OVERALL EVALUATION OF GEOTECHNICAL HAZARD
BASED ON FUZZY SET THEORY

VAN UU NGUYEN*

ABSTRACT

In land zoning and resources extraction industry, hazard delineation normally constitutes an essential element in the processes of planning and engineering operations. The basic requirements for hazard delineation are increased safety and effective operations, and the most common method of hazard delineation involving multiple criteria is the overlay method.

In this paper, fuzzy set operations are proposed as a tool to synthesize hazard indices of grid points or zones from a set of overlay plan maps. The proposed synthesis or aggregation procedure is based on the subjective nature of hazard index assessment and the multiple criteria associated with the overlay maps. These multiple criteria are normally complex for analysis by conventional mathematical approaches, and are assumed here to be generally non–interactive.

The procedure is simple to use, suitable for general multicriteria modelling and can be readily implemented on a computer.

Key words: landslide, stability analysis, statistical analysis, slope stability, fuzzy sets (IGC: E 6)

INTRODUCTION

Evaluation of potential hazards is perhaps a primary preoccupation of geotechnical engineering. With increasing world-wide trend of urbanisation, landslides continue to pose an ever present threat to human activities in hilly or sloping terrains. In the undertakings of resources extraction, offshore or underground, evaluation and delineation of hazard potential are often required for the formulation of investment strategy or the planning of safe and effective exploration or mining.

There are numerous approaches to hazard potential delineation. These approaches include on–ground monitoring, photogrammetry, satellite imagery, remote sensing techniques, deterministic, statistical and probabilistic analyses, overlay methods, and numerical modelling by the computer. A large body of geotechnical literature has been devoted to the development, field verification and application of these techniques. This paper is concerned mainly with the application of fuzzy set theory in the various classes of hazard potential delineation methods using factor overlay.

* Department of Civil and Mining Engineering, University of Wollongong, NSW 2500–Australia.
Manuscript was received for review on September 27, 1984.
Written discussions on this paper should be submitted before July 1, 1986, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.
TYPES OF HAZZARD DELINEATION BY OVERLAY

Landslide Hazards

Ward et al (1979) reviewed methods of evaluating landslide hazards in watersheds and concluded that the most common delineation method in use is factor overlay. The various factors used in the overlay as quoted by Ward et al included: topography, slope, erosional development and associated rock types, precipitation, rock strength, vegetation effects, and stream pattern.

The method essentially involves the determination or estimation of various factors that may trigger a landslide. Values for each individual factor are plotted on a transparency map. Individual maps are next overlayed on one another and subjective judgement is made to aggregate the weights contributed by all the individual factors to selected locations on the overlay or final hazard delineation map.

An overlay method of landslide potential classification based on the concept of classical set theory was proposed by Simons and Ward (1976). According to this method, landslide potential is classified in three different categories: high, medium, and low. A point on the map is to have a high landslide potential if it is located at an intersection of the subsets of all individual factors that could contribute to landslide. Similarly, it would have medium or low potential if it belongs to an intersection of a lesser number of factor subsets: or it is located entirely outside the union of all the subsets, respectively (Fig.1). Slope stability studies for the planning of additional urban development in the San Francisco Bay region were described by Nilsen and Brabb (1973). The main overlay maps used in the studies for evaluation of landslide potential include: a geologic map emphasising engineering properties of rock units including shear strength parameters, map of past landslide activities, map of clay mineralogy of soils and bedrock strata, and slope maps.

Hazard Analyses in Mining

Two areas of mining engineering that normally require the use of overlay methods in hazard delineation are: mine subsidence and stability of underground openings. The following description is only based on the latter for illustration.

Hazard analysis techniques for application in minimizing hazardous mining conditions have been described in detail by Ellison (1978), Ellenberger (1981) and Seegmiller (1983). Like the above overlay procedure for landslide potential evaluation, these mining overlay techniques involve the construction of a series of plan maps showing zones where each important geotechnical variable is exhibited. The plan maps with grids of geotechnical parameter values are then overlaid and subjective judgement is made to provide a composite picture of zones having the overlap of multiple geotechnical parameters. A numerical rating system can be established as a hazard index. The size and location of each hazard index zone may then be subjected to a cost multiplier. Zones which have high relative entry development or mining costs, as inferred from the hazard indices, may be abandoned or deferred for future mining. Zones with relatively low hazard costs could be mined initially. It is emphasised by Ellison (1978) that the synthesis of individual parameters is best obtained by a "round-table" group consisting of individuals with different disciplines, back-
grounds, experience and responsibilities to the project. Members of the panel giving subjective judgement on the overlay factors may thus include: the mining engineer, the hydrogeologist, the structural geologist, the geotechnical or strata control engineer, the safety officer, the economist and others. The synthesized results can naturally be processed and plotted by a computer. The method of hazard analysis, by factor overlay, could also be used to establish index maps for evaluating probable caving conditions behind longwall supports, estimating the magnitude of water inflow for different areas of the mine, and delineating zones of gas outburst potential.

