ESTIMATION OF SOIL PARAMETERS BASED ON MONITORED MOVEMENT OF SUBSOIL UNDER CONSOLIDATION

KATSUHIKO ARAI*, HIDEKI OHTA** and KEISUKE KOJIMA***

ABSTRACT

This paper presents a numerical procedure in which the soil parameters affecting the two dimensional consolidation are back-calculated from the data monitored in the field. By using a combination of finite element method and mathematical programming technique, the procedure allows to seek the unknown soil parameters which minimize the differences between calculated and measured quantities. Young's modulus, Poisson's ratio, and permeability coefficient in multiple soil layers are the soil parameters determined by the procedure. The validity of the proposed procedure is verified by calibrating the performance of the procedure being applied to hypothetical case studies. The applications to practical projects provide the interesting results such as the changing trend of soil parameter values during consolidation process. By applying the procedure at the early stage of consolidation, one may be able to predict the future settlements with a good accuracy.

Key words: computer application, consolidation, elasticity, finite element method, measurement, permeability, settlement, soft ground (IGC : E 2/H 0)

INTRODUCTION

The most difficult task in analysing the settlement due to consolidation of a soil deposit, is the determination of soil parameters affecting the progress of consolidation such as deformation moduli and permeability coefficient. The consolidation analysis, based on the soil parameters estimated by laboratory and by in-situ testing, does not provide the good predictions in many cases. In order to compensate for such deficiency, the revision of soil parameters is practiced during the construction phase, so that the settlements may be better predicted and agree with the observed

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ones. When finite element method (FEM) is employed to solve multi-dimensional consolidation problem, such revision of soil parameters requires too much effort in heuristic trials, especially in the case where the soil deposit consists of multiple layers. This paper presents a numerical procedure to practice automatically such revision in multi-dimensional consolidation analysis by the use of FEM and mathematical programming technique. When assuming the elastic behaviour of soil skeleton, the soil parameters to be found are Young's modulus, Poisson's ratio, and permeability coefficient. By the use of mathematical programming technique, this procedure continues to revise the unknown soil parameters in a way that the differences between the observed and the predicted values of settlement becomes sufficiently small. Arai, Ohta and Yasui (1983) proposed a numerical procedure to determine the elastic constants of soil deposit based on the informations from field observations. This paper extends the previous procedure to multi-dimensional consolidation problem. Gioda (1979) proposed a procedure for the inverse analysis which works well if the number of the soil parameters is not large. Asaoka (1978), Asaoka and Matsuo (1980) proposed another procedure of predicting the future settlement using a set of field settlement data obtained in the very beginning of construction phase. This procedure is backgrounded by a set of differential equations mathematically equivalent to the solution of Biot's equation of consolidation. The authors' procedure may have advantage over these two other methods for much faster and stabler computation caused by some mathematical treatment on one hand and for wider applicability caused by rather straight forward way of approach on the other hand. The applications of the authors' method to actual embankment projects, show the interesting results, for instance, the changing trend of soil parameters during consolidation process.

PROBLEM FORMULATION AND NUMERICAL ANALYSIS

Consolidation Analysis

Many numerical procedures for multi-dimensional consolidation analysis by the use of FEM may be classified into two types, i.e. the procedure by Christian and Boehmner (1970), and that by Sandue and Wilson (1969). The former treats stress, strain and pore water pressure as being constant throughout each element, whereas the latter evaluates both displacement and pore water pressure by the values at the nodal point. Based on the former, Akai and Tamura (1978) formulated the extended element stiffness equations as

\[ [K_e] \{u^{n+1}\} + \{L\} p^{n+1} = \{f^{n+1}\} \tag{1} \]

\[ \{L\}^t \{u^{n+1}\} - \alpha \cdot p^{n+1} + \sum_i (\alpha_i \cdot p_i^{n+1}) = V^n \tag{2} \]

where \([K_e]\): element stiffness matrix, \(\{u^n\}\): nodal displacements, \(p^n\): element pore water pressure, \(p_i^n\): pore water pressure of neighboring element (see Fig.1), \(\{L\}\): vector to calculate the element volume change from nodal displacements, \(\{f^n\}\): nodal loads, \(V^n\): volume change of the element, and \(n\) denotes the discretized time step. Eq. (2) is the equation of continuity to represent that the volume change of an element must be equal to the total flow rate from neighboring elements into the element. Referring to Fig.1, \(\alpha\) and \(\alpha_i\) in Eq. (2) are expressed as

