STOCHASTIC ANALYSIS OF PORE PRESSURE UNCERTAINTY FOR THE PROBABILISTIC ASSESSMENT OF THE SAFETY OF EARTH SLOPES

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ABSTRACT

A probabilistic slope stability model was developed that includes soil shear strength and pore pressure as random variables. A local estimation technique called the nearest-neighbor method was utilized for simulation of the spatial variability of saturated soil permeability values. The spatial variation in the soil permeability causes a variation in the position of the phreatic surfaces during steady-state seepage flow and, hence, pore pressures. A number of permeability sets were generated and the resulting series of phreatic surfaces were calculated. A series of slope stability computations was made combining the spatial variability of pore pressures and soil strength yielding a probability density function of the factor of safety. The probability of failure was computed from the mean and standard deviation of the factor of safety. The resulting probabilistic model was applied to a hypothetical embankment. In addition, a sensitivity study was performed on the influence of the variability of soil permeability. The results indicated that the variability of soil strength and soil permeability has a large influence on the outcome of the probabilistic analysis and that there is a large discrepancy in the magnitude of the probability of failure if the pore pressure uncertainty is excluded from the probabilistic slope stability analysis.

Key words : permeability, pore pressure, safety factor, seepage, slope stability, stability analysis, (spatial variability), (Monte Carlo) (IGC : E 6/E 13/H 4)

INTRODUCTION

In the last few years, it has become clear that the simulation of the groundwater flow, mass transport, and soil behavior can best

be accomplished by combining the deterministic and probabilistic framework. The hydrogeological environments are heterogenous (Lumb, 1966 ; Peck, 1967) and even in areas where considerable data are available, the

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exact spatial variation of the heterogeneous elements are never uncovered. The uncertainty associated with the spatial variability of soil parameters such as permeability and soil strength produces uncertainty in the hydraulic head, pressure head, the groundwater velocity, the total flow and the earth slope stability. In this paper, a local estimation technique called the nearest-neighbor method similar to kriging (Matheron, 1970; Delhomme, 1978) is utilized for the simulation of soil permeability values which affect the distribution of pore pressures during steady-state seepage. Combined with a probabilistic slope stability model representing spatial variability of soil strength parameters, c and $\tan \phi$, a new probabilistic procedure incorporating pore pressure uncertainty is formulated.

The purpose of this study is to formulate and apply a probabilistic model by which the safety of earth slopes can be modeled and systematically analyzed. The random variables in this study include shear strength (cohesion and $\tan \phi$) and pore pressures.

This study is limited to pore pressure uncertainty during steady-state seepage that results from the spatial variability of saturated soil permeability.

STOCHASTIC AND PROBABILISTIC ANALYSIS

An autoregressive scheme extended into the spatial domain which is a discrete block Monte Carlo technique called the nearest-neighbor method was used by Smith and Freeze (1979 a, b) to generate hydraulic conductivity values (soil permeability) in relation with groundwater analysis in confined flow. This method was also applied to analyze mass transport (Smith and Swartz, 1980, 1981) and to fit data on a sand deposit (Quadra Sand) in Canada (Smith, 1980). Procedures of probabilistic analysis and design of earth slopes have been varied. These procedures included the development of mathematical models that: (1) account for spatial variability and correlation structure

of soil properties (Vanmarcke, 1975, 1977 a, b, 1979, 1980; Anderson et al, 1981; Sharp et al, 1981); (2) integrate the individual components of uncertainties into an overall uncertainty (Tang et al, 1976; Yucemen et al, 1975); (3) systematically assess the statistics of the parameters involved (Yucemen, 1973; Wu and Kraft, 1970); (4) compute the probability of failure for homogeneous clay stratum (Alonso, 1976; Matsuo and Kuroda, 1974); (5) examine the additional probabilities of failure other than those due to a critical sliding surface (Morla-Catalan, 1974; Morla-Catalan and Cornell, 1976); and (6) use of a straightforward Monte Carlo simulation technique (Kraft and Mukhopadhyay, 1977).

THEORY

Generation of Permeability Realization

The nearest-neighbor method whose original concepts were developed by Smith and Freeze (1979 a, b) is based on subdividing the porous media into p equisized blocks with a single permeability value assigned to each block. Permeability values in neighboring blocks are generated. The simultaneous, nearest-neighbor, autoregressive relation in two-dimension can be written as:-

$$YS_{ij} = \alpha_x(YS_{i-1,j} + YS_{i+1,j}) + \alpha_y(YS_{i,j-1} + YS_{i,j+1}) + \varepsilon_{ij} \quad (1)$$

where YS_{ij} is the random variable satisfying the nearest-neighbor relation; ε_{ij} is an uncorrelated random variable associated with YS_{ij} ; α_x is an autoregressive parameter expressing the degree of spatial dependence of YS_{ij} on its two neighboring values in the x -direction; and α_y similarly expresses the dependence of YS_{ij} in the y -direction.

Fig. 1 schematically illustrates this process equation for the block model. In a stationary medium, Eq. (1) holds for every block. If $\alpha_x = \alpha_y$, the model is statistically isotropic.

Following Smith and Freeze (1979 b), consider now a two-dimensional system of permeability blocks, m rows and n columns. The process model for the entire p blocks (in rows, in columns) can be written as a

Fig. 1. Schematic illustration of the nearest-neighbor grid first-order model (after Smith and Freeze, 1979 b)

system of p linear equations :

$$\{YS\} = [W]\{YS\} + \eta\{\varepsilon\} \quad (2)$$

where $[W]$ is a matrix spatial lag operator of scaled weights, w_{ij} . These scaled weights are defined as (Smith, 1978) :

$$w_{ij} = \frac{w_{ij}^*}{r_b} \quad (3)$$

$$i=1 \dots p, \quad j=1 \dots p, \quad i \neq j$$

with $w_{ij}^* = \alpha_x$ if blocks i and j are contiguous in the x -direction

$w_{ij}^* = \alpha_y$ if blocks i and j are contiguous in the y -direction

$w_{ij}^* = 0$ if otherwise

r_b = total number of contiguous blocks surrounding block i .

The $[W]$ matrix indicates which blocks are linearly related to each other. The scaling is required to preserve the stationarity of the generated sequence.

