TESTING AND MODELLING OF 'MUNICH' SAND

F. SCHEELE*, C. S. DESAI** and A. MUQTADIR***

ABSTRACT

A general plasticity based constitutive model is developed for simulating hardening, stress path dependence, volume change and coupling of volume and shear responses of a cohesionless soil, called "munich" sand. The model parameters are evaluated from a comprehensive series of laboratory multiaxial tests on cubical specimens of the material. The proposed model is verified by comparing predictions with observed test results and with predictions of the test behavior by using the finite element method.

Key words : constitutive equation of soil, laboratory test, plasticity, sand, stress path, stress-strain curve, yield (IGC : D 6)

INTRODUCTION

Analysis and design of grouted anchors in cohesionless soils pose a difficult soil-structure interaction problem. Proper definitions of constitutive models for the soil as affected by factors such as initial state of stress, volume changes and stress paths, assume an important role in the analysis of such anchors.

Pullout tests of anchors in the field (Jelinek and Scheele, 1977 ; Scheele, 1981) show that the load displacement behavior is highly nonlinear. It is significantly influenced by the nonlinear stress-strain properties of the soil in which the grouted tendon is embedded. Therefore, a comprehensive laboratory study of the soil in which the anchors are

installed was conducted in order to characterize its constitutive behavior.

The soil was a gravelly sand, here called "Munich" sand, originating from the vicinity of Munich, West Germany.

The testing of anchors was performed at the University of Munich (Jelinek and Scheele, 1971 ; Scheele, 1983). Full scale anchors were installed in a large test pit in which the sand was compacted to a desired and uniform density. Specimens of the sand were brought to the University of Arizona for testing by using the truly triaxial or multiaxial device.

From a practical viewpoint, it is usually desirable to adopt comparatively simple laboratory tests to define the properties needed

* Alexander von Humboldt Fellow, W. Germany ; Department of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, AZ 85721, USA.

** Professor, Department of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, AZ 85721, USA.

*** Engineering Mechanics Research Corp., San Jose, California ; formerly Graduate student, Univ. of Arizona, Tucson, AZ 85721, USA.

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for analysis. However, it has been shown (Scheele, 1981) that use of the stress-strain relations based on the cylindrical triaxial test apparatus, only quantities such as ultimate skin friction along the grouted anchor can be obtained. Hence, it is felt that to analyze and understand the complex soil-structure interaction problem, more sophisticated and rigorous testing and modelling are required because the accuracy from a solution scheme such as the finite element method is limited if the constitutive behavior is not appropriately defined.

The behavior of the soil can be characterized as nonlinear and visco-plastic; the latter observation is based on the pullout test results of the anchors in the field (Jelinek and Scheele, 1977). It is affected by factors such as state of stress, stress path, volume change and coupling of volume and shear responses. Hence, it is desirable to develop a constitutive model that can account for these factors and is based on appropriate laboratory test program.

OBJECTIVE

The objective of this paper is to present details of testing under different stress paths by using the multiaxial device and application of a generalized plasticity based model and its verification.

Fig. 1. Grain size distribution for "Munich" sand

LABORATORY TESTING

Test Material

The material used in this investigation is a coarse to medium, well graded sand with about 3% silty particles. It is a washed out fraction (termed as 0/4 mm material) of a typical subgrade soil encountered in the southern part of Bavaria, West Germany. The grain size distribution of this material is shown in Fig. 1; also shown are the specific gravity, G_s , and the coefficient of uniformity, C_u .

The characteristic values of the moisture (w)-density (ρ) relationships at two different compaction efforts were

$$\text{Standard Proctor (Standard AASHTO)} \left\{ \begin{array}{l} \max \rho_d = 2.04 \text{ t/m}^3 \\ w_{opt} = 8.2\% \end{array} \right.$$

$$\text{Modified Proctor (Modified AASHTO)} \left\{ \begin{array}{l} \max \rho_d = 2.18 \text{ t/m}^3 \\ w_{opt} = 8.7\% \end{array} \right.$$

The natural water content of the sand is about $w = 4.1\%$.