![Fig. 1: Representation of equation of continuity](image)

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where $k$: permeability, $\Delta t$: increment of time step, and $\gamma_w$: unit weight of the water. Arai, Watanabe and Tagyo (1983) compared Akai and Tamura's procedure with Sandue and Wilson's one and reported that the former provides more stable solutions than the latter concerning the distribution of pore water pressures. Furthermore the former does not require the computation by double precision, when computing the inverse matrix of global stiffness matrix (see Arai, Watanabe and Tagyo, 1983). This advantage is effective to reduce the time of computation. For these reasons, the authors employ Akai and Tamura's procedure in analysing the multi-dimensional consolidation problem.

**Problem Formulation**

In consolidation process, assume that some vertical or horizontal displacements of soil deposit and some pore water pressures are observed with passage of time. The problem is to find Young's modulus $E$, Poisson's ratio $\nu$ and permeability $k$ by the back analysis of the field measurement data, assuming the elastic behaviour of soil deposit. Note that the elastic constants being considered here are in terms of effective stresses. This problem is formulated as an optimization problem to choose $E$, $\nu$ and $k$ which minimize the sum of squares of differences between measured quantities and those calculated from values of soil parameters assumed in heuristic trials, if one accepts the assumptions that the measured quantities can be equally weighted regardless of their quality or reliability of measurement and location of measuring point.

\[
J = \sum_{n=1}^{N} \sum_{i=1}^{N_t} (u_i^n - U_i^n)^2 + S \sum_{j=1}^{N_p} (p_j^n - P_j^n)^2
\]

where $J$: objective function, $N$: number of time steps, $N_t$, $N_p$: number of measured values of displacement, or of pore water pressure, $U_i^n$, $P_j^n$: measured values of displacement or of pore water pressure, $S$: scaling factor to equalize the order of magnitude of two terms, and $u_i^n$, $p_j^n$ are calculated as

\[
[u^n, p^n]^T = [K]^{-1} [F^n] \\
u^n = [u_i^n], \quad p^n = [p_j^n]
\]

where $[K]$: global stiffness matrix and $\{F^n\}$: vector of a known quantity, both of which are obtained by the superposition of Eqs. (1) and (2). When equalizing the weight of measured quantities at each step, it is possible to formulate as

\[
\text{minimize } J = \sum_{n=1}^{N} \left( \sum_{i=1}^{N_t} (u_i^n - U_i^n)^2 + S \sum_{j=1}^{N_p} (p_j^n - P_j^n)^2 \right)
\]

where $\Delta$ denotes the increment of each quantity. When assuming no change of soil parameters in each step of consolidation process, the extended element stiffness equations corresponding to such incremental formulation, are as follows.

\[
[K_e] \{\Delta u^{n+1}\} + (L) \Delta p^{n+1} = \{F^{n+1}\} \\\n(L)^T(\Delta u^{n+1}) - \alpha \cdot \Delta p^{n+1} + \sum_{i=1}^{N_t} \alpha_i \cdot \Delta p_i^{n+1} = \alpha \cdot p^n - \sum_{i=1}^{N_t} \alpha_i \cdot p_i^n
\]

Eqs. (5) and (7) require the field measurement data with an adequate time interval, such as $U_1^n$, $U_2^n$, ..., $U_{N_t}^n$. In actual cases, one may not be able to get the measured quantities at the all time steps. However it is not difficult to guess the missing quantities by adequate interpolations.

