CYCLIC UNDRAINED TRIAXIAL AND TORSIONAL SHEAR STRENGTH OF SANDS FOR DIFFERENT SAMPLE PREPARATION METHODS

FUMIO TATSUOKA*, KENZO OCHI**, SHINJI FUJI***
and MASAHIRO OKAMOTO**

ABSTRACT

The effects of sample preparation methods on the cyclic undrained stress–strain behavior of sands were investigated by means of triaxial and torsional shear tests using two kinds of clean sands and one kind of sand including fines, with a wide variation in the sample density. Four different sample preparation methods were adopted; air–pluviation, wet-tamping, wet–vibration, and water–vibration. The differences in the effects of sample preparation methods were found to be significant in both testing methods. However, specific ways in which the sample preparation methods affected the results were not consistent between the triaxial and torsional shear tests. This indicates that the process of estimating cyclic undrained simple shear strengths from triaxial strengths is not as simple as has previously been considered.

It was found that in torsional shear tests the shapes of strength curves in the planes of the cyclic stress ratio and the logarithm of the number of cycles $N_s$ are similar for different sample preparation methods, in the sense that the different curves collapse into a single curve when these curves are moved appropriately along the log $N_s$-axis. It was also found that for the same sample density the effects of the sample preparation method on the cyclic undrained stress–strain behavior decrease with the increase in strain. A new density index is proposed, from which the cyclic undrained behavior can be estimated for different sample preparation methods and for different kinds of sands until the shear strain becomes around 3% in double amplitude.

Key words: earthquake, liquefaction, repeated load, sand, special shear test, torsion, triaxial test (IGC : D6/D7)

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INTRODUCTION

To evaluate the liquefaction strength of a presently existing soil layer by performing laboratory cyclic undrained tests, it is crucial to obtain very high-quality undisturbed samples. On the other hand, laboratory cyclic undrained tests on reconstituted samples are still useful where information is required concerning the as-placed liquefaction strength of sand in soil layers which will be artificially made in the future. Furthermore, it is often required in such cases to determine in advance the design compaction density so that these soil layers will have sufficient resistance against design cyclic loadings.

It is widely considered that a laboratory simple shear simulation is a more appropriate measure of cyclic undrained strength than a triaxial simulation for level sand deposits and sand slopes under the plane strain condition. However, the triaxial apparatus is still much more popular and much easier to operate than the simple shear apparatus. Accordingly, detailed information is still required concerning the relationship between triaxial and simple shear strengths. On the other hand, only a limited amount of literature is available concerning this relationship and also the test conditions employed in these investigations were rather limited (Ishihara and Yasuda, 1975; Silver et al., 1980).

In an attempt to evaluate the as-placed liquefaction strength of artificially compacted deposits, it should be considered that the cause of the increase in strength by compaction are attributable, at least in part to: (1) the increase in density by compaction, (2) the increase in $K_0$-value by compaction, and (3) the change in the fabric of sand by compaction (Fig.1). Furthermore, the long-term time effect should be accounted for if necessary. Suppose that in Fig.1 the point designated by the letter $A$ represents the condition just after filling by pluviation through water and the curve $a-a'$ is the strength curve for the water-pluviated sand. The point designated by the letter $B$ represents the condition after compaction. In this study, the main investigation involved the effect of the change in the fabric of sand and the effect of the $K_0$-value was investigated only to a limited extent. The effect of long-term time consolidation was beyond the scope of this study.

As a test material, Sengenyama Sand was selected, since this sand has been widely used for constructing landfills in the Tokyo Bay area. In fact, this study was initiated to determine in advance the compaction criteria for landfills which were planned to be constructed in the central part of Tokyo Bay. In addition to Sengenyama Sand, TToyoura Sand was also used because this sand has been widely used in Japan for the study of sand liquefaction and because a large amount of comparable data is available. Only a limited number of triaxial tests were performed using Fujian Sand, the Chinese standard test sand (Wang et al., 1984). Presented herein are the summarized results of the research program which was carried out to obtain a general view of the effects of the method of compaction on the liquefaction strength.

Fig. 1. Schematic diagram showing probable increase in strength by compaction procedure

Fig. 2. Grading curves of test materials
TEST PROGRAM

The physical properties of the test materials used are shown in Fig. 2 (Note: some of the physical properties of Sengeniyama Sand reported previously by Tatsuoka et al., 1982 were incorrect and should be corrected). While Toyoura Sand and Fujian Sand have no fine contents, Sengeniyama Sand has 2.4% fine contents. To ensure the consistency of the properties of these sands during the period of this study, cyclic undrained triaxial and torsional shear tests under identical conditions were performed occasionally as a check, and it was confirmed that the variations in the results during this test program were negligible.

Fig. 3 shows the stress conditions during consolidation and cyclic loading employed in this study. For a cyclic triaxial test (this will be abbreviated as CTX in the following section), a sample was isotropically consolidated to an effective confining pressure $\sigma'_c$ of 1.0 kgf/cm$^2$ (98.1 kN/m$^2$) for Toyoura Sand and Fujian Sand or 1.333 kgf/cm$^2$ (130.8 kN/m$^2$) for Sengeniyama Sand, either value being equal to the mean effective principal stress at consolidation in the torsional shear test on the same kind of sand. The triaxial samples were 15 cm long and 7.5 cm in diameter with porous stones at both ends. 0.3 mm-thick latex rubber membranes were used in both the triaxial tests and the torsional shear tests. The frequency of cyclic loading was 0.5 Hz in the early stage of this investigation, but this was changed to 0.1 Hz in the later stage in order to record the outputs of the test more accurately. No effects of this change of frequency on the test results were found. In all of the tests, care was taken to maintain a constant amplitude of cyclic load and also to maintain the symmetry of the amplitude of cyclic load between the triaxial compression and extension conditions.

