shear wedge is obtained by substituting Eq. (5) in Eq. (6) and simplifying;

$$T_1^* = \frac{2\pi}{2.177} h^{3/4} \left[\frac{2}{3} \frac{AF}{\rho} \sqrt{\frac{1+2K}{3} \gamma' p_a} \right]^{-1/2} \quad (7)$$

It is observed that the fundamental period of an embankment, in which shear modulus varies with the square root of the effective confinement, increases with $3/4$ power of the height rather than linearly as in the conventional homogeneous shear wedge.

3 D Effect

The fundamental period expression in Eq. (7) has been derived on the basis of a plane strain shear wedge. To take into account the effect of the third dimension (i.e., width of valley) a correction factor may be applied. Such a correction factor would be a function of l/h ratio and the shape of the valley cross section. Shapes of real embankment cross sections across the valley are too complicated to be approximated by a simple geometric shape such as a triangle or a rectangle. If a single shape is to be selected a parabola is probably the closest. It has been shown earlier (Oner, 1980) that one may use an "equal-areas" criterion to convert all valley shapes into an equivalent parabola. This criterion yields

$$l/h = 1.5 A/h^2 \quad (8)$$

where A is the valley cross section area, h is the embankment height, and l/h is the

equivalent aspect ratio. For example the equivalent aspect ratio for a rectangular valley is 1.5 times the real l/h ratio and for a triangular shape it is 0.75 times that. This definition of equivalent aspect ratio can easily be tested using 2D and 3D finite element and shear wedge solutions. In the extreme, for example, one may try to predict the solution for a rectangle case from the solution of a triangle case, or vice versa. Such tests have shown that Eq. (8) is sufficiently accurate for practical purposes.

Once a parameter is defined to represent the valley shape a correction factor, say θ , can be derived such that

$$T_1' = \theta T_1^* \quad (9)$$

where T_1' is the fundamental period considering 3D effects, and T_1^* is the 2D solution (Eq. (7)). For the purpose of this study a simple expression has been derived using a simple, single degree of freedom model (Appendix B);

$$\theta = \frac{1}{\sqrt{1 + \frac{4}{(l/h)^2}}} \quad (10)$$

Finally, combining Eqs. (7) to (10), and simplifying, one obtains

$$T_1' = 4.653 h^{3/4} \left[\frac{AF}{\rho} \left[1 + \frac{4}{(l/h)^2} \right] \times \sqrt{(1+2K)\gamma' p_a} \right]^{-1/2} \quad (11)$$

Eq. (11) is an approximate expression for the fundamental period of a three-dimensional heterogeneous shear wedge. It can be used for three purposes:

(1) To estimate the natural period of embankments with known material properties, for small strain vibrations,

(2) To back calculate the dimensionless modulus coefficient, AF , for embankments for which fundamental period is known (Oner and Erdik, 1981), and

(3) To form the necessary analytical basis for a semiempirical natural period expression that can be useful for various practical purposes. This will be done in the following.

DATA BASE

There are basically two methods for determining the dynamic response characteristics of large embankment dams: (1) forced vibration testing, and (2) studying earthquake response. Forced vibration testing is perhaps the most commonly used method, the advantage being that it can be done any time as opposed to earthquake response observations. The number of large dams tested to date, however, is still very small.

In a forced vibration test vibration generators of double rotating-mass type are installed at the crest of the dam. A pair of vibration generators is preferred to increase the dynamic force amplitude and to be able to excite nonsymmetric modes. Excitation is commonly applied in transverse (upstream/downstream) direction with the exception of a few cases where longitudinal vibration has also been studied. Vibration generators are run over the frequency range of typically 1 to 10 Hz. The steady-state response of the dam at various frequencies is monitored by a set of accelerometers and/or seismometers placed along the crest and over the downstream slope. The data obtained is reduced into a number of natural frequencies and the corresponding mode shapes.

The author and his colleagues tested two large dams, a 62 m high zoned earthfill dam, Cubuk II, with a wide, compacted silty clay core and sandy gravel shells of alluvial origin, and a 200 m high rockfill dam, Keban, with steep (1:1.7) slopes built in a relatively narrow valley (Oner and Erdik, 1980; Erdik et al, 1980). Keban dam is the largest embankment dam tested by forced vibration method. The fundamental periods of Cubuk II and Keban dams have been found to be 0.44 and 0.61 sec, respectively, and the damping ratios obtained by bandwidth method (around the fundamental frequency peak) are 3% to 4% for both dams. Shear strain amplitudes induced in the embankments during testing have been estimated to be in the order of