The random function $\{\varepsilon\}$ forms an internally uncorrelated p member vector of random variables. Each term has a mean $\bar{\varepsilon}$ of zero and a variance $\bar{\varepsilon}^2$. These conditions are expressed mathematically by Smith and Freeze (1979 b). The variance $\bar{\varepsilon}^2$ will control the magnitude of permeability fluctuations but not the dependence between the random variables at neighboring points.

Rearranging, factoring, and solving for YS in Eq. (2), we get :

$$\{YS\} = ([I] - [W])^{-1} \eta \{\varepsilon\} \quad (4)$$

provided the inverse exists. For a p block system, the $(p \times p)$ matrix inverse $([I] - [W])^{-1}$ can be regarded as a filter operating on a random (p) vector $\{\varepsilon\}$ to yield an output (p) vector $\{YS\}$ with an internal correlation depending upon the form of the spatial lag operator $[W]$ and the autoregressive parameter α_x and α_y .

Smith and Freeze (1979 b) derived the mean and the covariance of the YS process. The mean of the YS process was given as :

$$E[\{YS\}] = 0 \quad (5)$$

The covariance of the YS process can be shown to be :

$$\text{Cov}[YS] = (([I] - [W])^T ([I] - [W]))^{-1} \eta^2 E[\{\varepsilon\} \{\varepsilon\}^T] \quad (6)$$

Let $[M]$ equal the result of the above matrix inverse in front of the expected value operator and let $[C]$ represent the covariance matrix. Then, the correlation matrix can be written as (Smith and Freeze, 1979 b) :

$$[R] = \frac{\bar{\varepsilon}^2 \eta^2}{\bar{Y} \bar{S}^2} [M] \quad (7)$$

The factor η is a premultiplier to yield the desired value of $\bar{Y} \bar{S}$, the standard deviation in the generated sequences. Let $\bar{\varepsilon}^2$ equal 1.0 in Eq. (7) and multiplying both sides by $[M]^{-1}$, we get :

$$[R][M]^{-1} = \frac{[I] \eta^2}{\bar{Y} \bar{S}^2} \quad (8)$$

Using this matrix equation and for a first-order nearest-neighbor model in a two-dimensional system of blocks of a statistically isotropic medium, the expression for η is given by Smith and Freeze (1979 b).

The nearest-neighbor filter Eq. (4), requires that both $\{\varepsilon\}$ and $\{YS\}$ sequences have a mean of zero. By adding the constant, $\bar{Y} \bar{S}$, i.e. the mean of YS , to each element of $\{YS\}$ and assuming the mean to be independent of the covariance process ; any mean for YS can be specified without affecting the second order moments. The system of equations for the nearest-neighbor can then be written as :

$$\{YS\} = \bar{YS} + [W]\{YS\} + \eta\{\varepsilon\} \quad (9)$$

However, $\bar{YS}$ is not added until after Eq. (4) is solved.

The permeability, K , is assumed to be lognormally distributed in this model. Attempts to fit permeability data to a probability density function have generally found the lognormal distribution to be satisfactory (Bakr, 1976). Freeze (1975), Smith and Freeze (1979 a, b) and Smith and Swartz (1980, 1981) and Smith (1981) have all assumed the permeability as lognormally distributed parameter. A permeability realization is formed by first generating a random sequence $\{\varepsilon\}$ from a normal distribution. The nearest-neighbor filter (Eq. 4) is then applied to yield a normally distributed, correlated sequence $\{YS\}$. A non-zero mean ($\bar{YS}$) is added to each member of $\{YS\}$. As base ten logarithms are used, finally the exponential transform :

$$K_i = \exp(2.36 YS), \quad i=1 \dots p \quad (10)$$

is applied to $\{YS\}$, yielding a lognormally distributed sequence $\{K\}$ of permeability values. The distributions of these sequences can be summarized as follows :

$$\varepsilon = N[0, \tilde{\varepsilon}] \quad (\text{normal distribution})$$

$$YS = N[\bar{YS}, \tilde{YS}] \quad (\text{normal distribution})$$

$$K = LN[\bar{K}, \tilde{K}] \quad (\text{lognormal distribution})$$

There is a loss in the strength of the correlation in the $\{K\}$ sequence compared to that of the $\{YS\}$ sequence due to the exponential transformation (Smith and Freeze, 1979 a, b). This loss is a function of both the variance $\tilde{YS}^2$ and the autoregressive parameters α_x and α_y according to Smith (1978). Matalas (1967) and Mejia and Rodriguez-Iturbe (1974) discussed the problem of preserving the autocorrelation structure through exponential transform.

In summary, the parameters needed to generate a permeability realization are the following : (1) the number of rows and columns forming the permeability field ; (2) the matrix of scaled weights specifying the neighbor dependence in the block system ; (3) the autoregressive parameters in the two coordinate directions ; (4) the mean permeability ;

and (5) the variance factor, η , to yield a specified permeability variance, $\tilde{YS}^2$.

The matrix of scale weights is set up by Eq. (3). The autoregressive parameter, α , is obtained by fitting the spatial variation of synthetic permeability sequences generated by the nearest-neighbor method to the actual spatial variation of the field data as illustrated in Smith (1980). The spatial variation of the permeability values can be characterized by the autocorrelation function. Freeze (1975) provides a table that summarizes the data collected on the standard deviation of permeability, $\tilde{YS}$. For permeability values log transformed in base 10, $\tilde{YS}$ generally lies between 0.2 and 2.0.

The estimated mean autocorrelation function in comparison with the theoretical autocorrelation function (Eq. 7) in the $\{K\}$ sequence, calculated over 300 realizations is plotted in Fig. 2. The estimator for the two-dimensional autocorrelation function is given by Agterberg (1970, 1974). In Fig. 2, the isotropic autoregressive parameters were set at 0.84 and 0.50.