Two kinds of cyclic torsional shear tests were performed, one for isotropically consolidated samples (Toyoura Sand only) and one for anisotropically consolidated samples (both Toyoura and Sengeniyama Sands). A torsional shear test sample was 10 cm long, 10 cm in outer diameter and 6 cm in inner diameter. A sample of Toyoura Sand consolidated isotropically to $\sigma'_c = 1.0$ kgf/cm$^2$ (98.1 kN/m$^2$) was cyclically loaded with a frequency of 0.5 Hz or 0.1 Hz. This kind of test will be called CTS-I (abbreviation of Cyclic undrained Torsional Shear test on Isotropically consolidated sample) in the following sections. No effect of this change of frequency was found. In most CTS-I, the height was free to change during cyclic undrained loading. In some tests on air-pluviated samples, the height of the sample was kept constant by clamping the loading ram vertically, but only a very small in-
crease in strength was induced by this clamping (see Fig. 14, Tatsuoka et al., 1982). The data reported herein are those obtained only by tests which did not use this kind of clamping. In the early stage of this investigation, in order to prevent slippage at the top and bottom ends of the sample, at each end six 1.5 mm-high stainless steel blades were fixed onto the surfaces of porous stone. However, it was found later during the investigation that even in the tests no blades were used—i.e. where the top and bottom surfaces of the sample were in contact with the porous stones—no slippage was observed, and the results were very similar to those obtained by the tests using these blades. Therefore, most of the data reported herein were obtained by tests which did not make use of these blades.

In the other kind of torsional shear test, the samples were anisotropically consolidated with the ratio of the effective horizontal stress to the effective vertical stress $K = \sigma_h / \sigma_v'$ being 0.5 for Sengenyama Sand and 0.52 for Toyoura Sand, where $e_v$ is the initial void ratio of the sample. The value $K = 0.52$ was the $K_v$-value measured on air-pluviated triaxial samples of Toyoura Sand (Okochi and Tatsuoka, 1984). The value $K = 0.5$ for Sengenyama Sand is close to the measured $K_v$-values for air-pluviated Sengenyama Sand, which are 0.44 for $D_v = 80\%$ and 0.47 for $D_v = 60\%$ (Tatsuoka et al., 1984a). The mean effective principal stress at consolidation $\sigma_{pv}' = (\sigma_{p1}' + 2 \sigma_{p2}')/3$ was equal to the value of $\sigma_v'$ in the corresponding triaxial test. The height of the sample was kept constant during cyclic undrained loading to maintain a constant cross-sectional area for the sample as in the field simple shear condition. This kind of test will be called CTS-A (abbreviation of Cyclic undrained Torsional Shear test on Anisotropically consolidated sample) in the following sections.

The methods used for maintaining a constant sample height in CTS-A and for applying the rotational movement to the loading ram were modified after this method.

Fig. 4. Schematic diagram of the modified torsional shear apparatus

Fig. 5. Typical record of CTS-A (cyclic undrained torsional shear test on anisotropically consolidated sample)

Fig. 6. Schematic diagram of four sample preparation methods
Table 1. Details of sample preparation methods

<table>
<thead>
<tr>
<th>Tamper</th>
<th>Footing (cm)</th>
<th>Weight (kg)</th>
<th>Free fall height (cm)</th>
<th>Number of layers*</th>
<th>Water content of sand (%)</th>
<th>Dead weight in averaged stress (kgr/cm²)</th>
<th>Number of layers*</th>
<th>Water content of sand (%)</th>
<th>Dead weight in averaged stress (kgr/cm²)</th>
<th>Number of layers</th>
<th>Sand</th>
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<td>CTX</td>
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<td>185.9</td>
<td>3.5</td>
<td>6</td>
<td>3</td>
<td>0.0286</td>
<td>10</td>
<td>3</td>
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<td>Saturated</td>
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<tr>
<td>CTS</td>
<td>1.645</td>
<td>185.9</td>
<td>3.5</td>
<td>6</td>
<td>3</td>
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<td>10</td>
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<td>0.0286</td>
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<td>Saturated</td>
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<td>8</td>
<td>0.0286</td>
<td>1</td>
<td>Saturated</td>
</tr>
</tbody>
</table>

NOTE: * The compacted surface of each layer was scarified.

¹ Only in some samples. Effects of number of layers 5 or 10 were found to be negligible.

² Diameter.

was reported for the first time elsewhere (Tatsuoka et al., 1982; Tatsuoka et al., 1983a, b). First the clamping device was moved to a level below the device in which the cyclic linear movement of wire is transformed to the rotational movement of the loading ram (Fig. 4). Secondly, the method of this transformation had been achieved with a rather complicated device in the previous version, but this was simplified by means of a ball spline bushing and its loading shaft, both of which are commercially available. It was found after these modifications that the vertical movement of the loading shaft under the action of its rotational movement has become smoother, and that the vertical settlement of the top of the specimen caused by its liquefaction in a CTS-A has been decreased significantly from the order of 0.1 mm in the old version to the order of 0.01 mm (as typically shown in Fig. 5); much closer to the ideal value of zero.

The following four sample preparation methods were employed (see Fig. 6 and Table 1):

1. **The air-pluviation method (AP)** consisted of pluviating air-dry sand into a mold from the nozzle of a tube, keeping the height of fall constant. Different density values were obtained by changing the height of fall. It can be considered that the fabric of a specimen produced by this method is similar to that of sand deposits formed by free falling of sand particles through air or water. Note that even a void ratio similar to the minimum void ratio can be achieved by this method.