**Numerical Analysis**

It is not easy to solve the formulated optimization problem analytically, because of the high nonlinearity in the objective function. There is a numerical procedure called a conjugate gradient technique proposed by Fletcher and Reeves (1964), which can be effectively used in this optimization problem. The detailed introduction of the procedure of this technique is not given here.
(see Arai, Ohta, and Yasui, 1983). The technique is outlined as follows. Calculate the gradient of objective function on decision variables to be determined. Set the initial values of decision variables. Repeat the iteration procedure to search the optimal solution of decision variables, which minimizes the objective function. The gradient of the objective function \( J \) on \( E \), which is necessary to apply the conjugate gradient technique, is calculated as

\[
\frac{\partial J}{\partial E} = 2 \sum_{n=1}^{N_H} \left[ \sum_{i=1}^{N_T} \left( u_i^n - U_i^n \right) \cdot \frac{\partial u_i^n}{\partial E} \right] + \sum_{j=1}^{N_T} \left( p_j^n - P_j^n \right) \cdot \frac{\partial p_j^n}{\partial E} \tag{10}
\]

\[
\frac{\partial u_i^n}{\partial E} = \left\{ \left[ K^{-1} \right] \frac{\partial E}{\partial E} \cdot \left[ F^{n+1} \right] \right\}_i + \left[ K^{-1} \right] \cdot \left( \frac{\partial \left[ F^{n+1} \right]}{\partial E} \right)_i \tag{11}
\]

where \( \{ \}_i \) represents the \( i \)-th element of the vector, and

\[
\frac{\partial [K]}{\partial E} = -\left[ [K]^{-1} \cdot \frac{\partial [K]}{\partial E} \cdot [K]^{-1} \right]^T \tag{12}
\]

Since the global stiffness matrix \([K]\) is the superposition of \([K_e]\), \([L]\), and \( \alpha \) in Eqs. (1) and (2), and since both \([L]\) and \( \alpha \) do not include \( E \), the term \( \frac{\partial [K]}{\partial E} \) in Eq. (11) is obtained by the superposition of \( \frac{\partial [K_e]}{\partial E} \), and

\[
\frac{\partial [K_e]}{\partial E} = \int_{\text{Vol}} [B]^T \cdot \frac{\partial [D]}{\partial E} \cdot [B] \, dv \tag{13}
\]

where \([B]\): matrix to calculate strains from nodal displacements, \([D]\): stress-strain matrix, and \( \text{Vol} \): volume of the element. When assuming the plane strain condition,

\[
\frac{\partial [D]}{\partial E} = \begin{bmatrix}
\frac{1}{1+\nu}(1-2\nu) & 0 \\
\nu/(1-\nu) & 0 \\
0 & 0
\end{bmatrix} \times
\begin{bmatrix}
1 & 0 \\
(1-\nu) & (1-2\nu)/2(1-\nu)
\end{bmatrix} \tag{14}
\]

\( \frac{\partial \left[ F^{n+1} \right]}{\partial E} \) in Eq. (11) is obtained essentially the same way. The gradient on \( \nu \) is identical to \( \frac{\partial j}{\partial E} \) except exchanging Eq. (14) for the following equation.

\[
\frac{\partial [D]}{\partial \nu} = \frac{E}{(1+\nu)^2(1-2\nu)^2}
\begin{bmatrix}
2\nu(2-\nu) & 0 \\
1+2\nu^2 & 0 \\
0 & -(1-2\nu)^2/2
\end{bmatrix} \tag{15}
\]

The gradient on \( k \) is also calculated in a similar manner. That is, \( \frac{\partial [K]}{\partial k} \) is obtained by the superposition of \( \frac{\partial \alpha}{\partial k} \) and of \( \frac{\partial \alpha}{\partial k} \) in each element stiffness equation, consulting Eqs. (1) and (2). It is easy to extend the present procedure to the case where the soil deposit consists of multiple layers, or to the case where the horizontal permeability is different from the vertical one.