Integral scale The integral scale, λ , is given for a continuous medium (Lumley and Panofsky, 1964) :

$$\int_0^\infty \rho(x) dx = \lambda \text{ in } x\text{-direction} \quad (11)$$

where $\rho(x)$ is the autocorrelation function

Fig. 2. Comparison of theoretical and sample autocorrelation function (ACF)

in x -direction. Thus, the integral scale is equal to the area beneath the autocorrelation function. In discrete representation, the autocorrelation function is a step function. The decay of the autocorrelation is due to the first order dependence in the model (Smith and Freeze, 1979 b). The integral (λ) can be interpreted as the average distance over which permeability values are correlated (Bakr et al, 1978). For a soil with statistically isotropic covariance function, the integral scales in the two coordinate directions are equal ($\lambda_x = \lambda_y = \lambda$). For each realization, the integral scale varies because the autocorrelation varies. Hence, the results are presented in terms of the estimate of the mean integral scale ($\bar{\lambda}$) obtained over all the realizations in a set of Monte Carlo runs with a specified set of $\bar{Y}S$, $\tilde{Y}S$ and α values as can be seen in Fig. 8, for instance.

Probabilistic structure The nearest-neighbor model can be formulated in terms of Markovian conditional probability or as a set of simultaneous joint probabilities (Brook, 1964). The later structure which is not a Markovian is adopted in this study. As a consequence, the permeability values we generate do not have a Markovian property. This departs from the autoregressive scheme in unilateral time series models. We assume that our spatial model operates in all directions.

Brook (1964) shows that if the dependence is unilateral (i.e. only between YS_i and YS_{i-1}), then the conditional and joint probabilities are easily related. However, in a bilateral scheme, casting the conditional probabilities in terms of a set of joint probabilities leads to a unilateral structure. Bartlett (1975) also showed that the spectrums of the two processes are not equal. Whittle (1954) and Bartlett (1975) show it is possible to convert multilateral simultaneous models to a unilateral autoregression with an equivalent correlation structure, but such a transformation only appears useful in one-dimension. Whittle (1954) questioned the use of the exponential as an autocorrelation function in two-dimensions because it

does not seem possible to construct simple two-dimensional stochastic process models that yield the exponential. Agterberg (1970) discussed this problem further. Rodriguez-Iturbe and Mejia (1974) used both an exponential function and a Bessel function to describe the distribution of precipitation in space. They found that the form of the correlation function did not seem to matter provided the scale of the function was adequately specified.

The question of scale In any discrete representation of a continuous medium such as the nearest-neighbor method used in this study, there is the problem of specifying the size of the block (Δx or Δy) that is used to characterize the spatial variations of permeability. At each point in the flow domain, it is assumed that the permeability is a random variable. By using single permeability values in each block, spatial variations on a scale smaller than the block size do not contribute to the variability of the resulting solution. Then, the Monte Carlo solution only approximates the true variability in the prediction. For accurate solution, the block model must reproduce all significant scales of heterogeneity that contribute to the uncertainties of the prediction at the scale of the model. In studying this problem, Smith and Freeze (1979 a, b) presented a simple criterion in that the block size should be chosen to preserve both the integral scale and approximate the spatial decay in the autocorrelation. For instance, they consider a problem of the effect on the hydraulic head by using different block sizes to represent a known spatial covariance function of permeability. Their results showed that the standard deviations in the hydraulic head do not depend on the block size provided a sufficiently accurate representation of the autocorrelation function is made. The integral scale (λ), however, can be viewed as an upper bound for the discretization.

A Model for Steady-State Seepage

This research utilized a finite element com-

puter program, SEEP (Florentine, 1975), for solving the steady-state porous media flow with free surface in two-dimensional region given by the following equation :

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial h}{\partial y} \right) = 0 \quad (12)$$

Note that the above equation reduces to the well-known Laplace equation for isotropic, homogenous medium. SEEP was derived from FREESURF-1 version initially coded by Neuman and Witherspoon (1970) and modified in this research. Because the coordinates of the finite element mesh are not a one to one correspondence with the nearest-neighbor blocks and since the coordinates of the finite element mesh change at each iteration, the permeability values needed to be adjusted. A computer program subroutine developed by Akima (1974) was utilized that converts the nearest-neighbor permeability values into permeabilities for each finite element.

THE PROBABILISTIC SLOPE STABILITY ANALYSIS

In performing a slope stability analysis, it is convenient to use a method of slices such as Bishop's Simplified Method (Bishop, 1955). Past experiences indicate that this method is generally superior to other simplified methods (Bailey and Christian, 1969). From the Mohr-Coulomb criteria, the soil strength for a width b (Fig. 3(a), (b)) is given as follows :

$$s_b = c_b + (\sigma - u_b)(\tan \phi)_b \quad (13)$$

By treating the normal stress, σ , as deterministic parameter while considering pore pressure, u , as additional random variable to the soil strength, the first-order approximation (Benjamin and Cornell, 1970) of the mean soil strength for the j th soil type with width, b , is as follows (Bergado, 1982) :

$$\bar{s}_{bj} = \bar{c}_{bj} + (\sigma - \bar{u}_{bj})(\bar{\tan \phi})_{bj} \quad (14)$$

where $\bar{c}_{bj}$ is the mean cohesion of the j th soil type averaged along the failure plane ; $\bar{u}_{bj}$ is the mean pore pressure of the j th

Fig. 3 (a). Typical cylindrical failure mass of an earth embankment (after Vanmarcke, 1977 b)

Fig. 3 (b). Typical cross-section of failure mass (after Vanmarcke, 1977 b)

soil type averaged along the failure plane ; $(\sigma \tan \phi)_{bj}$ is the mean frictional component of the j th soil type averaged along the failure plane ; and σ is the normal stress which is a deterministic parameter and a function of depth of the j th soil type.