Concentrating on a medium dense material ($D_r = 0.7$), specimens of initial density equal to $\rho = 2.03 \text{ t/m}^3$ were tested. This is the same density in the field at which the anchor test series was performed (Jelinek and Scheele, 1977; Scheele, 1981).

Experimental Procedure

The test samples [$4 \times 4 \times 4$ inch ($10 \times 10 \times 10$ cm)] were compacted at the above density and natural water content using a special compaction mold. After compaction, the samples were trimmed at the top and the latex membrane was sealed so the integrity of the sample could be maintained by means of a low negative pressure applied to the sample while inserting it into the center cavity of the frame of the multiaxial cubical device. Details of the test apparatus are described in Refs. (1, 20). After assembling the test cell and restoring atmospheric pressure conditions inside the specimen, all initial readings were recorded with three linear variable differential transformers (LVDT's) on each face.

All the tests started with the application of a hydrostatic pressure to a desired confin-

(a) Stress Paths in Triaxial Plane

(b) Stress Paths on Octahedral Plane

Fig. 2. Commonly used stress paths**Table 1. Details of tests (Scheele and Desai, 1983)**

Stress path	Density (t/m ³)	Confining Pressure (psi)
HC	2.01	—
CTC	2.00	3
CTC	2.03	6.5
CTC	2.04	10
CTC	2.03	13
CTC	2.03	20
CTC	2.03	30
RTC	2.03	13
RTE	2.03	13
CTE	2.04	10
CTE	2.03	30
TC	2.03	13
TE	2.03	13
SS	2.03	13
SS	2.03	20

1 psi = 6.89 kN/m²

ing pressure. The test device allows any arbitrary change of the three principal stresses. In general, the investigation of stress path dependency of a material is based on tests following a number of straight line stress

paths, represented schematically in Fig. 2. Table 1 shows details of the tests performed (Scheele and Desai, 1983).

A hydrostatic compression (HC) test, in which the three stresses were incrementally increased with intermediate unloading/reloading cycles, provided fundamental information for bulk or volumetric behavior of the sand. A wide variety of confining pressures, $\sigma_0 = 3$ (21) to 30 (207) psi (kN/m²), were chosen for a number of conventional triaxial compression (CTC) tests to study its influence on the material behavior and cover a wide range of stresses in order to simulate appropriately the in situ conditions of the sand near the anchor. All the other tests started from an initial equal all around pressure of 13 psi (90 kN/m²), except two conventional triaxial extension (CTE) tests with the confinements of 10 (69 kN/m²) and 30 psi (207 kN/m²), and two simple shear (SS) tests with 13 (90 kN/m²) and 20 psi (136 kN/m²) confinements.

The loading was applied gradually in increments of 2.5 or 5.0 psi (17 or 34 kN/m²), and each increment was maintained for about 15 minutes; this rate of loading is considered to be slow. In the tests performed, no attempt was made to reach a stable state of deformation by allowing sufficient time for an increment. This is partly because the time required for such stable states would be very large for the sand, and the resulting deformations would involve significant time dependent creep strains for which a modified model such as based on viscoplasticity may be more suitable.

For the tests performed here, it would be appropriate to consider stress-strain envelope corresponding to the states (point A, Fig. 3, which denotes a typical observed point) reached after 15 minutes for each increment; however, the strains after 15 minutes were very small compared to those that may occur if sufficient time until stable states was allowed. Hence, parameters computed from the envelope corresponding to maximum stresses (point A', Fig. 3) reached after various stress increments or those computed from the enve-

Fig. 3. Envelopes of stress-strain responses from conventional triaxial compression (CTC) tests, $\rho_0=2.03\text{ t/m}^3$; $1\text{ psi}=6.89\text{ kN/m}^2$

lope for states (point A, Fig 3) after 15 minutes were not significantly different. As indicated in Fig. 3, use of an average unloading slope with point A or A' indicates essentially the same plastic strains because the strains reached at the end of 15 minutes are very small. Thus, adoption of envelope corresponding to point A' is considered to approximately represent the behavior for the testing conditions considered.