**APPLICATIONS TO HYPOTHETICAL CASE STUDIES**

To verify that the proposed procedure is valid, its performances in hypothetical case studies are demonstrated here. The plane strain condition is assumed in all examples.

![Fig. 2. Model in Example 1](image)

**Fig. 2. Model in Example 1**

![Fig. 3. Calculated quantities in Example 1](image)

**Fig. 3. Calculated quantities in Example 1**
Table 1. Measured quantities in Example 1 \((n=1,2,\ldots,5)\)

<table>
<thead>
<tr>
<th>Case</th>
<th>Measured quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(V_1^n, U_1^n, V_2^n, P_1^n)</td>
</tr>
<tr>
<td>2</td>
<td>(V_1^n, U_1^n, V_2^n, V_3^n)</td>
</tr>
<tr>
<td>3</td>
<td>(P_1^n, P_2^n)</td>
</tr>
</tbody>
</table>

Fig. 4. Iteration behaviour in Example 1 (Case 1)

Fig. 5. Iteration behaviour in Example 1 (Case 2)

Fig. 6. Iteration behaviour in Example 1 (Case 3)

Example 1

Fig. 2 shows the model. Fig. 3 illustrates the results of the simple elastic finite element analysis. Now consider a geotechnical problem in which the soil parameters \(E\), \(\nu\) and \(k\) are unknown quantities to be found, and in which some of the displacements and of pore water pressures as shown in Fig. 3 are measured in the field. Table 1 shows the measured quantities in three hypothetical cases, where \(U_i^n\) or \(V_i^n\) is redefined as the horizontal or vertical displacement of nodal point \(i\), and \(P_j^n\) represents the pore water pressure of element \(j\) both at the \(n\)-th time step. Moreover Table 1 implies that one can get the measured quantities at each of the time steps, such as \(V_1^n=V_1^1, V_2^n, \ldots, V_1^n\). Figs. 4, 5 and 6 illustrate the iteration behaviour of the procedure using Eq. (5) or (7) as the objective function. The correct elastic constants are obtained in Case 1 and Case 2, whereas reasonable solutions are not obtained in Case 3 where only pore water pressures are specified as the measured quantities. Case 2 proves that the soil parameters affecting consolidation can be estimated only by monitoring the displacements in the field. The difference between the results by Eqs. (5) and (7) seems to be a little so far as concerning \(E\) and \(\nu\). However Eq. (7) may provide quicker convergence concerning \(k\).

Example 2

Fig. 7 illustrates the model which represents hypothetical embankment projects on soft soil deposits. Fig. 8 shows the results of an elastic finite element computation.
Some of such solutions are used as if they were the data of field measurement in the same manner as in Example 1. The locations of measuring points are shown in Fig. 7 and the measured time steps are in Fig. 8. Fig. 9 shows the performance of the proposed procedure with Eq. (7). As seen in Example 1, the input data of the field measurement consisting only of pore water pressures does not lead to the correct values of soil parameters. The usage of both displacements and pore water pressure as measured data, may not necessarily improve the accuracy of estimated values and the rate of convergence, comparing with the case where only displacement data are used. The measured data of pore water pressure may little contribute to a better performance, or makes the performance even less effective due to the increase in the number of terms to be summed up in the objective function, as far as employing the proposed procedure.

**Example 3**

Both the model and the input data are the same as those in Example 2 (see Figs. 7 and 8), in which the horizontal and vertical permeabilities are of the same value. However, it is assumed that the horizontal permeability may be different from the vertical one. The results based on Eq. (7), which are illustrated in Fig. 10, show that it is difficult to find the unique solution of the horizontal permeability. The calculation
should lead to a conclusion that both vertical and horizontal permeabilities are identical. However the actual performance of the calculation converges to the horizontal permeability which is largely underestimated compared with its true value. Similar results are obtained in other hypothetical case studies, in which horizontal permeability is different from vertical one. It is widely believed that one of the reasons of disagreement between calculated and observed settlements may be attributed to the anisotropy of permeability. However this result implies that the anisotropy of permeability may not be that important in this particular example of the multi-dimensional consolidation problem. That is, the vertical permeability is the dominant parameter concerning the seepage flow during consolidation. It seems possible to simulate the field consolidation process by selecting the appropriate isotropic permeability. This inference is supported by the actual case studies reported later.