It is proposed in the following, that overlay methods of delineating hazard potential in geotechnics can be rationally performed by the application of fuzzy set theory and multicriteria modelling.

FUZZY SET THEORY

Basic Description

The notion of fuzzy sets was first introduced by Zadeh (1965) to deal primarily with a real-life class, in which there may be a continuum of grades of membership. Fuzzy sets underlie much of our ability to make decisions based on information of a vague or imprecise nature. Take, for example, the identification of a highly swelling soil. To carry out the task, we may often ask ourselves "How swelling is highly swelling?" and it can be recognized immediately that such identification is largely subjective since it is inextricably linked with the acceptable consequences of swelling soil behaviour. The class or zone of highly swelling, in contrast with, say, a class of concrete mixers, is said to be a fuzzy set, which is a class admitting the possibility of partial membership in it. Through fuzzy set theory, descriptive words and phrases, such as "unstable" and "highly swelling soils", can be interpreted by the use of membership functions and linguistic variables that these membership functions serve to describe. Let

\[ U = \{ x \} \] denote a space of objects, or a universe of discourse, then a fuzzy set of \( A \) in \( U \) is a set of ordered pairs:

\[ A = \{ (x, \mu_A(x)) \}, \quad x \in A \text{ and } A \subseteq U \]

where \( \mu_A(x) \) is termed "the grade of membership of \( x \) in \( A \)", which may take on values in the range \( 0 \)–\( 1 \). For example, after a soil investigation, we conclude that the soil in the area is "highly swelling". "Highly swelling" is now a linguistic variable describing a subjective interpretation of soil swelling potential. If "highly swelling" is assigned with a central value of 0.7, the fuzzy set \( A \) describing "highly swelling soils" can be written as:

\[ A = \{ 0.5|0.4, \quad 0.6|0.8, \quad 0.7|1.0, \quad 0.8|0.8, \quad 0.9|0.5 \} \]

in which 0.5, 0.6, 0.7, 0.8 and 0.9 are the various values assumed by the linguistic variable "highly swelling": 0.4, 0.8, 1.0, 0.8 and 0.5 are the respective grades of membership; and "|" is a delimiter.

Fuzzy Set Operations

The basic mathematical operations on fuzzy sets can be summarised as follows:

* the union of two fuzzy sets \( A \) and \( B \), i.e. \( A \cup B \), results in membership levels:

\[ \mu_{A \cup B}(x) = [\mu_A(x) \vee \mu_B(x)] \]

\[ = \max[\mu_A(x), \mu_B(x)] \quad (1) \]

i.e. the support of membership of \( x \) in \( A \) or \( B \) is equal to the maximum value of the strength of belief that \( x \) lies within \( A \) or within \( B \) (\( \vee \) in fuzzy set theory is equivalent to the union symbol \( \cup \) and denotes "maximum of")
* the grade of membership for the intersection of $A$ and $B$ (i.e., $A \cap B$) is:

$$\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$$

where

$$\mu_A(x) = 1 - \mu_A(x)$$

* the operation “Not $A$”, or the complement of a fuzzy set $A$ is given by:

$$\bar{A} = \{x | \mu_A(x)\}$$

* the fuzzy relation $R$, from a fuzzy set $A = \{x | \mu_A(x)\}$ and a fuzzy set $B = \{y | \mu_B(y)\}$ is defined as a set having dual membership in both $A$ and $B$, with grade $\mu_R(x, y)$, i.e.

$$R = \int_{A \times B} [(x, y) | \mu_R(x, y)]$$

where

$$\mu_R(x, y) = \mu_A(x) \wedge \mu_B(y)$$

$$\mu_R(x, y) = \min[\mu_A(x), \mu_B(y)]$$

**Example**: Fuzzy set $A$ of “excessive footing settlement” and fuzzy set $B$ of “large number of wall cracks” can be expressed as:

- “Excessive” = $A = \{0.2, 0.1, 0.4, 0.3, 0.6\}$
- “Large” = $B = \{0.6, 0.4, 0.7, 0.8\}$

the fuzzy relation:

$$R = A \times B$$

has the membership evaluated as:

$$\mu_R(x_1, y_1) = \mu_B(0.2, 0.6) = \min[0.1, 0.4] = 0.1$$

$$\mu_R(x_1, y_2) = \mu_B(0.2, 0.7) = \min[0.1, 0.6] = 0.1$$

and

$$\mu_R(x_2, y_1) = \mu_B(0.3, 0.4) = \min[0.1, 0.4] = 0.1$$

etc.

and the full binary relation of “excessive footing settlement” and “large number of wall cracks” can be set up as:

$$y = \begin{bmatrix}
0.6 & 0.7 & 0.8 & 0.9 & 1.0 \\
0.2 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.4 & 0.3 & 0.3 & 0.3 & 0.3 \\
0.8 & 0.4 & 0.6 & 0.7 & 0.7 \\
1.0 & 0.4 & 0.6 & 1.0 & 0.7 & 0.6
\end{bmatrix}$$

The fuzzy composition of $A'$ and $R$ is a fuzzy subset $B'$:

$$B' = A' \circ R$$

where each element of $B'$ is constructed from:

$$\bigvee_{y}[\mu_{A'}(x) \wedge \mu_B(x, y)]$$

i.e. $\mu_B(y) = \max[\min(\mu_{A'}(x), \mu_B(x, y))]$

which can be interpreted as a matrix multiplication of $A'$ and $R$ in the fuzzy framework with multiplication replaced by min and addition by max.

**Example**: Suppose the above binary relation $R_{A \times B}$ applies to a residential area. After inspecting a footing foundation in the area, a geotechnical engineer gives his opinion of the extent of excessive settlement as:

$$[A'] = [0.2, 0.6, 0.1, 0.9, 0.3, 0.7, 0.8]$$

then, the expectation of observing a large number of cracks in the building has a membership matrix of:

$$[\mu_B] = \begin{bmatrix}
0.2 & 0.7 & 0.9 & 0.9 & 0.4 & 0.6 & 0.9 \\
0.1 & 0.1 & 0.1 & 0.1 & 0.4 & 0.6 & 0.7 & 0.8 \\
0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\
0.4 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\
0.4 & 0.6 & 0.7 & 0.8 & 0.7 & 0.6 & 0.6 & 0.6 \\
0.4 & 0.6 & 1.0 & 0.7 & 0.6 & 0.6 & 0.6 & 0.6 \\
\end{bmatrix}$$

where each element of $[\mu_B]$ is obtained by the min-max operations of fuzzy matrix multiplication, e.g., for the first element of the membership matrix:

$$\mu_{B'}(y) = \max[\min(0.2, 0.1), \min(0.2, 0.3), \min(0.7, 0.4), \min(0.9, 0.3), \min(0.9, 0.4)]$$

$$= \max[0.2, 0.4, 0.4, 0.4, 0.4] = 0.4$$

etc.

The fuzzy set $B'$ is:

$$B' = \{0.6, 0.4, 0.7, 0.6, 0.8, 0.9, 0.9, 0.7, 1.0, 0.6\}$$

i.e. Cracking is “0.6-severe” with a support of 0.4, “0.7-severe” with 0.6, etc.

**HAZARD DELINEATION WITH FUZZY SETS**

Let us now return to the universe of
geotechnical hazards, and recall that hazard
delineation by overlay technique involves the
construction of a series of plan maps, each
of which indicates a hazard potential contrib-
uted by a different criterion. Assessment
of the overall hazard potential by aggregating
the various hazard ratings or criteria is by and
large a subjective decision making process.
In such process, individual ratings are first
aggregated from field data, mapping, labora-
tory testing and analytical results and above
all, the resulting assessments by individual
members of an inter-disciplinary panel.
This first step of aggregation can be con-
veniently performed by arithmetic means,
probabilistic sums or fuzzy set operations.
For details, see Nguyen (1984).
The next and final step of hazard deline-
ation by fuzzy sets is aggregation of all the
known and available criteria contributing to
the likelihood and severity of hazard occur-
rence. This is best illustrated by an example
as follows:
Suppose we have established that landslide
potential in an area is mainly governed by
five factors:
(i) slope angle,
(ii) precipitation and run off pattern,
(iii) slakability of a bedrock stratum,
(iv) past landslide record, and
(v) shear strength parameters of surface
deposits.
Aggregation of the supports for each cri-
terion by the technical panelists resulted in
five transparency plan maps, each of which
indicates the degrees of support or belief
assigned to the grid points for a respective
hazard factor. The five criteria for the land-
slide hazard can be listed as:

\[ C_1 : \text{a high slope angle indicates high risk of instability. This criterion could be combined with the criterion on shear strength (} C_3 \text{) by factor safety calculations. In such case, a high factor of safety should correspond to a low hazard rating, and vice versa.} \]

\[ C_2 : \text{grid points belonging to a zone with high precipitation and poor drainage characteristics will have a high hazard rating.} \]

\[ C_3 : \text{a bedrock stratum with a high slakability index results in a high hazard rating,} \]

etc.
In a rating system that is numerically based
a rating can be derived by scaling from a
threshold value:

\[ (\text{Measured value}) \times \text{Rating} = \text{threshold value.} \]

e.g. If a factor of safety of slopes of 1.0
is considered the instability threshold, then an analytically determined value
of 1.25 for the factor of safety (F.S.)
of a slope grid point corresponds to a rating of

\[ \mu_{F.S.} = \frac{1.0}{1.25} = 0.80 \]

Let the set \( I = \{0, 0.1, 0.2, 0.3, \ldots, 1.0\} \) be the
set of decision alternatives, i.e. the set of
possible values of the overall or integrated
hazard index, from which we want to select
the best number that, in our opinion, can
satisfy all the five criteria stated above.
For simplicity, we reduce set \( I \) to a set of
only 6 elements:

\[ I = \{0, 0.2, 0.4, 0.6, 0.8, 1.0\} \]

Each grid point on the overlaid map of five
transparencies, has five sets of ratings, each
of which can be represented as a fuzzy set,
for example:

\[ C_1 = \{[0, 0.9, 0.2, 0.6, 0.4, 0.3, 0.60, 0.2, 0.8],[0.1, 1.0, 0] \}
\]
\[ C_2 = \{[0, 0.3, 0.2, 0.5, 0.4, 0.6, 0.6, 0.8, 0.8],[1.0, 1.0, 0.8] \}
\]
\[ C_3 = \{[0, 0.9, 0.2, 1.0, 0.4, 0.7, 0.6, 0.4, 0.8],[0.2, 1.0, 0] \}
\]
\[ C_4 = \{[0, 0.8, 0.2, 1.0, 0.4, 0.8, 0.6, 0.6, 0.8],[0.3, 1.0, 0.1] \}
\]
\[ C_5 = \{[0, 0.3, 0.2, 0.5, 0.4, 0.7, 0.6, 1.0, 0.8],[0.6, 1.0, 0.2] \}
\]
i.e. for criterion \( C_1 \) (slope angle), we believe
that the grid point has a hazard index of
0 with a strength of belief of 0.9, 0.2 with
0.6, 0.4 with 0.3, etc., or, in other words,
very low hazard by the slope criterion.
The established ratings for hazard index set
constitute a fuzzy binary relation:

\[
I = \begin{bmatrix}
0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 \\
1 & 0.9 & 0.6 & 0.3 & 0.2 & 0.1 & 0 \\
2 & 0.3 & 0.5 & 0.6 & 0.8 & 1.0 & 0.8 \\
3 & 0.9 & 1.0 & 0.7 & 0.4 & 0.2 & 0 \\
4 & 0.8 & 1.0 & 0.8 & 0.6 & 0.3 & 0.1 \\
5 & 0.3 & 0.5 & 0.7 & 1.0 & 0.6 & 0.2 \\
\end{bmatrix}
\]

\[R_{C \times I} = C\]