The first-order approximation of the variance of the shear strength depends on whether the cohesion, the frictional component, and the pore pressure are statistically independent or not. Matsuo and Kuroda (1974) and later on Matsuo (1976) have shown that cohesion and $\tan \phi$ are uncorrelated by varying water content and density. Lumb (1970, 1974) also showed that cohesion and $\tan \phi$ have negligible correlation. This statistical independence have also been assumed by Yucemen et al (1975), Dettinger and Wilson (1981) and Anderson et al (1981). In this study, it was conservatively assumed that the cohesion, frictional coefficient, and pore pressures are statistically independent in the sense that the covariance terms are neglected. Assuming statistical independence and applying the first-order approximation of the variance of s_{bj} (Benjamin and Cornell, 1970), Bergado (1982) obtained :

$$\tilde{s}_{bj}^2 = \tilde{c}_{bj}^2 + (\sigma - \bar{u}_{bj})^2 (\bar{\tan \phi})_{bj}^2 + (\bar{\tan \phi})_{bj}^2 \tilde{u}_{bj}^2 \quad (15)$$

where $\tilde{s}_{bj}^2$, $\tilde{c}_{bj}^2$ and $(\bar{\tan \phi})_{bj}^2$ are variances of the shear strength, cohesion, and frictional coefficient of the j th soil type of average width, b , respectively. Hence, the mean and variance of the factor of safety, respectively, becomes :

$$\bar{F}_b = \frac{r \Sigma [\bar{c}_{bj} + (\sigma - \bar{u}_{bj}) (\bar{\tan \phi})_{bj}] L_j + R_e}{M_{0,b}} \quad (16)$$

$$\tilde{F}_b^2 = \frac{r^2 \Sigma [\tilde{c}_{bj}^2 + (\sigma - \bar{u}_{bj})^2 (\bar{\tan \phi})_{bj}^2 + (\bar{\tan \phi})_{bj}^2 \tilde{u}_{bj}^2] L_j^2}{M_{0,b}} \quad (17)$$

where R_e is the contributions of end sections of the failure mass to the resisting moment ; and $M_{0,b}$ is the overturning moment of the failure mass of width b taken as deterministic quantity.

Using the mean pore pressure, $\bar{u}_{bj}$ and considering c_{bj} and $(\tan \phi)_{bj}$ as random variables, we can obtain the first-order approximation of the mean and variance of the factor of safety as follows (Bergado, 1982) :

$$\bar{F}_{bs} = \frac{r \Sigma \bar{c}_{bj} + (\sigma - \bar{u}_{bj}) (\bar{\tan \phi})_{bj} L_j + R_e}{M_{0,b}} \quad (18)$$

$$\tilde{F}_{bs}^2 = \frac{r^2 \Sigma [\tilde{c}_{bj}^2 + (\sigma - \bar{u}_{bj})^2 (\bar{\tan \phi})_{bj}^2] L_j^2}{M_{0,b}} \quad (19)$$

Similarly, using mean soil strength parameters, $\bar{c}_{bj}$ and $(\bar{\tan \phi})_{bj}$ and pore pressure, u_{bj} , as random variable, we obtain :

$$\bar{F}_{bu} = \frac{r \Sigma [\bar{c}_{bj} + (\sigma - \bar{u}_{bj}) (\bar{\tan \phi})_{bj}] L_j + R_e}{M_{0,b}} \quad (20)$$

$$\tilde{F}_{bu}^2 = \frac{r^2 \Sigma [(\bar{\tan \phi})_{bj}^2 \tilde{u}_{bj}^2] L_j^2}{M_{0,b}} \quad (21)$$

From Eq. (16), (18) and (20) we get :

$$\bar{F}_b = \bar{F}_{bu} = \bar{F}_{bs} \quad (22)$$

and using Eq. (17), (19), and (21) we obtain (Bergado, 1982) :

$$\tilde{F}_b^2 = \tilde{F}_{bu}^2 + \tilde{F}_{bs}^2 \quad (23)$$

where : $\tilde{F}_b^2$ = total variance of the factor of safety

$\tilde{F}_{bu}^2$ = the variance of the factor of safety which includes the pore

pressure uncertainty due to spatial variation of soil permeability

$\tilde{F}_{bs}^2$ = the variance of the factor of safety which includes the spatial variability of shear strength

THE PROBABILISTIC SLOPE STABILITY MODEL

In this paper, the model (Anderson et al, 1981) which was derived from the original concepts by Vanmarcke (1977 b) was utilized to obtain the combined variance of the factor of safety due to the model error and the spatial variability of the soil strength. The pseudo-three-dimensional probabilistic slope stability model developed in this study combined this variance with the one due to pore pressure uncertainty as a result of the spatial variation of the soil permeability, to compute the probability of failure. The resulting equation for the combined variance utilized in this study is given in Eq. (27).

Model error arises from various assumptions in the method of stability analysis including the assumptions to make the analysis statically determinant and assumptions of single-mode failure (i.e. probability of failure governed by that of critical failure surface). Whitman and Bailey (1967) also suggests that the plane strain factor of safety computed by Bishop's Simplified Method may be in error by as much as 5 percent.

There are eight main steps in the analysis. They are to evaluate : (1) the plane strain factor of safety, F ; (2) the resistance of the end areas, R_e ; (3) the critical width, b_c (Eq. 24) ; (4) the variance of the resisting moment by computing the three-dimensional variance reduction function for c and $\tan \phi$; (5) the variance of the factor of safety based on the spatial variability of soil shear strength (Eq. 19) ; (6) and include the contribution of the pore pressure uncertainty due to the spatial variability of soil permeability to the standard deviation of the factor of safety (Eq. 27) ; (7) and include the contribution of the model error of the standard

deviation of the factor of safety; (8) the probability of failure.

Bishop's Simplified Method of Slices (Bishop, 1955) is used to evaluate the mean plane strain factor of safety ($\bar{F}$), the normal stresses along the failure arc and other quantities required in probabilistic analysis. The contribution of the end sections of the failure mass to the resisting moment, R_e , is found by integrating the moment resistance on a differential element over the end area. A four point bivariate Gauss-Legendre numerical solution is used to evaluate the integrals (Carnahan et al, 1969) and is described in Anderson et al (1981). Due to the changing rates of $\bar{F}_{bs}$ with b , there is a critical width, b_c , at which the probability of failure is maximum. Vanmarcke (1977 b) developed an expression for b_c by evaluating the width at which the reliability index is minimized. The value of b_c is a function only of the end resistance, R_e , the mean resisting moment, $\bar{M}_r$, and the overturning moment, M_0 (Anderson et al, 1981).

