SHORT-TERM STABILITY OF REINFORCED EMBANKMENT OVER CLAYEY FOUNDATION

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ABSTRACT

Stability against deep-seated failure of an embankment over soft clayey foundation is minimal at the end of construction. Since this stability increases as the foundation consolidates, related design is controlled by short term constraints. Stability can be enhanced by using a reinforcing agent capable of carrying tensile force. Ideally, the reinforcement will bridge over a temporary unstable situation until the structure has gained the strength needed to ensure stability over the long-run. Consequently, an efficient and possibly economical structure is produced.

To analyze the stability effect of the reinforcement at the end of construction, a rotational failure mechanism is assumed. This mechanism allows the embankment to break rather steeply and enables the foundation to be squeezed out. The moment limiting-equilibrium equation is assembled rigorously for the assumed mechanism. The problem then is stated as a search for the required reinforcement tensile resistance so that specified safety factors with respect to the shear strength of the soils are exceeded while satisfying the moment equation. A computational scheme which follows the statement of the problem is introduced.

Nondimensional design charts, which provide an insight into the reinforcement effects, are presented. Results indicate that reinforcement has the potential to be effective only when the extent of the soft foundation is limited as compared to the embankment dimensions.

Key words : bearing capacity, earthfill, foundation, slip surface, slope stability (IGC : E 3/E 6/H 4)

INTRODUCTION

Occasionally there is a need to construct an embankment overlying a saturated clayey foundation. The clay may be underlain by a hard layer of soil or rock. Typically the embankment is comprised of free-draining soil. Adequate design of such an earth-

structure has to ensure, among others, proper stability against failure through the embankment and its saturated and relatively soft foundation. At the end of construction it is customary to assume that the foundation has not begun to consolidate (i. e., undrained conditions), thus it has not yet gained any strength. Conversely, the forces tending to

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cause failure are at their maximum extent, remaining constant throughout the life of the structure. Subsequently, the lowest margin of safety against deep-seated failure is at the end-of-construction. Design related to stability of such an embankment, therefore, is controlled by short-term constraints. Normally, a "total analysis" is carried out to determine the safety margin at the end-of-construction using the undrained shear strength $c_u(\phi_u=0)$ of the foundation. Often the long-term stability of this embankment, estimated via "effective analysis", is excessively high. Furthermore, in extreme cases it may be practically impossible to ensure the short-term stability while maintaining a prescribed embankment height with reasonable side slopes. It is therefore clear that the availability of an economical mean, bridging over the short-term potential instability, would provide a rational remedy to a possible inefficient structure.

One possible solution to the short-term problem may be obtained via installation of a reinforcing agent. Such reinforcement might be a geosynthetic (e.g., Ingold and Miller, 1986), a steel mesh (e.g., Fowler, Peters and Franks, 1986), fiberglass sheet, perforated sheet metal, etc. The reinforcement matrix, typically placed at the embankment-foundation interface, enhances stability through its ability to develop tensile resistance. If the long-term stability is adequate without reinforcement then the fact that the tensile resistance of the reinforcement may decrease with time (due, for example, to corrosion, excessive creep, stress relaxation) is not a major concern as long as the decrease rate is smaller than the stability gain due to consolidation. Thus one important objective of the reinforcement is to increase the short-term stability producing an economical structure, ideally having a uniform margin of safety against deep-seated failure during its entire service life.

The following is a presentation of a new method to assess the short-term stability of a reinforced granular embankment over a clayey foundation. It is based on the limit-

Fig. 1. Selected failure mechanism and basic definitions

equilibrium approach considering a rotational failure mechanism.

FORMULATION

Statement of the Problem

Fig.1 shows an embankment with face inclination i and height H underlain by a soil layer having thickness d . A reinforcing sheet, capable of developing a horizontal tensile force $t(x)$, is placed at the embankment-foundation interface. The embankment is comprised of a free-draining cohesionless soil having a unit weight of γ and friction angle ϕ . The foundation is a saturated isotropic and homogeneous clay with an undrained shear strength of $c_u(\phi_u=0)$.