Table 1. Data base

Dam Name	Type ⁽¹⁾	Height (m)	Crest Length (m)	Average Slope ⁽²⁾	l/h ⁽⁴⁾	Fundamental Period, T (sec)	Reference
Bouquet Canyon	E	63	363	3.00	5.74	0.45	Keightley (1964)
Brea	E	26.5	538	3.58	20.3	0.37	Abdel-Gaffar and Scott (1980)
Carbon Canyon	EH	30(31) ⁽³⁾	587	3.25	13.0	0.64	Abdel-Gaffar and Scott (1980)
Cubuk II	E	62	230	3.00	3.20	0.44	Oner and Erdik (1981)
Dry Canyon	EH	18(23)	148	2.53	8.09	0.52	Keightley (1964)
Keban	R	165 ⁽⁵⁾	610	1.69	3.22	0.61	Oner and Erdik (1981)
Kisenyama	R	95	225	2.48	2.68	0.37	Okamoto (1973)
Kuzuryu	R	128	355	2.25	2.77	0.37	Okamoto (1973)
Miboro	R	131	405	2.13	3.09	0.42	Okamoto (1973)
Rema	R	90	230	1.35	2.00	0.23	Paskalov et al (1980)
Rifle Gap	E	37(31)	457	—	12.5	0.32	Heller and Ahlberg (1973)
Sannokai	E	37	145	2.75	3.92	0.35	Okamoto (1973)
Santa Felicia	E	61(23)	389	2.81	5.88	0.61	Abdel-Gaffar and Scott (1979)
Tajik	EH	30	129	4.00	4.30	0.50	Atrakova et al (1973)
Tarumizu	R	39	137	3.05	2.00	0.26	Yanagisawa et al (1980)
Yanase	R	115	202	2.30	1.76	0.23	Okamoto (1973)
Yugo.	R	100	191	2.00	1.91	0.39	IZIIS (1978)

(1) E: Zoned earthfill, EH: Homogeneous earthfill, R: Rockfill

(2) Average of the upstream and downstream slopes

(3) Alluvium thickness, if not excavated

(4) See text for "effective" l/h ratio

(5) Neglecting concrete plug

10^{-4} to 10^{-5} percent by making use of shear wedge solutions and also by dividing the peak crest displacement by the height of the embankment. Thus the properties determined pertain to the "small-strain" range of the cyclic stress-strain relationship of embankment materials. Detailed information about the other characteristics of the dams, test equipment, procedure, and results may be found in the references cited.

Abdel-Gaffar and Scott (1979, 1980) have presented dynamic property data for three dams obtained by forced vibration testing and earthquake observations. These three dams are relatively small embankments with heights on the order of 30 m. In determining dynamic characteristics from earthquake records, Abdel-Gaffar and Scott have made use of shear wedge solutions and amplification spectra (i. e., the ratio of Fourier amplitude spectra of motions recorded at the crest and near the base of the dam).

Okamoto (1973) gives data on a number of dams, but only few of them have been found useful for the purpose of this study since data on embankment characteristics are not reported for the remaining ones.

Okamoto supplies data on four large rockfill dams with height on the order of 100 m, and one medium size zoned earthfill dam. Unfortunately the information presented by Okamoto are not quite detailed. The fundamental period data derived from earthquakes appears to be by observation of the predominant period of the crest motions recorded. The minimum values of the reported periods, determined from smaller earthquakes or by forced vibration testing, have been used in this study to make the data base consistent.

Other data collected from literature come from various sources (Keightley, 1964; Paskalov et al, 1980; Heller and Ahlberg, 1973; Atrakova et al, 1973; Yanagisawa et al, 1980). These data involve small to medium size (40 m to 60 m high) dams with the exception of two, a 90 m and a 100 m high rockfill dam in Yugoslavia. There is a large body of data in literature, in addition to the ones cited, on natural periods and mode shapes of dams. However, those data obtained solely from analyses, and those derived from nonlinear response during strong earthquakes have been

disregarded. The data base thus formed is summarized in Table 1.