$$b_c = \frac{R_e}{\bar{M}_r - M_0} \quad (24)$$

The value of b_c should be used to evaluate the mean and standard deviation of the factor of safety, $\bar{F}_b$ and $\tilde{F}_b$. It is interesting to note that b_c is not a function of the variance properties. The cohesion variance, $\tilde{c}_{bj}^2$, for a certain type of soil j over the failure surface (Fig. 3(a), (b)) can be stated in terms of the cohesion point variance, $\tilde{c}^2$ (Sharp, 1982; Anderson et al, 1984):

$$\tilde{c}_{bj}^2 = \Gamma_{c,z}^2(b) \Gamma_{c,l}^2(L) \tilde{c}^2 \quad (25)$$

where $\Gamma_{c,z}^2$ and $\Gamma_{c,l}^2$ are variance reduction function along the axis of the embankment and along the arc length, respectively. Similarly, Sharp (1982) and Anderson et al (1984) give the expression for the variance of $\tan \phi$ for a soil type j over the failure surface (Fig. 3(a), (b)) as:

$$(\tilde{\tan \phi})_{bj}^2 = \Gamma_{\tan \phi, z}^2(b) \Gamma_{\tan \phi, l}^2(L) (\tilde{\tan \phi})^2 \quad (26)$$

where $(\tilde{\tan \phi})^2$ is the point variance, $\Gamma_{\tan \phi, z}^2$ and $\Gamma_{\tan \phi, l}^2$ are variance reduction function along the axis of the embankment

and along the arc length, respectively. The pore pressure uncertainty is reflected in the combined variance of the factor of safety computed in Eq. (23). The contribution of the model error to the standard deviation of the factor of safety is included by using the plane strain factor of safety and a coefficient of variation of the factor of safety attributed to the model error (Cornell, 1971). Combining the effect of the model error and Eq. (23), the standard deviation of the factor of safety becomes (Bergado, 1982):

$$\tilde{F}_b = \sqrt{(V_F \bar{F})^2 + \tilde{F}_{bs}^2 + \tilde{F}_{bu}^2} \quad (27)$$

where V_F is the coefficient of variation designating model error; $\tilde{F}_{bs}^2$ is the variance of the factor of safety due to the spatial variability of soil strength; $\tilde{F}_{bu}^2$ is the variance of the factor of safety due to the pore pressure uncertainty that results from spatial variability of soil permeability. $\bar{F}$ is the mean plane strain factor of safety. The probability of failure (P_f) is defined as the probability that the factor of safety is less than one. For a sliding surface of width, b , the expression is as follows:

$$P_f(b) = P(F_b < 1) \quad (28)$$

The value of the probability of failure can be computed from the mean and the variance of the factor of safety assuming a normal distribution. A useful measure of the reliability is the reliability index, β_b , proposed by Vanmarcke (1977 a). It measures the distance between the mean factor of safety, $\bar{F}_b$, and its failure value of 1.0, in units of the standard deviation of the factor of safety, $\tilde{F}_b$. Following the computation of the mean and variance of the factor of safety, the reliability index, β_b , can be computed as follows:

$$\beta_b = \frac{\bar{F}_b - 1}{\tilde{F}_b} \quad (29)$$

where $\bar{F}_b$ and $\tilde{F}_b$ are the mean and standard deviation of the factor of safety of a sliding surface of width, b . Dwight (1961) gives several approximate methods to evaluate the probability integral in which, in this study, depends on the value of β_b .

APPLICATION OF THE PROBABILISTIC SLOPE STABILITY MODEL

The input required in generating each set of permeability values by the nearest-neighbor method includes the numbers of rows and columns of blocks (m , n) forming the permeability field, the block size (s), and the number of Monte Carlo runs (MC). The parameters of the permeability field required are the mean of the logarithm of permeability ($\bar{Y}_S$), the autoregressive parameter in x - and y - directions (α_x , α_y , respectively), and the parameter, η , that can be specified to yield any desired value of $\tilde{Y}_S$. The generated values presented herein were calculated from the system with $MC=300$, $m=10$, $n=20$, $s=10.00$ and $\bar{Y}_S=$

Fig. 4. Three-dimensional plot of permeability field, $\alpha=0.50$, $\tilde{Y}_S=0.46$

Fig. 5. Three-dimensional plot of permeability field, $\alpha=0.84$, $\tilde{Y}_S=0.46$

−2.0. A three-dimensional plot (Akima, 1974) for various values of α and $\tilde{Y}_S$ are

Fig. 6. Three-dimensional plot of permeability field, $\alpha=0.95$, $\tilde{Y}_S=0.46$

Fig. 7. Three-dimensional plot of permeability field, $\alpha=0.95$, $\tilde{Y}_S=0.60$

Fig. 8. Sample autocorrelation function for two-dimensional nearest-neighbor model

given in Fig. 4 to Fig. 7. The sample autocorrelation functions for α equals 0.50, 0.84 and 0.95 are plotted in Fig. 8.

Fig. 4 shows that at α equals 0.50 and $\tilde{Y}S$ equals 0.46, a series of high and low values of permeability may occur in adjacent blocks indicated by mountain peaks and valleys that lie adjacent to each other. This is because at low values of α , the distance of positive correlation is small, resulting in a "short memory" correlation function as shown in Fig. 8 at α equals 0.50.

At higher value of α , the three-dimensional plots in Fig. 5 ($\alpha=0.84$) and Fig. 6 ($\alpha=$

0.95) resemble a terrain with continuous mountain ranges and wider valleys rather than sharp isolated peaks. This characteristics of the generated permeability values indicates that similar magnitudes generally lie next to each other implying longer distance of positive correlation. This results in a "long memory" autocorrelation function which is plotted in Fig. 8 for α equals 0.84 and 0.95. When the standard deviation of permeability $\tilde{Y}S$ is increased from 0.46 and 0.60, the permeability plot is dominated by high peaks due to the large variability of adjacent permeability values as shown in

Fig. 9. Two-dimensional plot of 200 phreatic surfaces at $\alpha=0.50$, $\tilde{Y}S=0.46$

Fig. 10. Two-dimensional plot of 200 phreatic surfaces at $\alpha=0.84$, $\tilde{Y}S=0.46$

Fig. 7.