In order for the reinforced system, illustrated in Fig.1, to approach the state of total collapse, the strengths of the soils and reinforcement must be fully mobilized simultaneously. In design, however, one seeks a stable structure having adequate safety margins with respect to the various strength components. The art of selecting these margins is based mainly on how well the various materials involved are defined. Subsequently, an analysis is conducted on a structure comprised of fictitious materials having scaled-down strengths for which the state of limiting-equilibrium is realized

$$\phi_m = \tan^{-1}(\psi_m) = \tan^{-1}\left[\frac{\tan \phi}{F_\phi}\right] \quad (1)$$

$$c_m = \frac{c_u}{F_c} \quad (2)$$

$$t_m(x) = \frac{t(x)}{F_r} \quad (3)$$

where F_ϕ , F_c and F_r are the safety margins (i.e., factors of safety) with respect to the embankment's friction angle, the foundation's undrained shear strength, and the reinforcement's tensile resistance (or strength) at any location x , respectively. The subscript m symbolizes a mobilized component.

Note that only when $F_\phi = F_c = F_r = 1$, collapse indeed will occur. The fact that different safety factors may be selected for each material is consistent with common practice even when an unreinforced problem is considered.

Based on the above discussion, the problem can be stated as follows :

For a defined structural system (i.e., i, H, d, γ, ϕ and c_u), determine the required distribution of tensile resistance, $t(x)$, of a reinforcing sheet so that the prescribed safety factors F_ϕ , F_c and F_r are exceeded everywhere.

The required tensile resistance $t(x)$ must not exceed : (1) The internal strength of the reinforcement considering the required and allowable elongation needed to develop this strength as well as creep, corrosion, etc., and (2) The reinforcement pull-out resistance ("anchorage" strength) developing as a result of friction/adhesion with the soils. The frictional anchoring force is a function of the overburden pressure, which can be approximated, and the soil-reinforcement interface friction. The adhesive anchoring force is a function of the cohesion and clay-reinforcement interface properties. Both the interface coefficient of friction and adhesion can be determined experimentally. The overall pull-out resistance at a point is the minimum force generated on either side of that point, and it can be estimated via integration of frictional and adhesive restraining forces along the embedded sheet. It is assumed that if $t(x)$ equals the internal strength, it will realize only when both soils have reached failure. Otherwise, using the concept of limit-equilibrium may be unsafe since the reinforcement will break and t will drop to zero prior to the mobilization of ϕ and c_u rendering, in effect, an unreinforced

structure.

Failure Mechanism

Fig. 1 shows the selected rotational failure mechanism. This mechanism belongs to the family of log-spiral slip surfaces. Within the embankment [i.e., between points (2) and (3)] the slip surface is a log-spiral and within the foundation [i.e., between points (3) and (1)] it is a circular arc extending to depth d . Both the log-spiral and the circular arc are defined relative to the same center (x_c, y_c) . The reinforcement sheet is conservatively assumed to remain horizontal.

From the theoretical viewpoint of limit-analysis, the selected mechanism is kinematically admissible. Physically this mechanism enables the "stiff" material (i.e., embankment) to break rather steeply and the "soft" material underneath (i.e., foundation) to be squeezed out. As shown later on, the moment limiting-equilibrium equation for the sliding body defined by the selected mechanism can be assembled without resorting to any further assumption. This is a clear advantage when compared with other mechanisms used in non-rigorous methods. It should be pointed out that this mechanism (without reinforcement, though) was implicitly suggested by Baker and Garber (1978) who arrived at it through variational extremization.

Considering a polar coordinate system, which has its pole at (x_c, y_c) and conventions as shown in Fig. 1, the log-spiral portion of the slip surface is expressed as

$$r = A_2 e^{-\phi_m \beta} \text{ for } \beta_2 \geq \beta \geq \beta_1 \quad (4)$$

and the circular portion is

$$r = A_1 \text{ for } \beta_1 \geq \beta \geq -\beta_1 \quad (5)$$

where A_1, A_2 are unknown constants ; β is the independent variable in the polar system ; and $\phi_m = (\tan \phi / F_\phi)$.