SEMI-EMPIRICAL FORMULA

Derivation

The data base currently available is not statistically sufficient for deriving a purely empirical formula. At present a reliable formula can only be derived by making use of the theoretical formulation presented earlier (Eqs. (1) to (11)). For this purpose the coefficients and constants in Eq. (11) related to material characteristics may be grouped together. The effects of fill height and the third dimension can be taken as found previously. Thus, the form of the semi-empirical formula becomes:

$$T_1 = f_1(m) \cdot f_2(h) \cdot f_3(l/h) \quad (12)$$

where

$$f_2(h) = h^{3/4}$$

$$f_3(l/h) = [1 + 4/(l/h)^2]^{-1/2}$$

and $f_1(m)$ is an unknown function of material type to be determined empirically in which m denotes an arbitrary index for material type. In this study, the following values are assigned for the three basic groups of embankment materials:

$m=0$ for homogeneous earth dams

$m=1$ for zones earth dams

$m=2$ for rockfill dams

Values of $f_1(m)$ have been determined for the dams in Table 1, and the following exponential function has been found to fit the data closely:

$$f_1(m) = \exp(a \cdot m + b) \quad (13)$$

A least squares fit to the available data yields $a=0.555$ and $b=-3.206$. Substituting these in Eqs. (13) and (12) one obtains

$$T_1 = 0.0408 e^{-0.555m} h^{0.75} [1 + 4/(l/h)^2]^{-0.5} \quad (14)$$

in which h is in meters.

The comparison of observed and predicted fundamental periods shown in Fig. 2 indicates that the scatter is reasonably small in contrast to the simplicity of the semi-

Fig. 2. Comparison of measured and calculated periods

empirical expression.

Application

Recently data from earthquake observations and analyses of a large rockfill dam has become available (Mejia, Seed and Lysmer, 1982; Vrymoed, 1981). This gives an opportunity to test Eq. (14). The height of the embankment is 750 ft (230 m) and the crest length is 5150 ft (1700 m), and the valley cross section is nearly triangular with an equivalent $l/h \cong 5$ using Eq. (8). Using $m=2$ (for rockfill) Eq. (14) gives the "small-strain" fundamental period as 0.74 sec. From the three earthquakes recorded at the dam site and at the crest, Vrymoed reports that the fundamental period of the dam was 0.75 sec as observed from the free vibration parts of the recordings which presumably corresponds to small-strain range response, and the period increases to 0.80 sec due to nonlinearity during stronger shaking. Mejia et al also observe 0.8 sec fundamental period in their study of the strongest one of the three earthquakes recorded. In spite of the fact that this case represents an extrapolation the prediction is excellent.

CONCLUSIONS

A semi-empirical expression has been derived for the fundamental period of large embankment dams. Because of the scarcity of data the derivation has partly been based on an approximate analytical solution. Analytical solution is based on the assumption that the small-strain shear moduli of embankment materials vary with the square root of the effective confining pressure. A shear wedge idealization is then made in which material properties are averaged over a horizontal plane resulting in a simpler distribution that can be treated analytically. Natural frequencies, mode shapes, and participation factors have been presented which may be useful for hand calculations. The effect of the "third dimension" has also been considered in the simplified analytical expression for fundamental period. The expression has then been calibrated using the currently available empirical data. The accurate prediction of the period of a large dam, for which data has become available recently, shows that the semi-empirical formula derived can serve as a prediction tool as well as a framework for more refined expressions that may be developed in the future as a larger data base becomes available.

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APPENDIX A

Derivation of Eq. (6)

The analytical background of Eq. (6) and

Fig. 3. Heterogeneous shear wedge

other details of the solution are given below. Denoting the lateral displacement of the embankment at a depth y from crest at a time t by $u(y, t)$, the differential equation governing free vibrations can be written as (Fig. 3)

$$\frac{G_0}{\rho} \left[y^{1/2} \frac{\partial^2 u}{\partial y^2} + \frac{3}{2} y^{-1/2} \frac{\partial u}{\partial y} \right] = \frac{\partial^2 u}{\partial t^2} \quad (\text{A1})$$

Solution can be obtained by using the method of separation of variables, with

$$u(y, t) = U(y) \cdot V(t) \quad (\text{A2})$$

$$V(t) = V_0 e^{i\omega t}$$

$$x = y/h$$

where $V(t)$ represents a harmonic motion at a frequency ω . Substituting Eq. (A2) in Eq. (A1) and simplifying, one obtains the ordinary differential equation

$$\frac{G_0}{\rho h^{3/2}} \left[x^{1/2} \frac{dU}{dx} + \frac{3}{2} x^{-1/2} \frac{d^2 U}{dx^2} \right] = -\omega^2 U \quad (\text{A3})$$