2. **The wet-tamping method (WT)** is a method of compacting moist coarse grained material in which the material is placed in layers, with each layer compacted to a prescribed dry unit weight (Ladd, 1974; Silver et al., 1976). In this study, the density of each layer was controlled by adjusting the number of tamping blows with a constant free fall of 3.5 cm. This method was adopted as a simulation of field compaction of moist sand in layers by vertical tamping on the ground surface.

3. **In the wet-vibration method (WV),** a prescribed amount of moist sand was placed gently in a mold for each compaction layer. Then, either a solid cylindrically-shaped weight for triaxial tests or a hollow cylindrically-shaped weight for torsional shear tests was placed on each layer. Each layer was compacted by tapping the mold with a wooden hammer uniformly around the mold surface until the thickness of the layer was reduced to a prescribed value. The number of compaction layers was five or ten for Toyoura Sand and ten for Sengenyama Sand. No differences in cyclic strength values were found between the five-layered and ten-layered specimens of Toyoura Sand. The fabric of a test specimen produced by this method may be considered to be similar to that formed in field moist sand layers com-
pacted by a vibrational method.

In the water-vibration method (WAV), a prescribed amount of air-dry sand was poured through water in one layer for Toyoura Sand and Fujian Sand. For Sengenyama Sand, sand was pluviated through air in one layer and then saturated to prevent segregation. After placing the weight (the same kind used in the wet-vibration method) on the top surface of the sample, the specimen was compacted underwater by tapping the mold uniformly with a wooden hammer until the total thickness of the specimen was reduced to a prescribed value. This method was employed as a simulation of in-situ vibrational compaction procedures for submerged sand layers. Only triaxial tests were performed on samples made by this method for Fujian Sand.

In the wet-tamping method and the wet-vibration method, the compacted surface of each layer was scarified before placing the next layer. After being saturated, each test specimen was consolidated either isotropically or anisotropically. It was confirmed that the Skempton's B-value must be 0.98 or higher. The total number of tests was more than 800.

The cyclic triaxial test results were normalized using the conventional stress ratio $SR = \sigma_0/|\sigma'_e|$, in which $\sigma_0$ is the maximum single amplitude of cyclic deviator stress and $\sigma'_e$ is the effective isotropic consolidation stress. Note that $\sigma_0$ is defined as the single amplitude cyclic axial load divided by the cross-sectional area of the sample at consolidation. The cyclic torsional shear test results were normalized using the stress ratio $SR =\sigma_0/|\sigma_m'|$, where $\sigma_0$ is the maximum single amplitude of horizontal cyclic shear stress and $\sigma_m'$ is the effective mean principal stress at consolidation, which equals $\sigma'_e$ in CTS-1 or $(\sigma'_e + 2\sigma_m')/3$ in CTS-A. This normalization method was suggested by Ishihara and Li (1972) and Ishihara and Takatsu (1979), either to relate the triaxial strength with the torsional shear strength or to relate the torsional shear strengths for different values of $K = \sigma_m'/|\sigma_e|$. It will be shown, however, that this normalization method is effective only in a limited number of conditions. The cyclic undrained triaxial strength was defined for 2%, 5% and 10% double amplitude axial strain values. On the other hand, it can be shown that the maximum shear strain values defined as the differences between the major and minor principal strains $\varepsilon_1-\varepsilon_3$ are 1.5 times the axial strain values for undrained saturated triaxial samples. Therefore, for the purpose of comparing the triaxial strength with the torsional shear strength, the cyclic undrained torsional shear strength should be defined for 3%, 7.5% and 15% double amplitude maximum shear strain values. The shear strain values $\gamma$ obtained by dividing the average horizontal displacement at the mean radius of the sample on the top of the sample by the initial height of the sample is not exactly equal to the maximum shear strain value when the height of the sample changes and/or when the thickness of the cylinder wall changes during shearing as in CTS-1. However, it was confirmed that the difference between these two kinds of shear strain values, $\varepsilon_1-\varepsilon_3$ and $\gamma$, is negligible for the purpose of this study. Therefore, the torsional shear strengths were defined for double amplitude values of $\gamma$ of 3%, 7.5% and 15%.

The results of each kind of cyclic test were summarized as shown in Fig. 7. In

![Fig. 7. Typical results by torsional shear tests (CTS-I) on wet-vibrated Toyoura Sand](image-url)
this figure, $D_r$ is the relative density value obtained after consolidation and $N_e$ is the number of loading cycles where a double amplitude shear strain $DA$ of 15% is observed. For a value of $N_e$ less than 10, this value was obtained down to a one-tenth decimal place by interpolation, as follows: suppose that $DA=8\%$ at $N_e=7$ and $DA=14\%$ at $N_e=8$; then $N_e$ for $DA=10\%$ was obtained as $7+(10-8)/(14-8)=7.3$. Since a series of tests was performed for a similar cyclic stress ratio, purposely changing the density, a relationship between $D_r$ and $N_e$ could be defined for a prescribed value of cyclic stress ratio $SR$ as shown in Fig. 7. Using this relationship, the value of $N_e$ can be determined from given values of the other variables $D_r$ and $SR$ as well as the value of $D_r$ from the values of $N_e$ and $SR$. The values of $D_r$ and $N_e$ reported in the following sections were obtained as above. The results shown in Fig. 7 also indicate negligible effects of the number of compaction layers on the results of wet-vibrated Toyoura Sand samples.