Using the aggregating procedure proposed by Bellman and Zadeh (1970), we state that our decision on the choice of hazard index is based on the maximum membership grade of a decision set D satisfying all the five criteria:

\[D = \{C_1, C_2, C_3, C_4, C_5\}\]

\[D = \{C_1, C_2, C_3, C_4, C_5\}\]

i.e. the chosen index \(I_i\) has a membership grade of:

\[
\mu_{I_i} = \bigwedge_{t=1}^{6} \left[ \bigvee_{k=1}^{5} \mu_{C_k} \right] = \bigwedge_{t=1}^{6} \left[ \bigvee_{k=1}^{5} \mu_{C_k} \right]
\]

where \(\mu_{C_k}\)'s are elements of the binary matrix \(R_{C \times I}\) above. We have therefore:

\[
\mu_{D} = \{\min (0.9, 0.3, 0.9, 0.8, 0.3), \min (0.6, 0.5, 1.0, 1.0, 0.5), \min (0.3, 0.6, 0.7, 0.8, 0.7), \min (0.2, 0.8, 0.4, 0.6, 1.0), \min (0.1, 1.0, 0.2, 0.3, 0.6), \min (0, 0.8, 0, 0.1, 0.2)\}
\]

and

\[D = \{0, 0.3, 0.2, 0.5, 0.4, 0.3, 0.6, 0.2, 0.8, 0.1, 1.0, 0\}\]

which yields:

\[
\mu_{I_i} = 0.5 \quad \text{and} \quad I_i = 0.2
\]

The hazard index for the grid point obtained from the five criteria by fuzzy set operations is 0.2 with a strength of belief of 0.5. By repeating the same procedure for other grid points we can establish a grid or contour map of hazard indices for the whole area considered.

**PRACTICAL EXAMPLE**

Multicriteria evaluation techniques for quantifying landslide hazards, similar to the overlay method, have been extensively developed mainly by Japanese workers (e.g. Kubomura and Takei, 1971; Haruyama and Kitamura, 1984; Kawakami and Saito, 1984). These techniques are based on the theory of quantification put forward much earlier by Hayashi (1950, 1952), and comprise four different methods of quantifying categorical data: method I for numerical external criteria, method II for qualitative external criteria, and methods III and IV for quantification problems in the absence of external criteria. "External criteria" are those criteria that are derived from past observations, for example, data of landslide slopes and stable slopes (e.g. Kawakami and Saito, 1984).

It can be recognized that Hayashi's methods of quantification bear strong resemblance to the various classes of techniques available for clustering and factor analysis commonly used in social sciences today (e.g. Lorr, 1983; Michaud, 1983). These multicriteria evaluation techniques normally treat the different criteria or categories as statistical variables with varying degree of interdependence or correlation among them. It is considered that these techniques are very powerful in the grouping classification of the various criteria, and yet, from a theoretical point of view, the aggregation of these criteria, most of which are largely subjective, by arithmetic (i.e. summation of scores) or probabilistic (i.e. unions and intersections) operations may not be totally appropriate.

A practical example is used in the following to illustrate the viability and simplicity of fuzzy set aggregation method, as compared with the conventional scores summation procedure, in the context of landslide hazard delineation. The basic criteria used in evaluating landslide risk, as published by Haruyama and Kitamura (1984), are used again for the illustration. Table III of Haruyama and Kitamura's paper is reproduced in Table 1 here. According to Haruyama and Kitamura's analysis, numerical values of 0 to 4 can be assigned to eight different categories effecting landslide risk, namely: elevation, contour density, relief energy, stream density, vertical type of slope, horizontal type of slope, geology, and ground cover. The
Table 1. The scores of risk degree for each item by grouping classification (reproduced from Table III, Haruyama and Kitamura (1984))

<table>
<thead>
<tr>
<th>Item</th>
<th>Score</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevation</td>
<td>0-149</td>
<td>150-249</td>
<td>250-359</td>
<td>400-599</td>
<td>600-</td>
<td></td>
</tr>
<tr>
<td>Contour density</td>
<td>0-6</td>
<td>7-9</td>
<td>10-11</td>
<td>12-18</td>
<td>19-</td>
<td></td>
</tr>
<tr>
<td>Relief energy</td>
<td>0-49</td>
<td>50-99</td>
<td>—</td>
<td>100-149</td>
<td>150-</td>
<td></td>
</tr>
<tr>
<td>Stream density</td>
<td>0-5</td>
<td>—</td>
<td>6-10</td>
<td>11-30</td>
<td>31-</td>
<td></td>
</tr>
<tr>
<td>Vertical type of slope</td>
<td>—</td>
<td>Convex and Complex</td>
<td>Straight</td>
<td>Concave</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geology</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Ground cover</td>
<td>Housing lot and Orchard</td>
<td>Forest</td>
<td>—</td>
<td>Grassland</td>
<td>Bare land</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Classification of landslide risks from the total scores (reproduced from Table IV, Haruyama and Kitamura (1984))