Rearranging the equation with the abbreviation

$$\alpha^2 = \omega^2 \rho \frac{h^{1.5}}{G_0} \quad (\text{A4})$$

the governing differential equation simplifies to

$$\sqrt{x} U'' + \frac{1.5}{\sqrt{x}} U' + \alpha^2 U = 0 \quad (\text{A5})$$

in which primes denote differentiation with respect to x . The general solution of this differential equation is

$$U = x^{-1/4} J_{1/3}(\lambda x^{3/4}) \quad (\text{A6})$$

where $J_{1/3}$ is the Bessel function of the first kind of order $1/3$. The λ parameter is determined by the boundary condition $U=0$ at $x=1$ at the base of the dam, which then gives the values of λ as the

zeroes of $J_{1/3}$ function. Each λ_n value determines a mode shape, U_n , and a natural frequency ω_n . Natural frequency, ω_n , for mode number n can be calculated by Eq. (A 4) and the following relationship between λ_n and α_n :

$$\alpha_n = \frac{3}{4} \lambda_n \quad (\text{A7})$$

The zeroes may be evaluated either numerically computing $J_{1/3}$ function from infinite series, or by making use of the tables available. Here the Bessel function was first expressed in terms of Airy functions and tables were used (Abramowitz and Stegun, 1972). The first four roots determines in this manner are

n	λ_n	α_n
1	2.9025	2.1769
2	6.0328	4.5246
3	9.1707	6.8780
4	18.5907	13.9434

Substitution of these zeroes in Eq. (A 6) yields mode shapes. First three mode shapes are shown in Fig. 4. Modal participation factors needed for simplified

Fig. 4. First three mode shapes of heterogeneous shear wedge

response analyses have also been computed using the basic principles,

$$\Gamma_n = \frac{\int_0^1 U(x) m(x) dx}{\int_0^1 U^2(x) m(x) dx} \quad (\text{A8})$$

where $m(x)$ is the mass of a horizontal slice of the wedge of thickness dx which increases linearly with x . Thus $m(x)$ in Eq. (A 8) may be replaced by x cancelling the linear proportionality factor. The modal participation factors found in this manner are 1.86, 1.70, and 1.53 for the first three modes.

APPENDIX B

Derivation of Eq. (10)

A simple single degree of freedom model that represents a shear wedge within a triangular valley is shown in Fig. 5. Model shown in Fig. 5 represents one half of an embankment separated along the symmetry plane perpendicular to dam axis. The idealization consists of a lumped mass, m , and a shear spring, k . If z denotes the distance of the crest mid-point to the abutment plane, then the lumped mass can be calculated as

$$m = \beta^3 \frac{z}{3} A_b \rho \quad (\text{B1})$$

Fig. 5. Single degree of freedom model for the derivation of 3D correction factor

in which β is an as yet undetermined fraction of z defining the portion of the embankment that acts as lumped mass, A_b is the area of the base of the separated mass, and ρ is the mass-density.

To derive the spring constant expression, let the distance of the center of gravity of the mass to the crest mid-point be denoted by a fraction α of the distance z . Then the sloping mid-plane of the spring portion is at a distance ξz where $\xi = (1 + \alpha)/2$ (Fig. 5). The force in the spring can conveniently be computed at this mid-plane as:

$$\tau_m \cdot A_m$$

where $\tau_m = \gamma_m G$

$$A_m = \xi^2 A_b$$

and γ_m represents the shear strain at the mid-plane. Assuming the shear spring deforms uniformly the displacement of the center of gravity of the mass is

$$\gamma_m (1 - \alpha) z$$

Finally, the ratio of the force and the displacement computed above will be the spring constant:

$$k = G \frac{\xi^2 A_b}{(1 - \alpha) z} \quad (\text{B2})$$

where G is the shear modulus, and ξ and

α are auxiliary geometric parameters shown in Fig. 5 which can be related to β parameter.

Once m and k are defined, natural period can easily be calculated as

$$T_{3D} = 2\pi \sqrt{\frac{\beta^3 (1 - \alpha)}{3 \xi^2}} \frac{z}{v_s} \quad (\text{B3})$$

where $v_s = \sqrt{G/\rho}$, shear wave velocity. Expressing z in terms of l and h and imposing the condition $l/h \rightarrow \infty$ representing the plane-strain (2D) condition,

$$T_{2D} = 2\pi \sqrt{\frac{\beta^3 (1 - \alpha)}{3 \xi^2}} \frac{h}{v_s} \quad (\text{B4})$$

One may equate this expression to the conventional shear wedge solution and evaluate β . Instead, the "period ratio," θ , is computed here by its definition in Eq. (9), i. e., as the ratio of T_{3D} to T_{2D} ;

$$\theta = \frac{1}{\sqrt{1 + \frac{4}{(l/h)^2}}} \quad (\text{B5})$$

A comparison of this equation with 2D and 3D finite element solutions for embankments in parabola shaped valleys has shown that it represents the 3D effect quite accurately (Oner, 1980).