**TEST RESULTS**

**Toyoura Sand:** Figs. 8(a) through 8(d) show the relationships between the cyclic stress ratio and the logarithm of the number of loading cycles where a 10% double amplitude axial strain was observed in CTX for different consolidated relative density values for four different sample preparation methods. It may be noted by comparing these figures that the effect of sample preparation methods is significant, as has been reported by Ladd (1974) and Mulilis et al. (1977). Furthermore, it may be noted that for each sample preparation method the strength curves for different values of $D_r$ have a similar shape having the maximum curvature at a stress ratio of around 0.23. For the range of stress ratio less than this value these curves have a smaller slope, whereas for the range of stress ratio larger than this value these curves have a larger slope. Thus, this kind of stress ratio will be called the critical stress ratio $SR_{cr}$, which was found to be rather unique in each of the cyclic triaxial tests ($\sigma_{d1}/\sigma_{d2}=0.23$) and the cyclic torsional tests (CTS-I and CTS-A) ($\tau_{e0}/\sigma_{m1}=0.3$). These features may imply the following: when the stress ratio of a given sinusoidal loading is larger than the critical stress ratio, whether or not cyclic undrained failure is induced in a given sand sample depends mainly on its number of loading cycles, and is less sensitive to the stress ratio value. On the other hand, when the stress ratio is less than the critical stress ratio, whether or not cyclic undrained failure is induced depends mainly on the stress ratio, and the number of loading cycles is of secondary importance. Accordingly, it seems that the number of loading cycles corresponding to the critical stress ratio for a given strength curve may represent the overall features of the strength curve better than such conventional strength indices as the stress ratio for which a given strain value is induced at a certain number of cycles—for example, at 10 or 20 cycles. The number of loading cycles corresponding to the critical stress ratio will be called the critical number of loading cycles: $(N_e)_{cr}$. This point will be discussed again in a later section.

To see in more detail the effects of different sample preparation methods, Figs. 9(a) through (c) were prepared, where for $D_r=75\%$ the strength curves for the four sample
Fig. 9. Stress ratio versus $N_c$ to (a) 2%, (b) 5% and (c) 10% DA axial strains by triaxial tests on Toyoura Sand, $D_r=75\%$

preparation methods are directly compared. It may be seen that, the other factors being equal, the air-pluviated samples are the weakest, the water-vibrated samples are of intermediate strength and the wet-tamped samples and the wet-vibrated samples have the greatest strength among these kinds of samples.

Similar results obtained by torsional shear tests on isotropically and anisotropically consolidated samples are shown in Figs. 10 through 13. It may be seen that in the torsional shear tests as well as in the triaxial tests the effects of sample preparation method on cyclic undrained strength are significant. It may also be seen that in the torsional shear tests and also in the triaxial tests the air-pluviated samples are the weakest, the water-vibrated samples are of intermediate strength and the wet-vibrated samples are the strongest. However, the test results show that the wet-tamped samples are not necessarily the strongest in the torsional shear tests. These results show that the sample preparation method may affect the

Fig. 10. Stress ratio versus $N_c$ to 15% DA shear strain by CTS-I on Toyoura Sand

Fig. 11. Stress ratio and $N_c$ to (a) 3%, (b) 7.5% and (c) 15% DA shear strains by CTS-I on Toyoura Sand, $D_r=75\%$

Fig. 12. Stress ratio versus $N_c$ to 15% DA shear strain by CTS-A on Toyoura Sand
Fig. 13. Stress ratio versus $N_c$ to (a) 3\%, (b) 7.5\%, and (c) 15\% DA shear strains by CTS-A on Toyoura Sand, $D_v=75\%$

cyclic undrained strength in somewhat different ways between in the triaxial tests and in the torsional shear tests. This point will be discussed again in a later section.

**Sengenyma Sand**: The results for Sengenyma Sand are summarized in Figs.14 through 17. It may be seen from these figures that for Sengenyma Sand, just as in the case of Toyoura Sand, the effects of sample preparation method are significant. In general, the air-pluviated samples of Sengenyma Sand are the weakest and the wet-vibrated samples are the strongest, the other parameters being equal. However, several features different from those for Toyoura Sand may be seen in these figures. First, at one relative density value, the strength of Sengenyma Sand is generally less than that of Toyoura Sand for any sample preparation method in both triaxial and torsional shear tests. This point is clearly seen in Figs.18(a) through (d) and in Figs.19(a) through (d). It would appear from these results that the relative density value used so far in this study is not a good index to represent the density condition of different kinds of sand with respect to cyclic undrained strength. The values of $e_{max}$ and

Fig. 14. Stress ratio versus $N_c$ to 10\% DA axial strain by triaxial tests on Sengenyma Sand

Fig. 15. Stress ratio versus $N_c$ to (a) 2\%, (b) 5\%, and (c) 10\% DA axial strains by triaxial tests on Sengenyma Sand, $D_v=75\%$

Fig. 16. Stress ratio versus $N_c$ to 15\% DA shear strain by CTS-A on Sengenyma Sand
Fig. 17. Stress ratio versus $N_c$ to (a) 2%, (b) 7.5%, and (c) 15% DA shear strains by CTS-A on Sengenyama Sand, $D_r = 75$

$e_{\text{min}}$ used for calculating relative density values were obtained following the method specified in 1979 by the Japanese Society of Soil Mechanics and Foundation Engineering. These values were measured under no vertical stress using air-dry sand. $e_{\text{max}}$ was obtained by gently pouring air-dry sand into a stainless-steel mold and $e_{\text{min}}$ was obtained by tapping a mold in which air-dry sand has been placed by spooning in 10 layers. The mold has inner dimensions of 6 cm in diameter and 4 cm in height. However, the values of $e$ used for calculating relative density values were those measured after consolidation, using test samples prepared by a method which may have been different from that used in obtaining the values of $e_{\text{max}}$ and $e_{\text{min}}$. Therefore, it seems that there are no sound reasons for using the relative density value defined above as a measure of sample density from which cyclic undrained strength is uniquely defined for various sample preparation methods and various kinds of sand.