Example 4

Fig.7 shows the model, in which the soil deposit consists of two different layers. $E_i$, $v_i$ and $k_i$ denote respectively the Young’s modulus, Poisson’s ratio and permeability of the $i$-th layer. The input data are the same as in Example 2. As seen in Fig.11, reasonable parameter findings are made by the proposed procedure with Eq. (7), in spite of the limited number of input data while having many unknown soil parameters in these two layers.

Example 5

In order to get an idea of the minimum number of input data, i.e. the field measurements needed for reasonable estimates of soil parameters, the proposed procedure is applied to several cases with the various sets of the distribution and the number of measuring points. The model is the same as the one in Example 2 (see Figs. 7 and 12). Table 2 shows the measured quantities used as the input data in each case. Such cases are hypothesized by...
Table 2. Measured quantities and performances of the proposed procedure in Example 5: uniform soil deposit

<table>
<thead>
<tr>
<th>Case</th>
<th>Instruments</th>
<th>Estimated</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A  B  C  D  E  F</td>
<td>E  ν  k</td>
<td>E  ν  k</td>
</tr>
<tr>
<td>1</td>
<td>*  *  *  *  *  *</td>
<td>97  0.34 10.0 6 4  —</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>100 0.33 8.7 3 3  3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>100 0.34 9.4 2 6  7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>101 0.33 8.5 2 8  8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>100 0.33 8.7 4 6  7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>100 0.33 8.6 4 6  6</td>
<td></td>
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<tr>
<td>7</td>
<td>99  0.34 7.8 4 6  10</td>
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<tr>
<td>8</td>
<td>100 0.33 8.7 4 6  6</td>
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<td>9</td>
<td>101 0.33 10.0 4 6  —</td>
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<tr>
<td>10</td>
<td>100 0.33 9.0 4 7  8</td>
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<tr>
<td>11</td>
<td>100 0.33 8.5 2 4  9</td>
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<tr>
<td>12</td>
<td>100 0.33 8.4 2 5  8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>100 0.34 9.9 4 6  —</td>
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<td>100 0.34 9.1 4 7  6</td>
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<td>100 0.33 8.6 4 6  7</td>
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<td>100 0.33 9.0 4 6  8</td>
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<td>18</td>
<td>101 0.34 9.7 4 7  —</td>
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<tr>
<td>20</td>
<td>100 0.33 8.7 2 5  8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correct</td>
<td>100 0.33 8.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Instrument A: settlement gauges beneath the center of embankment, B: settlement gauges beneath the toe of embankment, C: settlement plates in the loaded area, D: settlement plates outside the loaded area, E: inclinometers, F: lateral movement indicators. * : employed as input data. Estimated: estimated value at the 10th iteration step, E: Young's modulus (tf/m²), ν: Poisson's ratio, k: permeability (x10⁻⁶ m/day), and Iteration: number of iterations required for 90% convergence (1 tf/m²=9.8kPa).

assuming the locations of measuring instruments such as shown in Fig.12 and Table 2. Table 2 shows the performance of the proposed procedure as well: the soil parameter values estimated in the 10th iteration step. Reasonable estimates of soil parameters are obtained in all of the cases in Table 2, in which the instrumentation patterns are thought to be the conventional ones seen in the usual engineering practice.