<table>
<thead>
<tr>
<th>Sum of score</th>
<th>Level of risk degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-9</td>
<td>0</td>
</tr>
<tr>
<td>10-16</td>
<td>I</td>
</tr>
<tr>
<td>17-23</td>
<td>II</td>
</tr>
<tr>
<td>24-29</td>
<td>III</td>
</tr>
</tbody>
</table>

Summation of all the scores corresponding to the eight categories for each mesh will determine which class of risk level the mesh belongs to (Table 2).

Inspection of Table 1 suggests strongly that each individual numerical value or description corresponding to a criterion, obtained for a particular mesh element, can fall within the ranges near the boundaries between any two of the five different score classes, and yet the class rating or score for the criterion assumes a uniform value regardless of any judgement on the accuracy of field measurements or observation. It is thus stressed that the proposed fuzzy set application in this instance attempts to address a fundamental question. The question is that in the case the engineer is unduly concerned about uncertainty arising from quantitative and, particularly, qualitative data of the various categories falling into inter-class regions, and given that the scores summation method would result in the same classification as in the case of crisp data involving a much higher degree of certainty on the data, how would the engineer account for this type of uncertainty in his procedure of deriving a landslide hazard index or classification?

In the framework of fuzzy sets, we can now transform the five different scores (0, 1, 2, 3, and 4) into four different risk degrees of landslide potential (0, I, II, and III), as shown in Table 4, and apply the Bellman–Zadeh aggregation procedure (i.e. the min–max scheme) to select an optimum classification from the degrees of support assigned to the various classes. The degrees of support or membership grades are given in lieu of, and based directly on the numerical value or qualitative description proposed by Haruyama and Kitamura in Table 1. It is observed that the effect of replacing 5 different scores by the final four risk levels in criteria aggregation means the lumping of those criteria having scores of 0 or 1, and is thus not as ideal as that in a situation having equal number of transformations, such as fuzzification of the CSIR rock mass classification system (Nguyen, 1985b). It is shown in the following, however, such replacement does not affect the final classification in any significant way. The risk level obtained by fuzzy aggregation is essentially the same as that by the scores summation procedure, and yet, the fuzzy scheme can incorporate uncertainty of numerical or qualitative data, particularly those lying on the boundaries between different risk levels or scores. For example, referring to the Haruyama–Kitamura table, a stream density of 10 would be assigned with a score of 2,
whereas in the fuzzy rating procedure, we would give the stream density a degree of support of, say, 0.90 for a class corresponding to a score of 2, a membership grade of 0.82 for a class corresponding to a score of 3, and 0.60, i.e. lesser support for a class corresponding to scores of 0 and 1.

In this example, suppose for a certain mesh element, we have obtained the following data, numerical and qualitative, corresponding to the eight categories of Table 1:

(i) Elevation: 240 m  
(ii) Contour density: 9  
(iii) Relief energy: 60 m  
(iv) Stream density: 10  
(v) Vertical type of slope: slightly concave, or in fuzzy grammar, Concave[1.00, 0.80, Very concave]0.70.  
(vi) Horizontal type of slope: Straight (i.e. straight [0.75, etc])  
(vii) Geology: Type B  
(viii) Ground cover: Grassland to bareland.

The scores corresponding to the eight items or categories of Table 1 can be obtained for the mesh element, and are shown in Table 3. From Table 3 the sum of scores is 12 indicating that the landslide potential for this mesh element belongs to Class I, according to the Haruyama and Kitamura grouping classification (Table 2).