COMPARISONS BETWEEN TRIAXIAL AND TORSIONAL SHEAR STRENGTHS BASED ON STRESS RATIO VALUES

Ishihara and his colleagues (Ishihara and Li, 1972; Ishihara and Yasuda, 1975; Ishihara and Takatsu, 1979) showed that for a loose sample ($D_r = 55\%$) of Fuji river Sand prepared by a method similar to the water-vibration method used in this study, the triaxial strength and the torsional shear strength of isotropically consolidated samples with $K = \sigma_{\text{int}} / \sigma_{\text{ext}} = 1.0$ and the strength values of anisotropically consolidated samples with $K$ of 0.5 and 1.5 are related to each other as

$$\frac{(\sigma_{\text{int}}^2 / 2 \sigma_{\text{ext}}^2}_{\text{CTX}} = \frac{(\tau_{\text{int}} / \sigma_{\text{ext}})_{\text{CTX}}}{(\tau_{\text{int}} / \sigma_{\text{ext}})_{\text{CTS-A}}}$$

in which $(\sigma_{\text{int}}^2 / 2 \sigma_{\text{ext}}^2)_{\text{CTX}}$, $(\tau_{\text{int}} / \sigma_{\text{ext}})_{\text{CTS-A}}$ and
(τ_{cy} / σ_{m}^{r})_{CTS-A} are the stress ratios at which initial liquefaction was observed in the twentieth cycle. Concerning the relationship between (τ_{cy} / σ_{m}^{r})_{CTS-I} and (τ_{cy} / σ_{m}^{r})_{CTS-A}, it was found by Tatsuoka et al. (1984b) that Eq. (1) is generally valid for the data obtained for Toyoura Sand, whereas for dense air-pluviated or wet-tamped samples the value of (τ_{cy} / σ_{m}^{r})_{CTS-A} is larger than the value of (τ_{cy} / σ_{m}^{r})_{CTS-I}, with the other values being the same. However, it seems that this discrepancy noted above is not materially large when compared with that between (σ_{d} / σ)_{CTX-A} and (τ_{cy} / σ_{m}^{r})_{CTS-A} as described below.

The torsional shear strength by a CTS-A is often estimated from the triaxial strength by using the equation

\[ (τ_{cy} / σ_{m}^{r})_{CTS-A} = c_{1} (σ_{d} / 2σ)_{CTX} \]  

(2)

where (τ_{cy} / σ_{m}^{r})_{CTS-A} and (σ_{d} / 2σ)_{CTX} are defined for a certain strain value in a prescribed number of loading cycles and c_{1} is the correction factor. It can be shown that when Eq. (1) is valid the correction factor c_{1} equals (1 + 2K_{S})/3 for K_{S}-consolidated samples. On the other hand, a value of 0.57 has been proposed for c_{1} by De Alba et al. (1976) and Seed (1976), based on their large-scale simple shear tests and triaxial tests on Monterey No. 0 Sand. When K_{S} = 0.4, c_{1} = (1 + 2K_{S})/3 = 0.6, which is very close to 0.57. Therefore, the proposal for c_{1} by De Alba et al. (1976) and Seed (1976) can be considered essentially similar to that proposed by Ishihara et al.: i.e. c_{1} = (1 + 2K_{S})/3.

Figs. 20(a) and (b) and Figs. 21(a) and (b) show the values of c_{1} obtained as c_{1} = (τ_{cy} / σ_{m}^{r})_{CTS-A} (σ_{d} / 2σ)_{CTX} plotted against the consolidated relative density for failure defined for a double amplitude shear strain of 3% or 15% (or axial strain of 2% or 10%) in the twentieth cycles. These test results show that the value of c_{1} is generally higher than (1 + 2K_{S})/3 or (1 + 2K)/3. In particular, the value of c_{1} is considerably larger than 1.0 for loose Sengenyama Sand. This implies that in a case in which correction factor of c_{1} = (1 + 2K_{S})/3 is used in Eq. (2), the torsional shear strength by CTS-A is greatly underestimated. For Sengenyama Sand, the relation c_{1} = (1 + 2K_{S})/3 is approximately valid only in dense samples. Accordingly, it can be concluded that the estimation of the simple shear strength from the triaxial strength for a given sand element is not as simple as suggested previously by either Seed and his colleagues or Ishihara and his colleagues. It can also be said that in most cases their methods may be conservative.

It is very likely that in this study the effects of the membrane penetration on the measured liquefaction strength are greater in the torsional shear tests than in the triaxial tests. This is because the ratio of the lateral surface area of the sample in contact with the rubber membrane to the sample volume of a hollow cylindricl sample used for CTS is around two times that of a solid cylindricl sample used for CTX. Therefore, it is likely that the values of c_{1} shown in Figs. 20 and 21 will become

![Fig. 20. Correction factor c_{1} for failure defined as 3% double amplitude shear strain in the 20th cycle for Toyoura Sand and Sengenyama Sand](image-url)
Fig. 21. $c_i$ for failure defined as 15% double amplitude shear strain in the 20th cycle (refer to Fig. 20)

slightly smaller as a whole when the correction for the membrane penetration is applied. However, the trend of the effects of sample preparation methods on the values of $c_i$ seen in Figs. 20 and 21 may be principally unchanged even after this correction, since the effects may not be so different among different sample preparation methods.