The stress-strain response envelopes for six CTC tests at different confining pressures, presented in Fig. 3, give an idea of the in-

Fig. 4. Volumetric responses from conventional triaxial compression (CTC) tests ($1\text{ psi}=6.89\text{ kN/m}^2$)

Fig. 5. Octahedral shear stress vs strain results from different stress paths, $\sigma_0=13\text{ psi}$, $\rho_0=2.03\text{ t/m}^3$

fluence of initial hydrostatic pressure on the stress-deformation and ultimate shear strength of the sand. The samples exhibited a high degree of strain isotropy in the horizontal, x -(2) and y -(3) direction; the deviator stress was applied in the z -(1) direction (vertical). It was observed that the unloading moduli (slopes of unloading/reloading curves) increased somewhat with the increase of the confining pressure, supporting the well-known fact that the elastic modulus of a granular material is a function of the confining pressure. In Fig. 4, the volumetric response for the above tests are shown in the conventional ϵ_v - ϵ_1 space where ϵ_v and ϵ_1 = volumetric and axial strain, respectively. Here it is evident that dilatation is reduced with the increase of confining pressure.

Typical results for conventional triaxial compression (CTC), triaxial compression (TC), simple shear (SS) and triaxial extension (TE) tests are plotted as τ_{oct} - γ_{oct} relation in Fig. 5. In the early stages, the curves show a similar slope. Beyond this stress level and near the ultimate yield conditions, the behavior is significantly different, however. These relationships demonstrate the high dependency of the material behavior on the stress paths. It will be necessary to select a constitutive model which can provide a proper simulation under these

combined stress and stress path conditions.

CONSTITUTIVE MODELLING

Generalized Model

It has been generally accepted that the behavior of soils is too complex to be modeled adequately by linear and nonlinear elasticity, and improved theories of modelling are necessary. Constitutive laws based on the theory of plasticity can describe effectively the behavior of geological materials in spite of some inherent assumptions and limitations (Desai and Siriwardane, 1984).

Recently, a new generalized constitutive model using a general procedure (Desai, 1980 ; Desai and Faruque, 1983, 1984 ; Faruque and Desai, 1985) has been developed which can eliminate some of the limitations of other available plasticity based models (Sandler and Rubin, 1979 ; Schofield and Wroth, 1968). Desai (1980) proposed yield function as a complete polynomial in terms of the three invariants J_1 , J_2 and J_3 of the stress tensor, σ_{ij} , as

$$F(J_1, J_2, J_3) = \alpha_0 + \alpha_1 J_1 + \alpha_2 J_2^{1/2} + \alpha_3 J_3^{1/3} + \alpha_4 J_1^2 + \alpha_5 J_1 J_2^{1/2} + \alpha_6 J_2 + \alpha_7 J_1 J_3^{1/3} + \alpha_8 J_2^{1/2} J_3^{1/3} + \alpha_9 J_3^{2/3} + \alpha_{10} J_1^3 + \alpha_{11} J_1^2 J_2^{1/2} + \alpha_{12} J_1 J_2 J_3^{1/3} + \alpha_{13} J_1 J_2 + \alpha_{14} J_1 J_3^{1/3} + \alpha_{15} J_2 J_3^{1/3} + \alpha_{16} J_2^{3/2} + \alpha_{17} J_2^{1/2} J_3^{2/3} + \alpha_{18} J_3 + \alpha_{19} J_1 J_2^{1/2} J_3^{1/3} + \dots \quad (1)$$