Referring to Table 1 and the results summarized in Table 3, we can replace the scores by membership grades set up as a fuzzy binary relation shown in Table 4. The membership grades or degrees of support given in Table 4 are subjectively derived with the aid of linear scaling from a threshold or by using triangular fuzzy or possibilistic distribution (e.g. Nguyen, 1985 b). For example,

- in the item of contour density, we have a numerical value of 9 belonging to Class 0 but bordering Class I. Noting that Class I has a central value between 10-11. i.e. 10.5, the degree of membership for this item in Class I is roughly:

\[ \mu_{(ii)} = 9/10.5 = 0.85 \]

- in the stream density item (iv), the field data was obtained as 10, which corresponds to a score of 2 in the scoring method. Uncertainty from this item arises from the fact that the data on stream density lies on the upper bound of Class I having a range 6-10. The central stream density for this Class is 8 (Table 1). If we sketch a fuzzy (or possibilistic) distribution of a triangular type with maximum support (\( \mu = 1.0 \)) at density equal to 8, and a support of 0.82 at the two bounds, 6 and 10, then the membership grade or our degree of support for this stream density item to belong to Class 0 or II would be low, and can be read off from the triangular possibilistic distribution as 0.60 and 0.55, respectively. It is recalled that a possibilistic distribution is a plot of membership grade or degree of support \( \mu(x) \) versus the values of \( x \), in a similar fashion as the well-known probability distribution.

- For item (v), Vertical Type of Slope, the field engineer recorded “Straight” as the

<table>
<thead>
<tr>
<th>ITEM</th>
<th>CLASS</th>
<th>0 (0-1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>0.70</td>
<td>0.74</td>
<td>0.53</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>0.88</td>
<td>0.85</td>
<td>0.60</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>0.80</td>
<td>0.60</td>
<td>0.48</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>(iv)</td>
<td>0.60</td>
<td>0.82</td>
<td>0.55</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>(v)</td>
<td>0.40</td>
<td>0.80</td>
<td>0.90</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>(vi)</td>
<td>0.75</td>
<td>0.65</td>
<td>0.60</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>(vii)</td>
<td>0.70</td>
<td>0.80</td>
<td>0.90</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>(viii)</td>
<td>0.50</td>
<td>0.70</td>
<td>0.90</td>
<td>0.80</td>
<td></td>
</tr>
</tbody>
</table>

Min. 0.40 0.60 0.48 0.20
Max. 0.60
vertical type of slope, yielding a score of 2, no matter how “straight” the slope is. Suppose the field engineer also made an observation that although the slope is vertically straight it is also slightly concave. In the fuzzy framework, we account for this type of uncertainty, arising from human perception and subjective judgement, by assigning a degree of support of 0.8 for the slope being truly straight, 0.9 for straight and slightly concave, and very low degrees of support (0.4) for being convex and complex. - etc.

By continuing the same procedure, we can assign the degrees of support or membership grades to the binary relation of “Item versus Classification” (Table 4) from field data, summarized in Table 3, using our expert judgement on every bit of data. Aggregation of these membership grades is then performed by taking the minima down the columns, and maximum along the final row of the resulting minima, yielding an optimum membership grade of 0.60 corresponding to Class I. The result is thus identical to that obtained by scores summation, and yet the min-max procedure can incorporate our expert judgement reflecting subjective uncertainty on the numerical and qualitative descriptions for the various items.

DISCUSSION

The major objective of this paper is to draw the attention of geotechnical engineers involved in the practice of landslide hazard delineation to the potential applicability of fuzzy sets as a tool to assist in the procedure of hazard index aggregation or hazard classification with incorporation of expert knowledge.

It has been shown that fuzzy set operations can provide a simple and “risk-aversion” procedure in aggregating a hazard index from a number of overlay maps each depicting a different factor that may contribute to triggering a landslide. By means of fuzzy sets, the proposed procedure admits the complexity and imprecise nature of the types of information from which the overlay ratings, are derived, and the theoretical difficulties in aggregating both quantitative and qualitative data involving elements of subjective uncertainty. Determination of most of these ratings is, very often, cost ineffective or even infeasible by conventional analytical techniques.

The fuzzy procedure essentially involves assignment or input of membership grades or degrees of support to the binary relation between failure-triggering factors and the risk classification. It should be stressed however that the use of fuzzy sets is not proposed here to replace multivariate or discriminant analysis (e.g. Lorr, 1983; Michaud, 1983) in setting up the binary relation or in identifying the principal components responsible for landslides. Similar to a multivariate analysis however the salient feature of factor aggregation by fuzzy sets is the non-interactivity among the factors underlying fuzzy set operations (e.g. Dubois and Prade, 1980). It is recalled that in a multivariate procedure, an axis rotation scheme is customarily used to obtain uncorrelated variates required in the analysis. The fuzzy set procedure further emphasizes the human perception nature or expert judgement, the qualitative criteria used in landslide hazard delineation, and the theoretical basis for aggregation of these criteria.