At the present stage of the investigation, it is not possible to derive the necessary conditions for the back analysis concerning the arrangements or locations of measuring instrument. It may be concluded that the usual ways of instrumentation provide the sufficient informations for the back analysis, when assuming the elastic behaviour of soil deposit, and when assuming the uniformity of soil deposit. However, when the soil deposit consists of multiple layers, there are cases where the solutions obtained by the proposed procedure do not have such a high accuracy as those on the uniform soil deposit (see Table 3). This tendency is remarkably seen in the estimation of permeability. Such deficiency in soil parameter finding by the proposed procedure happens to arise due to the long measuring time steps chosen in the above example, since at the time steps illustrated in Fig.8, the considerable progress of consolidation has already been made. It is seen in the applications to the actual multi-layered soil deposits described in the next section that the reasonably good results are obtained by the proposed procedure based on the input data monitored in commonly adopted instrumentation systems. The other cause of such difficulty encountered in these examples may be interpreted as follows. Eq. (7) is a scalar function which is the sum of the differences between calculated and measured quantities. If only the value of J given by Eq. (7) becomes smaller, the magnitude of the partial differences between them may not be so important. This effect reduces the uniqueness and the accuracy of the values of soil parameters in the multi-layered soil deposit. In such a case, it is important to distribute the measuring points so that the measured quantities may
sufficiently specify the deformation of each layer, and so that the number of measured quantities in each layer may be well balanced among the layers.

APPLICATIONS TO ACTUAL CASE STUDIES

In applying the proposed procedure to the actual case studies, the estimation of soil parameters may be attempted sometime during the consolidation process. The back analysis must be made by the use of the measured quantities up to the time of attempting calculation. In this paper the soil deposits are assumed to behave elastically. However this assumption may not be fully satisfied by real soil deposits. To overcome this, two methods are adopted in the applications of the proposed procedure. One method is as follows (method I): The total process of consolidation is divided into several pieces of period. And in each period the values of soil parameters are assumed as being constant. The soil parameters in each period are estimated by the back analysis based on the data measured in the corresponding period. If the trend of soil parameters changing with the progress of the consolidation process is found, it is possible to predict the subsequent consolidation settlements with a good accuracy. The other method is as follows (method II): The soil parameters are supposed to be constant from the initial stage of consolidation to the time at which the back analysis is made. The soil parameters up to this time are estimated by the use of the whole data measured up to this time. The subsequent settlements after this time are predicted by the similar manner in the method I. When employing the method I, the usage of Eq. (7) must be abandoned, because the values of soil parameters in a certain period may be different from the one in the next period, while Eqs. (8) and (9) are derived by assuming no change in the values of soil parameters during consolidation process. To unify the numerical procedure for easier comparison, Eq. (5) must be adopted both in the method I and II. It should be noted that Eq. (5) is given in terms of the total magnitude of displacement rather than the increment of displacement, and hence is convenient in avoiding the difficulty arising from the smooth differentiation of the objective function.

Example 6

A trial embankment for Tomei Express Way construction, Aiko, Kanagawa, is chosen as an example. By use of the data observed during the project, Shoji and Matsumoto (1976) carefully examined the techniques of determining the elastic constants and permeability from the results of laboratory tests such as conventional triaxial tests and oedometer tests. Fig. 13 shows the profile of soil deposit, which consists of three different layers. The locations of measuring points are also shown in Fig. 13. The detailed properties of soil deposit are given by Shoji and Matsumoto (1976). The performances of the proposed

![Figure 13. Finite element model in Example 6]
procedure in Examples 1 through 5 have proved that the soil parameters affecting consolidation can be estimated only from the data of displacement monitored with passage of time. Considering this, only the data of displacement are used here as the input data in the back analysis, since the values of the field measurement of pore water pressure may appear to be unstable and unreliable as compared with those of displacement. The total process of consolidation is divided into four periods as shown in Fig. 14. The time steps, at each of which the measurement data are specified for consolidation analysis, are also shown in Fig. 14. Fig. 15 shows the values of soil parameters estimated by the methods I and II, as well as those estimated heuristically by Shoji and Matsumoto (1976). The calculated displacements based on the soil parameters obtained from the methods I and II, are shown in Fig. 14 together with those measured in the field. The solid and the broken line in Fig. 14 represent the future displacements which are predicted by use of the soil parameters estimated by the methods I and II respectively. In these computations of the future displacement, the soil parameters obtained from the back analyses are used without modifications. For instance, the predicted displacements from the period 3 to 4 are obtained by use of soil parameters estimated in the period 2. This is because the specification of the trend of soil parameter changing with passage of time is difficult. This subject must be investigated in the future study on a number of the actual case studies. The examination of the relations between the estimated soil parameters and the state
variables such as mean effective stress, shear stress, etc. in many case studies, would clarify the general tendency of the change in soil parameters during consolidation.