For each set of permeability values, a phreatic surface was calculated by the finite element solution (Neuman and Witherspoon, 1970) of flow through porous media. By repeating the process over the series of runs, a series of phreatic surfaces were calculated. Fig. 9 to 12 show the two-dimensional plots of the phreatic surfaces in which each was calculated from the generated permeability values using α of 0.50, 0.84 and 0.95 with $\tilde{Y}S$ values of 0.46 or 0.60.

In Fig. 9 ($\alpha=0.50$), the location of the

phreatic surfaces occur as a narrow band of lines in a two-dimensional plot. This indicates less uncertainty in the prediction of phreatic surfaces. The lower uncertainty is due to the variance reduction effect. As the α value increases, the uncertainty in the prediction of phreatic surface locations increases. This is due to the occurrence of similar magnitudes of permeability values in adjacent blocks which would tend to pull the results further away from the mean value. This increase in the uncertainty is indicated by the wider bands of the phreatic

Fig. 11. Two-dimensional plot of 200 phreatic surfaces at $\alpha=0.95$, $\tilde{Y}S=0.46$

Fig. 12. Two-dimensional plot of 200 phreatic surfaces at $\alpha=0.95$, $\tilde{Y}S=0.60$

Fig. 13. Phreatic surface using smaller elements (solid line) compared to phreatic surface using larger elements (dotted line)

Fig. 14. Three-dimensional plot of permeability field, $\alpha=0.95$, $\tilde{Y}\tilde{S}=0.60$

surface locations as shown in Fig. 10 ($\alpha=0.84$) and Fig. 11 ($\alpha=0.95$). Fig. 12 shows an even wider band of phreatic surface locations when the standard deviation of the logarithm of permeability, $\tilde{Y}\tilde{S}$, is increased to 0.60. This is due to the large variability in the magnitudes of the generated permeability values that were used in obtaining the phreatic surfaces.

Several of the solutions appear to have produced impossible phreatic surfaces as illustrated in the dotted curve in Fig. 13. In this case, the generated set of permeability values for the embankment (Fig. 14) has a zone of low permeability values between

zones of high permeability values. Under these conditions, the large finite elements (sizes of blocks) near the exit points where large hydraulic gradients occur, produced inaccuracies in the finite element solution. When smaller element sizes (smaller blocks) were used, the finite element solution produced the phreatic surface shown by the solid curve in Fig. 13. Since this curve appears to be an average of the dotted curve representing the phreatic surface (and only differs near the problem zone), the factor of safety computed using the latter curve would not be significantly different from the one computed using the former curve.

The statistical soil strength properties used in this study were obtained from an actual mine tailings dam (Anderson et al, 1981). The soil parameters were obtained from an extensive subsurface investigation in Cooper City No. 2 tailings embankment dam near Miami, Arizona, U.S.A. The tailings dam was constructed using the upstream method of construction. The coarser material that settled out near the discharge lines termed dike material is referred to as soil 1 and the finer material in the inner portion of the tailings embankment is referred to as soil 2. Five conventional soil borings to depths ranging from 50 ft (15.24 m) to 214 ft (65.24 m) and 77 continuous cone penetrometer

soundings to depths ranging from 40 ft (12.20 m) to 221 ft (67.38 m) were made to characterize the soil properties (Fig. 15). The continuous cone penetrometer soundings were made with an electric cone penetrometer advanced at 2 cm/s. Grain size analysis tests were performed on all samples. The tailings material generally classifies as a non-plastic silty fine sand. The percent passing the No. 200 sieve (0.075 mm) typically ranged from about 20% to 50%. The direct shear test results indicate that the material is generally cohesionless. For conversion of the penetrometer data into the friction angle data, three methods were utilized (Aryani, 1982), namely: the bearing capacity equation, Schmertmann equation, and Utah State University (USU) equation. The bearing capacity equation estimates the lowest mean friction angle in both materials; and the USU equation estimated the highest. The dike material (soil 1) has a greater shear strength and lower variance than the tailings material (soil 2). A more detailed description of the subsurface investigation and results of the laboratory tests are given in Anderson et al (1981) and Aryani (1982). The soil properties included the mean and variance of $\tan \phi$ and autocorrelation function in x -, y - and z - directions for $\tan \phi$. Table 1 shows the values

of the mean, variance and autocorrelation estimates of $\tan \phi$ that were adapted in this study. The horizontal (x -direction) autocorrelation functions were obtained from the results of the sets of sounding shown

Fig. 15. Cone sounding locations for horizontal autocorrelation analysis (after Anderson et al, 1981)

Table 1. Statistical data used in stability analysis (after Anderson et al, 1981)

Dike						Tailings					
x		y		z		x		y		z	
ρ	Log (ft)(m)	ρ	Log (ft)(m)	ρ	Log (ft)(m)	ρ	Log (ft)(m)	ρ	Log (ft)(m)	ρ	Log (ft)(m)
1	0	1	0	1	0	1	0	1	0	1	0
0.55	6(1.8)	0.95	0.125(0.04)	0.29	25(7.6)	0.55	6(1.8)	0.915	0.125(0.04)	0.46	25(7.6)
0.28	100(30.5)	0.32	1.0(0.3)	0	300(91.5)	0.28	100(30.5)	0.774	0.25(0.08)	0.09	220(67.07)
0.05	275(83.8)	0.16	1.5(0.46)	0	400(122.0)	0.05	275(83.8)	0.545	0.5(0.15)	0	650(198.17)
0	400(122.0)	0.123	2(0.6)			0	400(122.0)	0.358	1(0.3)	0	850(259.15)
0	450(137.2)	0.11	3(0.9)			0	450(137.2)	0.23	3.4(1.04)		
			3.6(1.1)					0.205	8(2.44)		
			5.0(1.5)					0.17	30(9.15)		
								0	136(41.5)		
								0	140(42.7)		
$\bar{\phi} = 35.28^\circ$						$\bar{\phi} = 33.60^\circ$					
$(\tan \phi)^2 = 0.004863$						$(\tan \phi)^2 = 0.002921$					
$\gamma = 117 \text{ pcf (18.4 kN/m}^3\text{)}$						$\gamma = 110 \text{ pcf (17.3 kN/m}^3\text{)}$					

in Fig. 15. The data were pooled (Anderson et al, 1981) in soil 1 and soil 2 by considering the values of $\tan \phi$ at elevation increments of 3.0 feet (0.92 meters) to insure independence of the data sets. The soundings that were used to determine the mean and the variance of the shear strength were also employed to determine the vertical (y -direction) autocorrelations.