The first step used in the proposed aggregation procedure is to select membership grades associated with each hazard criterion. By scaling out the membership grade or rating from a threshold, the procedure does not preclude various mathematical and statistical ways of objectively deriving a safety index, i.e. the threshold or range of numerical values based on which a membership grade can be obtained. The practical estimation of membership functions or possibilistic distributions is still a major area of research for workers in the field of artificial intelligence and expert systems. For engineering purposes at present, apart from threshold scaling, two other popular strategies, i.e. exemplification and statistical scaling, can
be used. Exemplification is basically a survey of opinions whereby grade values of certain linguistic variables, such as "important", "critical", can be obtained. Statistical scaling involves constructing a set of statistical data in the form of a histogram. The membership grades or ratings are however scaled in a different manner to that used in frequency estimation. For membership grading, the histogram is normalized through an affine transformation that brings the highest ordinate to 1, whereas in frequency estimation, the whole area made up by the histogram is brought to 1 (e.g. Dubois and Prade, 1983). In this paper, we have highlighted through the practical example, the use of a triangular possibilistic distribution, under the basic principle of statistical scaling and scaling of degree of support from a certain threshold to assign the various membership grades. In the case high reliability of membership grade assignment is required, aggregation of membership grades for each individual hazard criterion is performed as the next step, and the aggregation is normally obtained from a technical and interdisciplinary panel, again by using the pessimistic "minimum rule" (e.g. Nguyen, 1985a).

Two more points should be emphasized in fuzzy aggregation of a hazard index. First, in the examples above, equal importance has been assigned to the rating for each individual item. Should a particular item deserve a different degree of emphasis, it can be assigned with a weight \( w \) to its membership grade \( \mu \) and modified by an exponential transformation:

\[
[\mu] \Rightarrow [\mu^w]
\]

in which the weight exponent \( w \) is greater than 1.0 for major influence and less than 1.0 for minor influence. Second the ratings assigned for every item should be of comparable magnitudes to avoid any bias tendency towards a particular criterion having low ratings. For example, in Table 4 above, the minima obtained were not derived from a single item but from many different items, i.e. 0.40 from item (v), 0.60 from item (iii), etc. For more description of these fuzzy characteristics, see Nguyen (1985a, 1985b).

Finally, a brief comparison should be made between probabilistic analysis, and fuzzy set application to clarify the proposed min-max aggregation procedure. Probability theory is an established discipline used in geotechnical engineering to quantify uncertainty of a statistical or frequency-related type, such as parameter variability and uncertainty, or the likely frequency of occurrence of a hazard or undesirable event. Fuzzy set theory on the other hand refers to a different type of uncertainty embodying imprecise data, human errors and omissions, and is closely linked with human perception or expert judgement. In the current literature of fuzzy sets, the theory is gaining the new name of possibility theory with many characteristics being developed parallel to those of probability theory. It is emphasized again that possibilistic operations as proposed here serve to overcome many difficulties in the modelling of subjective uncertainty and aggregation of a set of mixed data, quantitative and qualitative, involved in hazard delineation.

In a probabilistic framework, we model "AND...AND" by a joint probability, and "OR...OR" by the probability union. In possibility theory, "AND...AND" is represented by "The Minimum of...", and "OR...OR" "The Maximum of". The possibilistic approach underlies much of human risk-aversion attitude. For example, landslide can occur due to heavy rainfall AND concave slope AND soft rock stratum AND etc. This can be said to be perceived in a risk-aversion manner by using the "MINIMUM of" our degrees of belief on each of these potentially causal factors. In fact this step of aggregation using MINIMUM is becoming almost a universal method of modelling the linguistic AND in medical expert systems. For more details on the sensitivity of the aggregation process and its sensitivity with varying degrees of factor ratings, the readers are referred to Nguyen (1985a) and Nguyen
and Ashworth (1985).

CONCLUSION

Through examples of landslide hazard delineation, it has been shown that fuzzy set operations for multicriteria modelling are applicable in various classes of geotechnical hazard delineation using overlays. These applications include: mining hazard analysis, earthquake zoning, landslide hazard delineation, etc.

The proposed fuzzy procedure is simple to use, gives excellent agreement with the conventional scores summation method, and yet it can incorporate expert judgement on the subjective uncertainty of many criteria required in the aggregation process. The overlay method based on fuzzy aggregation procedure can be readily implemented on a computer.

REFERENCES