CRITICAL NUMBER OF LOADING CYCLES

The cyclic undrained strength has been conventionally represented using a stress ratio value for which initial liquefaction or a certain value of strain is observed at a certain number of loading cycles. However, in some cases this method can sometimes be quite misleading. For example, the points designated by the letters A, B and C in Fig. 8(a) correspond to the cyclic stress ratio values at which a double amplitude axial strain of 10% was observed at the fifteenth cycle in cyclic triaxial tests on the air-pluviated Toyoura Sand samples having relative density values of 85%, 80% and 60%. These stress ratio values will be denoted as $SR_A$, $SR_B$ and $SR_C$. It may be seen that the difference between $SR_A$ and $SR_B$ is large, on the order of around 0.5, while the difference in the general features between these two strength curves for $D_r = 80\%$ and 85\% is not so large. In contrast, it may also be seen in Fig. 8(a) that the difference between $SR_B$ and $SR_C$ is small, around 0.15, but that the difference in the general features between these two strength curves for $D_r = 60\%$ and 80\% is not so small. Furthermore, it may be seen in Figs. 18(a) through (d) that when the sand is loose, the stress ratio increases at a very small rate along a strength curve as relative density increases, whereas when the sand is dense the stress ratio increases at a very large rate as the relative density increases. This kind of discussion is also valid for the torsional shear tests. In particular, it may be seen in the data obtained by this study that when the stress ratio value $\sigma_{sp}/2\sigma'$ is larger than around 0.23 in the triaxial tests or when the stress ratio value $\tau_{sp}/\sigma_m'$ is larger than around 0.3 in the torsional shear tests, the change of the strength in terms of stress ratio—either by a small change of relative density at a fixed number of loading cycles or by a small change of the logarithm of the number of loading cycles for a fixed density value—can be very large. Therefore, since stress ratio values larger than these critical values shown above can be too sensitive to test variables, the comparisons of these values may be somewhat misleading.

On the other hand, it was found that when the number of loading cycles $N_e$ is divided by $\langle N_e \rangle_{err}$, strength curves such as those shown in Figs. 8 through 17 collapse into a single curve for the same strain value in each of the triaxial and torsional shear tests (Figs. 22 and 23). Note that in the torsional shear tests the shape of the normalized strength curve is largely independent of the sample density, the kind of sand
and the sample preparation method within the limit of test condition employed, whereas this shape is a function of the strain value for which the strength curve is defined. It may be seen in Fig. 22 that for the triaxial tests the normalized strength curve is rather unique irrespective of the sample density and the kind of sand, whereas the shape of the curve for $SR > (SR)_{ct}$ is somewhat different for the different sample preparation methods. Consequently, it seems that in the case of the torsional shear tests the strength curve for a certain cyclic strain value in terms of $SR = \tau_{cy}/\sigma_{mc}'$ and $\log N_c$ can be readily defined only from the value of $(N_c)_{ct}$. Therefore, it seems that as a single strength index for cyclic undrained torsional shear tests using sinusoidal loadings the index $(N_c)_{ct}$ is more appropriate than the cyclic stress ratio value defined for a certain cyclic strain value observed at a certain number of loading cycles, especially for denser samples. Further research will be needed to clarify why the normalized strength curves for $SR > (SR)_{ct}$ is affected by the sample preparation method in the triaxial tests but not in the torsional shear tests.

### DENSITY INDEX AS A VARIABLE OF LIQUEFACTION STRENGTH

In order to get a better insight into the

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**Fig. 22.** Normalized cyclic stress ratio $SR/(SR)_{ct}$ versus normalized number of loading cycles $N_c/(N_c)_{ct}$ for (a) 2% and (b) 10% DA axial strains in triaxial tests ($SR = \sigma_{dy}/2\sigma_c'$ and $(SR)_{ct} = 0.23$)

**Fig. 23.** Normalized cyclic stress ratio $SR/(SR)_{ct}$ versus normalized number of loading cycles $N_c/(N_c)_{ct}$ for (a) 3% and (b) 15% DA shear strains in torsional shear tests ($SR = \tau_{cy}/\sigma_{mc}'$ and $(SR)_{ct} = 0.3$)
effects of sample preparation methods and the kind of sand on the liquefaction strength, several attempts were made to find out whether or not the liquefaction strengths obtained by this study for the different sample preparation methods and for the different kinds of sand can be uniquely related to an index or indices representing the density condition of the samples. First, the minimum void ratio was redefined as follows.

It seems that the most meaningful role of the minimum void ratio would be to represent the smallest void ratio which can be achieved for a given element of sand. Therefore, it was considered that the minimum void ratio should be redefined for each consolidated sample. For example, in the water-vibration method the density of the sample is controlled by adjusting the number of mold-tapping blows (see Fig. 6). Fig. 24 shows the relationships between the void ratio of triaxial samples after consolidation \(e_c\) and the number of blows.\(^1\) In measuring the values of \(e_c\) shown in Fig. 24 the triaxial samples were consolidated at the same confining pressure as for the cyclic undrained tests in this study. It may be seen that the value of \(e_c\) does not decrease with the further increase in the number of blows when this number becomes larger than a certain value (say 500). In this study the lower limit value of \(e_c\) was defined at the