Here $\alpha_i (i=0, 1, 2, \dots, n)$ = response functions and the (fractional) powers of J_1 , J_2 , J_3 are chosen to render the expressions dimensionless. Truncated forms of this equation can provide various acceptable functions to represent yield functions for given materials. One such function

$$F \equiv \alpha_6 J_2 + \alpha_9 J_1 J_3^{1/3} = 0 \quad (2)$$

was analyzed because it showed invariant values at ultimate conditions, irrespective of the stress path (Desai, 1980 ; Desai and Faruque, 1983, 1984 ; Desai and Siriwardane, 1984). The models proposed by Lade (1975) and Matsuoka and Nakai (1974) based on behavior of given geological materials are contained in Eq. (1). The ultimate condition is defined as the asymptotic values to a stress-strain curve. The conventionally used fail-

ure and critical states are either lower than or coincide with the ultimate state. At this time, this model describes the rate independent inelastic behavior of soils which are isotropic initially as well as during plastic deformation. The performance of this model under various types of loadings of the "Munich" sand will be examined.

One of the possible yield conditions, F , from the generalized model is expressed in terms of the invariants J_1 , J_2 and J_3 , as $F \equiv J_2 + \alpha J_1^2 - \beta J_1 J_3^{1/3} - \gamma J_1 - k^2 = 0$ (3) where α , β , γ , k are material response functions, and J_2 = second invariant of the deviatoric stress tensor. The terms α and γ are related to the frictional behavior of soil,

Fig. 6. Yield surface in various stress spaces

and k is a measure of the cohesive strength of the material. For convenience, α , γ and k are assumed to be constants associated with the ultimate yield envelope, and β is treated as a growth or evolution function and, as a simplification, it is expressed as $\beta = \beta(\xi)$, where ξ , the growth parameter, is the total plastic strain trajectory :

$$\xi = \int (d\epsilon_{ij}^p d\epsilon_{ij}^p)^{1/2} \quad (4)$$

where $d\epsilon_{ij}^p$ = plastic strain increment tensor.

The advantage of the single function plasticity model is the continuity and convexity of the yield functions describing hardening behavior of the soil, expanding or contracting isotropically about the hydrostatic axis in principal stress space (see Fig. 6(a)). In Fig. 6(b), the yield functions for various β are shown for a given value of J_{3D} , the third invariant of the deviatoric stress tensor, exemplifying the convex nature of the surface in the $J_1 - \sqrt{J_{2D}}$ space.

The explicit forms of the growth function β were proposed (Desai and Faruque, 1983, 1984) as

$$\beta = \beta_u \left(1 - \frac{\bar{\beta}_a}{\xi^\eta} \right) \quad (5a)$$

and

$$\beta = \beta_u \left[1 - \frac{\beta_a}{\xi^{\eta_1} (1 - \beta_b r_D^{\eta_2})} \right] \quad (5b)$$

Fig. 7. Plot of r_v vs r_D for different stress paths

where $\beta_u = 3\alpha$, which is the value of β at ultimate yielding. The ratio $r_D = \xi_D/\xi$ describes the deviatoric response of the material during plastic straining. The choice of r_D was guided by the plots of observed behavior under different stress paths in terms of r_v vs r_D , Fig. 7; here $r_v = \xi_v/\xi$ and ξ_v and ξ_D = trajectories of volumetric and deviatoric plastic strains, respectively. It can be seen from Fig. 7 that, irrespective of the stress path followed, the relation between r_v and r_D can be assumed to be the same. Thus, use of r_D , r_v or both in the growth function can allow for the effect of stress paths and coupling of volumetric and shear responses.

Growth functions in Eq. (5a) and (5b) are derived on the basis of laboratory test behavior. First the constants α , γ and k associated with the ultimate surface are determined (see subsequent description). Then, for a given stress-strain curve, the values of ξ and stress components (σ_1 , σ_2 , σ_3) are found at various points. These are then substituted in Eq. (3) to evaluate the corresponding values of β . The plots of β vs ξ are found to be nonlinear bounded by an ultimate asymptotic value $\beta_u = 3\alpha$. The functions in Eq. (5) are then derived on the basis of these plots of β vs ξ .