Probability of Failure

The probability of failure for the embankment section of Fig. 16 was analyzed using the probabilistic model developed in this study. This dam section is assumed to be the maximum cross-section of a hypothetical zoned embankment with a free-draining toe drain and a free-draining upstream outer shell. The upstream portion of the dam has a 2.5 to 1.0 slope and the downstream portion has a 2.0 to 1.0 slope. The height of the dam is assumed to be 33.5 meters. The water level of the impounded water is assumed to be 30.5 meters above the base of the dam. The embankment consists of two zones, soil 1 and soil 2, which are both cohesionless. Soil 1 has a total unit weight of 18.4 kN/m^3 and soil 2 has a total unit weight of 17.3 kN/m^3 . The mean friction angles of soil 1 and soil 2 are 35.2° and 33.6° , respectively.

In this study, the pore pressure uncertainty due to the spatial variation of soil permeability is analyzed in two-dimensions. It is

assumed that pore pressures are perfectly correlated in the third dimension. This assumption is made considering that during steady-state seepage condition, there is not much variation in the shape and elevation of the phreatic surface in the third dimension (along the dam axis).

The slope stability analysis was performed using a model error of 5 percent. This model error was chosen using engineering judgment and was based on the magnitude used by Anderson et al (1981) where analyses of different cases by increasing the model error from 2 to 5 percent were made. Model errors of different magnitudes have also been used (Yucemen et al, 1975; Tang et al, 1976). The magnitude of the model error is subjective and depends on the type of analysis being performed and the amount of conservatism to instill in the analysis and design. Table 2 shows the results of the probabilistic analysis including pore pressure uncertainty for various α and $\tilde{Y}S$ values. Also, given in Table 2 are the results obtained when considering only soil shear strength uncertainty. These results (Table 2) should not be interpreted in terms of their absolute values, rather, these results should be interpreted in terms of their relative values. That is, the increase in the probability of failure when the uncertainty due to pore pressure is included in the analysis.

The results in Table 2 may be described, for instance, at a radius, R , of 37.3 meters

Fig. 16. Embankment geometry, grid location, and failure arcs

Table 2. Summary of results of this study using a model error of 5%

Center :	R				D				U				S			
X = 73.17 m Y = 85.37 m	R = 30.85 m				R = 37.34 m				R = 46.00 m				R = 54.88 m			
P_f = Probability of Failure	$\bar{F}_b$	$\bar{F}_b$	β_b	P_f	$\bar{F}_b$	$\bar{F}_b$	β_b	P_f	$\bar{F}_b$	$\bar{F}_b$	β_b	P_f	$\bar{F}_b$	$\bar{F}_b$	β_b	P_f
β_b = Reliability Index																
$\bar{F}_b$ = 3-D Std. Dev. of F.S.																
$\bar{F}_b$ = 3-D Mean of F.S.																
Tan ϕ only as random variable	1.366	0.11	6.674	$\frac{1.24}{(10^{-11})}$	1.800	0.1433	11.224	$\frac{1.56}{(10^{-29})}$	2.200	0.183	13.179	0.000	2.480	0.210	14.115	0.000
$\alpha = 0.50$ $\tilde{Y}S = 0.46$	1.366	0.11	6.654	$\frac{1.43}{(10^{-11})}$	1.790	0.147	10.939	$\frac{3.76}{(10^{-28})}$	2.164	0.211	11.424	$\frac{1.58}{(10^{-30})}$	2.450	0.228	12.995	$\frac{6.50}{(10^{-39})}$
$\alpha = 0.84$ $\tilde{Y}S = 0.46$	1.366	0.11	6.654	$\frac{1.43}{(10^{-11})}$	1.770	0.163	9.885	$\frac{2.43}{(10^{-23})}$	2.150	0.230	10.465	$\frac{6.23}{(10^{-26})}$	2.44	0.240	12.334	$\frac{2.96}{(10^{-35})}$
$\alpha = 0.95$ $\tilde{Y}S = 0.46$	1.366	0.11	6.654	$\frac{1.43}{(10^{-11})}$	1.775	0.162	9.956	$\frac{1.19}{(10^{-23})}$	2.120	0.241	9.988	$\frac{8.64}{(10^{-24})}$	2.48	0.250	11.840	$\frac{1.212}{(10^{-32})}$
$\alpha = 0.95$ $\tilde{Y}S = 0.60$	1.366	0.11	6.654	$\frac{1.43}{(10^{-11})}$	1.740	0.182	8.847	$\frac{4.48}{(10^{-19})}$	2.153	0.265	9.071	$\frac{5.9}{(10^{-20})}$	2.451	0.268	11.040	$\frac{1.23}{(10^{-28})}$

with a center of rotation at $x=73.2$ meters and $y=85.4$ meters (Fig. 16), the probability of failure is found to be 4.5×10^{-19} for α equal 0.95 and $\tilde{Y}S$ equals 0.60. When using the soil shear strength as the only random variable (pore pressure treated as deterministic), a probability of failure of 1.56×10^{-29} is obtained. P_f becomes larger when the spatial variability in the permeability values which is reflected in the uncertainty of the phreatic surface locations, is included in the probabilistic slope stability analysis. The results in Table 2 indicated that large discrepancies in the magnitude of P_f may occur due to pore pressure uncertainty. Therefore, including pore pressure uncertainty in the probabilistic slope stability analysis has an important influence on the engineering judgment during the analysis and design of earth slopes. Otherwise, the results may be severely limited.

It is important to note that the failure surfaces defining maximum probability of failure and minimum factor of safety do not coincide (Sharp et al, 1981). Hence, the engineering significance of the analysis in this study is to demonstrate the influence of pore pressure and soil strength uncertainty on the probability of failure assuming failure planes of different radii not necessarily corresponding to minimum factor of safety. The standard procedure suggested by Anderson et al (1981) is to find first the failure surface with minimum factor of safety and

then proceed to obtain the probability of failure using this failure plane.