<p>| Table 2. The modified minimum void ratios for consolidated test samples. (e_{min}^*) |
|---------------------------------|----------------|----------------|----------------|
| SAMPLE PREPARATION METHOD | TOYOURA SAND | SENGOKU YAMA SAND | FUJIAN SAND |</p>
<table>
<thead>
<tr>
<th>CTX</th>
<th>CTS</th>
<th>CTX</th>
<th>CTS</th>
<th>CTX</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIR-PLUVIATED</td>
<td>0.607</td>
<td>0.600</td>
<td>0.535</td>
<td>0.531</td>
</tr>
<tr>
<td>WET-TAMPERED</td>
<td>0.646</td>
<td>0.644</td>
<td>0.547</td>
<td>0.542</td>
</tr>
<tr>
<td>WET-VIBRATED</td>
<td>0.648</td>
<td>0.645</td>
<td>0.574</td>
<td>0.574</td>
</tr>
<tr>
<td>WATER-VIBRATED</td>
<td>0.629</td>
<td>0.625</td>
<td>0.551</td>
<td>0.549</td>
</tr>
</tbody>
</table>

The number of 1,000 blows. This lower limit value will be referred to as the modified minimum void ratio and will be denoted as \(e_{min}^*\). Relationships similar to those shown in Fig. 24 were also obtained and the value of \(e_{min}^*\) was defined for consolidated hollow cylindrical samples as used in the undrained cyclic torsional shear tests. For Toyoura Sand the values of \(e_{min}^*\) for isotropically and anisotropically consolidated samples were found to be essentially the same. The values \(e_{min}^*\) thus obtained are listed in Table 2. For air-pluviated samples, the value of \(e_{min}^*\) was defined for the fall height of pluviation beyond which no further increase in \(e_c\) was observed with the increase in fall height. For wet-tamped samples, the value of \(e_{min}^*\) was defined at the number of tamping strokes beyond which no further increase in \(e_c\) was observed with the increase in the number of strokes. For wet-vibrated samples, the value of \(e_{min}^*\) was defined as for water-vibrated samples. It may be seen from Table 2 that for each sample preparation method, in the value of \(e_{min}^*\) of any one kind of sand, there is a slight difference between the triaxial sample (CTX) and the torsional shear sample (CTS). It seems that this small difference is due mainly to the different sample dimensions which may cause (i) different degrees of error in the volume change measurement due to the membrane penetration and the bedding error, and (ii) differences in the manner of packing of particles. Nevertheless, it may be seen that the differences in \(e_{min}^*\) between the different sample preparation methods are much larger than those between any two kinds of samples (i.e. the CTX and CTS
samples). It may be seen that for both kinds of sands and for both kinds of samples the value of $e_{\text{min}}^*$ is the lowest for air-pluviated samples, whereas for wet-vibrated samples it is the highest.

An attempt was made to redefine the relative density by using $e_{\text{min}}^*$. In this case, the maximum void ratio $e_{\text{max}}^*$ should be redefined as well as $e_{\text{min}}^*$, but it was found difficult to clearly define $e_{\text{max}}^*$ for sample preparation methods other than the air-pluviation method. An effort was made to define the value of $e_{\text{max}}^*$ for the wet-tamped or wet-vibrated samples by measuring the void ratio of a consolidated sample which was prepared by gently placing moist sand into a mold without any further tamping or without tapping the mold. And for the water-vibrated sample an attempt was made to define the value of $e_{\text{max}}^*$ by measuring the void ratio of a consolidated sample which was prepared by placing sand underwater into a mold. However, it was found that the void ratio of a consolidated sample thus prepared can be very different from one sample to another, with the result that it is very difficult to reliably define the value of $e_{\text{max}}^*$. Accordingly, it was necessary to give up the attempt to redefine the relative density as $(e_c - e_{\text{min}}^*)/(e_{\text{max}}^* - e_{\text{min}}^*) \times 100\%$.

After several other attempts, it was finally found that when the difference $e_c - e_{\text{min}}^*$ is the same, the cyclic undrained resistance, which is defined either for a double amplitude axial strain of 2% or a double amplitude shear strain of 3%, is rather similar for the different kinds of sand and the different sample preparation methods in each of the triaxial and torsional shear tests. Note that the value of $(e_c - e_{\text{min}}^*)$ is almost free from the error induced by membrane penetration and bedding error, since both kinds of error are similarly involved in both $e_c$ and $e_{\text{min}}^*$. Fig. 25 shows the relationships for the triaxial tests between $(e_c - e_{\text{min}}^*)$ and the number of loading cycles at which a double amplitude axial strain of 2% is induced when $\sigma_{3p}/2\sigma_c' = 0.23$. The value of $(e_c - e_{\text{min}}^*)$ will be referred to as the modified volume decrease potential, following Ishihara and Watanabe (1976), and will be denoted as $V_d^*$. It may be seen in Fig. 25 that a rather unique relationship can be defined for all the data obtained by this study. It was also found that when similar relations are plotted for a double amplitude axial strain of 10% such a unique relationship is not obtained. Fig. 26 shows, for the torsional shear tests, the similarities in the relationship between $V_d^*$ and the number of loading cycles at which a double amplitude shear strain of 3% is induced when $\tau_{3p}/\sigma_{3p}' = 0.3$. A rather unique relationship may be seen for these data as well. It was further found that for a double amplitude shear strain of 15% such a unique relation as that seen in Fig. 26 is not obtained. Consequently, it seems that the stress-strain behavior of saturated sands under cyclic undrained loadings is strongly controlled by the index $V_d^*$ when the induced double amplitude shear strain is at least less than 3%. It was noted that the shear strain is of the order of 3% in double amplitude at the cycle when the maximum excess pore pressure in a cycle becomes similar to the