Determination of Constants

There are seven and nine material constants associated with the model with growth functions in Eqs. (5a) and (5b), respectively; two for elastic behavior and five or seven

Table 2. Material constants for "Munich" sand

Parameter	Magnitude	Physical Significance
E	16000(psi)	Elastic Modulus
ν	0.36	Poisson's Ratio
α	0.212	Material Parameters associated with the ultimate yielding
γ	3.67(psi)	
k	0	
$\bar{\beta}_a$	0.0055	Hardening Parameters in growth function, Eq. (5a)
η	0.74	
β_a	0.000278	Hardening Parameters in growth function, Eq. (5b)
η_1	1.161	
β_b	0.805	
η_2	0.704	

1 (psi) = 6.89 (kN/m²)

for plastic behavior. Table 2 gives the values of the constants ; brief descriptions for the procedures for finding them are given below.

The elastic parameters E and ν , where E is the Young's modulus and ν the Poisson's ratio, are determined from the CTC tests and the HC test. Average values for E and the bulk modulus K were obtained from the slopes of the unloading/reloading curves as-

Fig. 8. Plot of $\ln \left(1 - \frac{\beta}{\beta_u} \right)$ vs $-\ln (\xi_v)$ for HC Test

Fig. 9. Plot of $n \left[1 - \frac{\beta_a}{\xi^{\eta_1} \left(1 - \frac{\beta}{\beta_u} \right)} \right]$ vs $-n(r_D)$

Fig. 10. Plot of $\ln \left(1 - \frac{\beta}{\beta_u} \right)$ vs $-\ln (\xi)$

Fig. 11. Comparison of predictions and observations for CTC test, Eq. (5a) ; $\sigma_o = 6.5 \text{ psi}$

suming elastic responses of the samples.

The parameters associated with ultimate yielding α , γ and k were evaluated on the basis of the ultimate stresses for each test plotted in the $J_1 - \sqrt{J_{2D}}$ space. Since the ultimate surface curve passes through the origin, the value of k was assumed to be zero. Now, a least square procedure is used on the ultimate envelope to find α and γ .

For $r_D=0$, Eq. (5b) represents hydrostatic behavior, and β_a and η_1 are found from test results from hydrostatic behavior. Fig. 8 shows a plot of $-\ln(\xi_v)$ vs $\ln(1-\beta/\beta_u)$ for the sand. The intercept along the ordinate gives β_a and the slope of the straight line gives η_1 .

Fig. 12. Comparison of predictions and observations for TC test, Eq. (5a); $\sigma_0=13$ psi (1psi=6.89kN/m²)

Fig. 9 shows a plot of $-\ln(r_D)$ vs $\ln[1-\beta_a/\{\xi^{\eta_1}(1-\beta/\beta_u)\}]$ from various stress paths. An average straight line is drawn and its intercept along the ordinate gives β_b and its slope gives η_2 .

For the growth function in Eq. (5a), the values of $\bar{\beta}_a$ and η_1 are found from average plot of $-\log(\xi)$ vs $\ln(1-\beta/\beta_u)$ from different stress paths, Fig. 10.

Verification of Test Results

In order to verify the proposed constitutive model, a number of typical examples for back-predictions of the material responses using Eq. (5a) and (5b) are presented. Such comparisons are shown for various stress paths: CTC(Fig. 11(a), (b)), TC (Fig. 12(a),

Fig. 13. Comparison of predictions and observations for SS test, Eq. (5b); $\sigma_0=20$ psi (1psi=6.89kN/m²)

Fig. 14. Comparison of predictions and observations for CTE Test, Eq. (5a) ; $\sigma_0 = 10 \text{ psi}$ ($1 \text{ psi} = 6.89 \text{ kN/m}^2$)

(b)), SS (Fig. 13(a), (b)) and CTE (Fig. 14 (a), (b)). Here unloading-reloading are considered to be linear elastic.