Sensitivity Study

In applying the pseudo-three-dimensional probabilistic slope stability model developed in this study, the magnitude of the probability of failure, P_f , of an embankment depends on the mean and variance of the factor of safety and the model error. The mean and variance of the factor of safety depends on the mean and variance of the soil shear strength with their associated variance reduction functions, the mean and variance of the pore pressure or the phreatic surface location, and the embankment geometry. In an earlier study (Anderson et al, 1981), a sensitivity study was performed to observed the influence of the probabilistic strength parameters and the types of uncertainties i.e. model error, on the probability of failure. In this study, a sensitivity study was performed using the developed probabilistic model to investigate the effect of the spatial variability of the soil permeability on the probability of failure. In describing the spatial variability of permeability values, the influence of the autoregressive parameter, α , and the standard deviation, $\tilde{Y}S$, were considered. The embankment section in Fig. 16 was utilized.

Influence of the standard deviation of permeability The results in Table 3 show the dependence of the probability of failure,

P_f , on the standard deviation of the logarithm of permeability, $\tilde{Y}S$. P_f increases when $\tilde{Y}S$ changes from 0.46 to 0.60 for various radius of the assumed failure arcs. As the heterogeneity of the soil increases (larger $\tilde{Y}S$), so does the uncertainty associated with predicting the location of the phreatic surfaces, which results in an increase in the uncertainty of the predicted factor of safety and, thus, higher P_f .

Influence of the autoregressive parameter
The autoregressive parameters, α , indicates the correlation between neighboring values of permeability. The uncertainty in the predicted factor of safety increases with the increase in the parameter, α . This increase leads to a higher probability of failure, P_f . Table 4 shows an increase in P_f for various radii of the assumed failure arc when α increased from 0.50 to 0.95. At low values of α , the magnitude of P_f , is low due to

the effect of variance reduction. At higher α , where a series of high or low permeability values occurs in adjacent nearest-neighbor blocks, the solution of the porous media flow equation giving the phreatic surface locations may move away from its mean location. Thus, a higher probability of failure would result. Comparing Fig.9 and Fig.11 showing the phreatic surface for α equals 0.50 and 0.95, respectively, confirms clearly the influence of the α parameter.

SUMMARY AND CONCLUSIONS

A pseudo-three-dimensional probabilistic slope stability model has been developed to perform a probabilistic analysis including soil shear strength and pore pressure as random variables. The pore pressure uncertainty was due to the spatial variation of saturated soil permeability. The model developed in

Table 3. Comparison of probability of failure, P_f , and reliability index, β , at different standard deviation of YS -process, $\tilde{Y}S$

Center : x = 73.17 m y = 85.37 m	R A D I U S							
	R = 30.85 m		R = 37.34 m		R = 46.00 m		R = 54.88 m	
P_f = Probability of Failure β_b = Reliability Index	β_b	P_f	β_b	P_f	β_b	P_f	β_b	P_f
$\alpha = 0.95$, $\tilde{Y}S = 0.46$ $\bar{\lambda}_x = 21.26$, $\bar{\lambda}_y = 19.77$	6.654	1.43×10^{-11}	9.956	1.19×10^{-23}	9.9875	8.64×10^{-24}	11.840	1.21×10^{-32}
$\alpha = 0.95$, $\tilde{Y}S = 0.60$ $\bar{\lambda}_x = 20.03$, $\bar{\lambda}_y = 18.47$	6.654	1.43×10^{-11}	8.847	4.48×10^{-19}	9.071	5.90×10^{-20}	11.040	1.23×10^{-28}

Table 4. Comparison of the values of probability of failure, P_f and reliability index, β_b , for different values of the autoregressive parameter, α

Center : X = 73.17 m Y = 85.37 m	R A D I U S							
	R = 30.85 m		R = 37.34 m		R = 46.00 m		R = 54.88 m	
Probability of Failure, P_f	β_b	P_f	β_b	P_f	β_b	P_f	β_b	P_f
Reliability Index, β_b								
$\alpha = 0.50$ $\tilde{Y}S = 0.46$ $\bar{\lambda}_x = 7.62$ $\bar{\lambda}_y = 7.52$	6.654	1.43×10^{-11}	10.939	3.76×10^{-28}	11.424	1.58×10^{-30}	12.995	6.50×10^{-39}
$\alpha = 0.95$ $\tilde{Y}S = 0.46$ $\bar{\lambda}_x = 21.26$ $\bar{\lambda}_y = 19.77$	6.654	1.43×10^{-11}	9.956	1.19×10^{-23}	9.988	8.64×10^{-24}	11.840	1.212×10^{-32}

this study demonstrates the following :

(1) The simultaneous, autoregressive, nearest-neighbor model provides an adequate generation technique for permeability realization satisfying a known space law in a stationary medium.

(2) The results of the probabilistic analysis will be severely limited if the pore pressure uncertainty due to the spatial variation of soil permeability is not taken into account. Larger magnitudes of the probability of failure may occur due to the uncertainty of the pore pressure in addition to the uncertainty of the soil shear strength.

(3) An increase in the variance of the permeabilities of an earth slope indicating an increase in the heterogeneities of the soil consisting the earth slope, leads to an increase in the uncertainty of the factor of safety, and thus, an increase in the probability of failure.

(4) An increase in the autoregressive parameter, α , indicating larger distances of correlation of permeabilities of an earth slope, leads to an increase in the probability of failure.

The hypothetical embankment analyzed in this study was limited to cohesionless soils and to a particular cross-section. For future research, the analysis of embankments of different cross-sections and consisting of different soil types, especially c - ϕ soils, is recommended. The simulation technique used in this probabilistic model may also be applied to generate distributions of structural discontinuities in jointed rocks and to generate the grain size distributions or porosities of rockfills.

NOTATION

- α =an autoregressive parameter expressing the degree of spatial dependence
- β =reliability index
- I^2 =variance reduction function
- ε =uncorrelated random variable with mean zero and variance ε^2
- ρ =autocorrelation function
- σ =total stress at a point normal to failure surface

$P_f(b)$ =probability of failure of a mass of width b
 YS =the random variable satisfying the nearest-neighbor relation

$\bar{YS}$ =mean of YS

$\hat{YS}$ =standard deviation of YS .

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