![Fig. 25. $(N_c)_{cr}$ for 2% DA axial strain versus the modified volume decrease potential $V_d^* = e_c - e_{\text{min}}^*$ for triaxial tests.](image-url)
initial effective pressure; this is conventionally called the onset of initial liquefaction. Therefore, as has been pointed out by Ishihara and Watanabe (1976), the initial liquefaction characteristics reflect the negative dilatancy (or the contractancy) characteristics, which may be reflected in the density index $V_d^*$. On the other hand, it may be conceivable that the cyclic undrained stress–strain behavior after the onset of initial liquefaction is not related to the density index $V_d^*$, since this behavior is closely related to the positive dilatancy characteristics during cyclic undrained loading; this behavior is often called the cyclic mobility. It was found that the additional number of loading cycles which is needed to increase the double amplitude axial strain from 2% to 10% in triaxial tests or to increase the double amplitude shear strain from 3% to 15% in torsional shear tests is not a function of $V_d^*$ but is a function of the void ratio $e_c$ of the sample (Figs. 27 and 28). One might conclude from these figures that the effects of the sample preparation methods have disappeared from
the relations shown. It may, however, be seen that these relations are different for different kinds of sand. It was also found that these relations depend on the kind of sand even when the relative density $D_r = (e_e - e_{\text{min}})/(e_{\text{max}} - e_{\text{min}}) \times 100(\%)$ is used in place of $e_e$ as the density index. The results shown in Figs. 27 and 28 seem to show that for each kind of sand the stress-strain behavior at cyclic shear strains larger than 3% is a function of the density value itself, which may be represented by dry density or void ratio or relative density, and that the behavior is independent of the initial fabric within the sample, which may be different for each different sample preparation method. Such characteristics as described above seem natural since the initial fabric produced during sample preparation likely undergoes some change due to cyclic straining. Therefore it seems natural that the effects of sample preparation methods on the cyclic undrained stress-strain characteristics are large at smaller cyclic strain values but that these effects will tend to disappear with the increase in cyclic strain values.

On the other hand, it was found from another series of investigation by the authors that for torsional shear tests the resistance against a given time history of irregular cyclic shear stress series can be well correlated with the $(N_e)_{cr}$ of the sand element concerned. This point will be reported on in detail elsewhere by the authors (Tatsuoka et al., 1986). The averaged curves shown in Figs. 26 and 28 may be used to estimate the values of $(N_e)_{cr}$ only from the void ratio $e_e$ and its minimum value $e_{\text{min}}$. However, further research will be needed to clarify how and why the curves shown in Fig. 28 depend on the kinds of sand; with this clarification these curves can be put to more general use.

CONCLUSIONS

On the basis of the limited number of the tests performed in this study, the following conclusions can be derived:

(1) The cyclic undrained torsional shear strengths were strongly affected by sample preparation methods as well as the cyclic undrained triaxial strengths. Among the four different sample preparation methods employed, the wet-vibration method generally provided the strongest samples and the air-pluviation method generally provided the weakest samples; the other two methods, the wet-tamping method and the water-vibration method, provided the strongest samples in some cases or the weakest samples in other cases, or samples having intermediate strength in the remaining cases.

(2) The characteristics of the effects of the sample preparation methods on cyclic undrained strength were not necessarily the same for the two kinds of sand tested or for the two kinds of tests (i.e. the triaxial and torsional shear tests). Accordingly, the ratio of strength represented in terms of stress ratio value between the triaxial test and the torsional shear test was found not to be unique, but was a complicated function of many factors: the kind of sand, the sample preparation method, the relative density, the strain value for which failure is defined and so on. Therefore it can be concluded that the accurate estimation of the simple shear strength from the triaxial strength is in general rather difficult. To estimate a simple shear strength it seems better to perform a simple shear test if possible. For this purpose a cyclic torsional shear test on a $K_e$-consolidated sample (CTS-A, described in this paper) may be appropriate.

(3) When the strength is expressed as $\tau_{cv}/\sigma_{my}$, the strength of Toyoura Sand as determined by a torsional shear test on an isotropically consolidated sample was generally similar to that of an anisotropically consolidated sample, except for dense samples prepared either by the air-pluviation method or by the wet-tamping method.

(4) For the purpose of representing the overall features of the strength curve on a stress-ratio and $\log N_e$ plane, the critical number of loading cycles $(N_e)_{cr}$ was found to be a better strength index than the conventional stress ratio index defined as the
stress ratio for which liquefaction is induced at a certain number of loading cycles. \((N_c)_{cr}\) is defined in this study as the value of \(N_c\) at which a certain strain value is observed for the critical stress ratio \((SR)_{cr}\). \((SR)_{cr}\) is defined as the stress ratio at which the slope of the strength curve has the largest curvature, which was found in this study to be \(\sigma_d/2\sigma'_c = 0.23\) in the triaxial tests and \(\tau_{cv}/\sigma_{im} = 0.3\) in the torsional shear tests on both anisotropically and isotropically consolidated samples (CTS-I and CTS-A).

(5) For the torsional shear tests the normalized relationship between \(SR\) and \(\log (N_c/(N_c)_{cr})\) was found to be independent of the sample density, the sample preparation methods and the kinds of sand employed in this study. However, this relationship was found to be a function of the strain value for which the failure of the sample is defined. Similar trends were found for the triaxial test, except that the normalized relationship when \(SR > (SR)_{cr}\) is somewhat affected by the sample preparation method.

(6) In each of the triaxial and torsional shear tests, the stress-strain behavior until the double amplitude shear strain reached the order of 3% was a function of the difference between the void ratio of the test sample \(\epsilon_1\) and its smallest value \(\epsilon_{min}^*\), which is called the modified volume decrease potential \(V_{d}^*\). The value of \(\epsilon_{min}^*\) is strongly affected by the sample preparation method.

(7) It was found that in both the triaxial and torsional shear tests the effects of the sample preparation methods on the cyclic undrained stress and strain behavior will disappear with the increase in cyclic strain after the double amplitude shear strain becomes larger than around 3%.

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