The remarks derived from studying Example 6 are as follows: 1) As shown in Fig.15, Young's modulus estimated by the method I decreases with passage of time. This indicates that the yielding of clay with the increase in shear stress is more significant than the strengthening due to consolidation. 2) As seen in Fig.14, the considerable differences are recognized between the future displacements predicted by the method I in the period 1 and those monitored in the field. The reasons for a such a discrepancy may be as follows. The proposed procedure indicates that the displacements in the period 1, which is the period of loading (see Fig.14), are close to those under undrained condition. The permeability in the period 1 is calculated as a very low value which happens to be the best fit parameter. Such a low permeability makes the predicted progress of consolidation in the period 2 through 4 much slower than the actual one as shown in Fig.14. In order to predict the future displacements adequately, the proposed procedure may require the measurement data at a few subsequent time steps after the load becomes constant. This inadequate result may be attributed to the nonuniqueness of soil parameters due to too small number of measurement data available, as stated in Example 5. 3) The estimated values of permeability at the early stage of consolidation appears to be unreliable, especially in the layers remote from the drainage boundary. When the drainage can be permitted only from the upper surface as in Fig.13, little water flow occurs from lower layers at the initial stage of consolidation. Very little displacement in the lower layers in the early stage of consolidation provides insufficient information to estimate the reasonable value of permeability. 4) The calculated displacements obtained by the method II in the period 4, where the soil parameters are assumed to be constant throughout the consolidation process, agree fairly well with most of the displacements measured in the field as shown in Fig.14. Though such an agreement may not be expected in other cases, it is interesting that the behaviour of soil deposit is well approximated by a simple elastic model. 5) Fig.16 shows the iteration behaviour in applying the method II in the period 4. The computation time required for ten iteration steps in this case using the method II in the period 4 is about 50 seconds when using FUJITSU FACOM M200, while in case of using the method I in the period 1 it takes 18 seconds. The difference in computation time in these two cases is attributed to the number of recalculation of the inverse of the stiffness matrix. The stiffness matrix includes the soil parameters and the time increment as its components. The choice of different length of time interval as in case of the method II for the period 4 requires the recalculation of inverse matrix each time depending on the time interval resulting much longer computational time.

Example 7

The performance of a trial embankment for Douo Express Way placed at Toyohoro,
sand layers, i.e. the third and the fifth layer. The number and the location of the measuring points in Fig.18 are not enough to provide the sufficient information needed in specifying the deformations of these sand layers. In back analysing this Example 7 the soil parameters of the sand layers are assumed to be the ones estimated from Cone resistance $q_c$ and are kept constant throughout the back analysis. Moreover Poisson’s ratios of all the layers are assumed to be 0.33, because the reasonable estimate of Poisson’s ratio may be difficult to achieve by the following factors: 1) The number of input data is insufficient compared with the number of unknown soil parameters to

Hokkaido is studied as an example. Fig.17 shows a brief description of subsoil properties. The finite element mesh formation is illustrated in Fig.18. As shown in Fig.18, the soil deposit includes two

![Fig. 17. Soil properties in Example 7](image)

![Fig. 18. Finite element model in Example 7](image)

![Fig. 19. Estimated values of soil parameters in Example 7](image)

![Fig. 20. Calculated and measured displacements in Example 7](image)
be estimated. 2) There is a limitation to approximate the behaviour of extremely soft soil such as in the upper soft layer by the elastic model.