The predictions and observations of the stress-strain responses in Figs. 11(a), 12(a), 13(a) and 14(a) indicate very good agreement. It may be noted that the material constants used were computed as average values from various tests.

Concerning the simulation of the volumetric behavior (Fig. 11 (b), 12 (b), 13(b), 14 (b)), excellent agreement is observed in the initial range of curves that represents the compressive response. The subsequent behavior involving volume increase (dilation) is predicted satisfactorily in the triaxial com-

pression, simple shear and extension tests (Fig. 12(b), 13(b), 14(b)). The correlation in this range for the CTC (Fig. 11(b)) is not that satisfactory, however.

Some of the main reasons for this divergence can be attributed to the nonassociative and (induced) anisotropy characteristics of cohesionless Munich sand. It is possible to modify the model by introducing a correction to the yield function F to develop plastic potential function, Q (Desai, Frantziskonis and Somasundaram, 1985 ; Desai and Siriwardane, 1980, 1984 ; Desai, Somasundaram and Faruque, 1985). An expression for the correction function can be obtained by modifying the growth function, β , as

$$\beta = \beta_F + f(r_v) \quad (6a)$$

where β_F = growth function in F , and r_v = ratio of the trajectories of volumetric and total plastic strain trajectories. This approach can be developed for the Munich sand and can provide improved predictions (Frantziskonis et al, 1985).

For the studies in (Desai, Frantziskonis and Somasundaram, 1985) and (Frantziskonis et al, 1985), an extended and alternative form of F is used as follows :

$$F = J_2 D - (-\alpha J_1^n + \gamma J_1^2) (1 - \beta S_r)^m = 0 \quad (6b)$$

where α , n , β and m are response functions. Note that the meanings of these functions are different from those in Eq. (3). Assuming α as the growth function, which is different from β in Eq. (3), the correction is introduced by modifying α to $\bar{\alpha}$ as

$$\bar{\alpha} = \alpha + \kappa(\alpha_I - \alpha) (1 - r_v) \quad (7)$$

where α_I = growth function at the start of nonassociativeness and κ = material constant found from volumetric response. As a simplification, it is found from the slope of $\varepsilon_v - \varepsilon_1$ plot near the ultimate (Frantziskonis et al, 1985).

The model has been implemented in a three-dimensional finite element program which allows solution of various boundary value problems such as simulation of the laboratory tests and field problems like piles and anchors. The predictions have been ver-

ified with respect to laboratory and field behavior of these problems; details are given elsewhere (Desai, Muqtadir and Scheele, 1985; Muqtadir, 1984).

CONCLUSIONS

This paper describes a comprehensive multiaxial laboratory testing of a sand (called "Munich" sand), conducted to obtain realistic representation of the soil behavior subjected to loading under various stress paths. On the basis of the tests, a generalized plasticity model is adopted, and its constitutive contents are evaluated. Accuracy and validity of the model are demonstrated by comparing back-predictions with the observations. This model is found to provide a satisfactory simulation of the behavior of the sand and, thus, applied in connection with a finite element code, offers a promising approach to predicting the behavior of ground anchors in cohesionless soils. Improvements in the model can be obtained by introducing corrections that can account for nonassociative behavior and (induced) anisotropy.

The proposed model involves only a single yield surface in contrast to the multisurface models such as the critical state (Schofield and Wroth, 1968) and cap (Sandler and Rubin, 1979) models and that proposed by Lade (1975). As a result, it avoids the singularity at the intersection of the (yield) cap and the failure surface thereby eliminates computational difficulties near the intersection. The hardening behavior is expressed in terms of the total plastic strain trajectory and the ratio r_D ; hence, it includes the effect of stress path and coupling of volume and shear responses. As a consequence, it provides improved predictions of the volume changes as compared to those from the previous models in which hardening is expressed only in terms of the volumetric plastic strain.

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