The parametric equations relating points on the slip surface in the two coordinate systems are

$$x = x_c + r \sin \beta \quad (6)$$

$$y = y_c - r \cos \beta \quad (7)$$

Moment Limiting-Equilibrium Equation

As being implied in Fig. 1, it is assumed that the failure surface extends between the crest elevation (i. e., x_2, H) and the toe elevation (i. e., $x_1, 0$). Moreover, for convenience, the origin of the cartesian coordinate system is selected at the toe with the x -axis opposing the anticipated slide direction and the y -axis opposing gravity. Based on the above, assuming that the slip surface function (Eq. (7)) is univalued (i. e., for each x -ordinate a single value of y exists) and following Baker's (1981) technique, one can assemble the moment limiting-equilibrium equation written about (x_c, y_c)

$$\begin{aligned} M = & \int_{x_1}^{x_3} c_m [(y - y_c) - (x - x_c) y'] dx \\ & + \int_0^{x_3} \gamma y_c (x - x_c) dx \\ & + \int_0^{x_2} \gamma (\bar{y} - y_c) (x - x_c) dx \\ & - \int_{x_3}^{x_2} \gamma (y - y_c) (x - x_c) dx - t_m y_c = 0 \quad (8) \end{aligned}$$

where $y' = dy/dx$ and $\bar{y}$ is the slope surface function, i. e.,

$$\bar{y} = 0 \text{ for } x \leq 0 \quad (9)$$

$$\bar{y} = [\tan(i)]x \text{ for } [H/\tan(i)] \geq x \geq 0 \quad (10)$$

$$\bar{y} = H \text{ for } x \geq [H/\tan(i)] \quad (11)$$

Note that by virtue of the selected failure mechanism the foundation's unit weight does not appear in the moment equation; i. e., since the circular arc extends between β_1 and $-\beta_1$ (Fig. 1), the moment components, written about (x_c, y_c) , resulting from the foundation's self weight cancel out. Furthermore, since the slip surface belongs to the log-spiral family, the resultant of each elemental normal force on this surface and its associated frictional force must pass through the center of rotation (x_c, y_c) . Consequently, the normal stress distribution over the slip surface is not needed for Eq. (8).

Observing Eq. (8) one realizes that the moment equilibrium equation can be easily modified to accommodate c_u which changes with depth (i. e., substitute $c_m = \text{constant}$ by

$c_m(y) = c_u(y)/F_c$). Such modification has no effect on the mechanism's geometry and it should model reality, where c_u is typically increasing with depth, more accurately. Furthermore, Eq. (8) can be modified to include the effect of any number of reinforcing sheets, located at elevations y_j where j indicates sheet number; i. e., replace the term $(t_m y_c)$ by $\Sigma[t_m(y_c - y)]_j$. The above modifications were not included in the present formulation for the sake of brevity; their effects, however, will be reported in a follow-up publication.

At this stage it is useful to introduce the following nondimensional notations

$$N_m = \frac{1}{\gamma H} \frac{c_u}{F_c} \quad (12)$$

$$T_m = \frac{1}{\gamma H^2} \frac{t}{F_r} \quad (13)$$

$$X = \frac{x}{H} \quad (14)$$

$$Y = \frac{y}{H} \quad (15)$$

Dividing Eq. (8) by γH^3 and substituting the nondimensional notation result in

$$\begin{aligned} \bar{M} = & \int_{X_1}^{X_3} N_m [(Y - Y_c) - (X - X_c) Y'] dX \\ & + \int_0^{X_3} Y_c (X - X_c) dX \\ & + \int_0^{X_2} (\bar{Y} - Y_c) (X - X_c) dX \\ & - \int_{X_3}^{X_2} (Y - Y_c) (X - X_c) dX - T_m Y_c \\ = & 0 \quad (16) \end{aligned}$$

where $\bar{M} = M/\gamma H^3$ and

$$\bar{Y} = 0 \text{ for } X \leq 0 \quad (17)$$

$$\bar{Y} = [\tan(i)]X \text{ for } [1/\tan(i)] \geq X \geq 0 \quad (18)$$

$$\bar{Y} = 1 \text{ for } X \geq [1/\tan(i)] \quad (19)$$

Introducing Eqs. (4) through (7) and (17) through (19) into Eq. (16), transforming dX and Y' to the polar coordinate system and carrying out the integration give