Fig. 19 shows the estimated values of soil parameters in each period by the use of the method I, as well as those by the method II where the soil parameters are assumed to be constant throughout the consolidation process. As being similar in Example 6, Fig. 19 shows that Young’s modulus may decrease with the progress of consolidation. The permeability changes very little except in the first layer, since the soil deposit contains two sand layers to provide quick dissipation of pore water pressure. Fig. 20 compares the displacements monitored in the field with the displacements calculated by using these soil parameters obtained from the back analysis. The calculated displacements by assuming the soil parameters to be constant throughout the consolidation process (method II), are in good agreement with most of the measured displacements, as being similar in Example 6. It is omitted here to investigate the compatibilities of the future displacements predicted by the present procedure with the observed ones.

CONCLUSIONS

The numerical procedure proposed in this paper can successfully estimate the soil parameters affecting consolidation by the back analysis of field monitoring data. By using a combination of finite element method and mathematical programming technique, the procedure allows to stepwisely correct the unknown soil parameters up to a stage of good convergence in a way that the differences between calculated and measured quantities decrease sufficiently. The soil parameters estimated by the proposed procedure are Young’s modulus, Poisson’s ratio, and permeability in multiple soil layers under load. By applying the procedure at the early stage of consolidation, one may be able to predict the future settlement with a good accuracy. Moreover by applying the proposed procedure to the actual construction works completed in the past, one may obtain new informations concerning the trend of soil parameters changing during the consolidation process.

The concluding remarks obtained by applying the proposed procedure to hypothetical case studies are as follows: 1) The measured displacements are useful in estimating the soil parameters, whereas measured data only of the pore water pressure do not provide reasonable estimates of soil parameters. 2) When assuming the anisotropy of permeability, the proposed procedure may not provide a unique estimate of horizontal permeability. This implies that the anisotropy of permeability may not be so important in the particular examples of the multi-dimensional consolidation as to be believed up to now. It seems possible to simulate the field consolidation process by selecting the appropriate isotropic permeability. This inference is supported by the actual case studies. 3) When assuming the uniform soil deposit, the number and the location of measuring points chosen in usual field monitoring, may provide the correct estimates by the proposed procedure. It is not possible to indicate the minimum number and optimal distribution of measuring points. The more monitored data are given, the quicker convergence in iteration the proposed procedure may have. 4) The proposed procedure is applicable to the multi-layered soil deposit. However, depending on the number and on the location of measuring points, there are cases where the proposed procedure does not provide such good estimates as those in uniform soil deposit, especially concerning permeability. It is important to plan the distribution of the measuring points so that the measured data may well specify the deformation of each layer, for instance, by studying beforehand a hypothetical case in which the subsoil profile and soil parameters of each layer are assumed to be as close as in the field to be monitored.

When applying the proposed procedure to
the actual case studies of the consolidation process in the fields, two methods are developed to predict the future displacements. This is the consequence of a trial of taking the fact that the actual soil deposits are not ideal elastic media into consideration. The method I estimates the values of soil parameters in each period by use only of the data measured in the period. The method II estimates the soil parameters representative through the consolidation process prior to the period, by assuming the soil parameters being constant from the initial stage up to the period. The concluding remarks obtained from two actual case studies are as follows: 1) The proposed procedure using either of the method I or II provides the reasonable estimates of soil parameters, the displacements calculated by which are well consistent with most of the field measurements in actual construction projects. 2) The values of soil parameters estimated either by the method I or II, considerably changes their values with the progress of consolidation. For instance, Young's modulus estimated by the method I appears to decrease with time due to the yielding of soil deposit. However it is not possible to evaluate the general tendencies of changes in soil parameters until the case studies on a number of sites are carried. This must be investigated in the future through many case studies. 3) To obtain a precise prediction of the future displacements, the procedure may require the measurement data at a few subsequent time steps in which the load is kept constant. 4) When the drainage is permitted only from the ground surface, the estimated permeability at the early stage of consolidation appears to be unreliable, especially in lower layers. 5) As far as two case studies concern, the calculated displacements by assuming the soil parameters to be constant throughout the consolidation process, considerably agree with most of the displacements measured in the field.

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