$$\bar{M} = m_1 + m_2 N_m + m_3 T_m = 0 \quad (20)$$

where

$$\begin{aligned}
 m_1 = & A_2^3 \left\{ \frac{\cos^3 \beta}{3} + \frac{\psi_m}{1+9\psi_m^2} (\sin \beta - 3\psi_m \cos \beta) \right\} \\
 & \times e^{-3\psi_m \beta} \Big|_{\beta_1}^{\beta_2} + \frac{A_2^2 Y_c}{2} e^{-2\psi_m \beta} \sin^2 \beta \Big|_{\beta_1}^{\beta_2} \\
 & + \frac{X_c^2}{2} - \frac{A_2^2}{2} e^{-2\psi_m \beta} \sin^2 \beta_2 \\
 & - \frac{[\cot(i)]}{6} [3X_c - \cot(i)] \quad (21)
 \end{aligned}$$

$$m_2 = 2\beta_1 A_1^2 \quad (22)$$

$$m_3 = A_1 \cos \beta_1 \quad (23)$$

where the constants A_1 and A_2 are the constants defined in Eqs. (4) and (5) divided by H .

Eq. (20) can be rewritten in a way which fits the statement of the problem and the subsequent solution

$$T = \frac{t(x)}{\gamma H^2} = -F_r \frac{m_1 + m_2 N_m}{m_3} \quad (24)$$

Observing Eqs. (21) through (24) one sees that for a given problem (i.e., given $i, \gamma, \phi, H, F_\phi, c_u, F_c, d$ and F_r) the following unknowns exist: $X_c, Y_c, A_1, A_2, \beta_1, \beta_2$ and $t(x)$. Thus a framework for a numerical procedure is created; i.e., at any interface location X , find $\max(t)$ which satisfies Eq. (24) for all possible combinations of the parameters $X_c, Y_c, A_1, A_2, \beta_1$ and β_2 . This procedure is a result of the moment limiting-equilibrium equation and it complies with the statement of the problem.

Geometrical Relationships

To develop an explicit solution procedure, some geometrical relationships which result from the selected failure mechanism are highlighted. First, however, the nondimensional parametric equations for each segment of the slip surface are rewritten [Eqs. (4) through (7)]

$$\left. \begin{aligned} X &= X_c + A_2 e^{-\psi_m \beta} \sin \beta \\ Y &= Y_c - A_2 e^{-\psi_m \beta} \cos \beta \end{aligned} \right\} \text{ for } \beta_2 \geq \beta \geq \beta_1 \quad (25)$$

$$\left. \begin{aligned} X &= X_c + A_1 \sin \beta \\ Y &= Y_c - A_1 \cos \beta \end{aligned} \right\} \text{ for } \beta_1 \geq \beta \geq -\beta_1 \quad (26)$$

Based on simple trigonometry and through observation of Fig. 2, one can state that

Fig. 2. Geometry of the problem

$$A_1 = \frac{D}{\tan(\beta_1/2) \sin \beta_1} \quad (27)$$

where $D = d/H$.

Since it was assumed that the slip surface emerges at the toe elevation (i.e., $Y_1 = 0$), it follows from Eq. (26) that

$$Y_c = A_1 \cos \beta_1 \quad (28)$$

At point (3) (Fig. 2) the radius of the circle equals that of the log-spiral, i.e.,

$$A_2 = A_1 e^{\psi_m \beta_1} \quad (29)$$

Since it was assumed that the slip surface extends from the crest elevation (i.e., $Y_2 = 1$), it follows from Eq. (25) that

$$Y_c - A_2 e^{-\psi_m \beta_2} \cos \beta_2 = 1 \quad (30)$$

For the numerical procedure it is convenient to replace Eq. (30) with a combination of Eqs. (27) through (30)

$$\begin{aligned} D [e^{-\psi_m \beta_1} \cos \beta_1 - e^{-\psi_m \beta_2} \cos \beta_2] \\ - e^{-\psi_m \beta_1} \tan \frac{\beta_1}{2} \sin \beta_1 = 0 \end{aligned} \quad (31)$$

Recall that the slip surface was restricted to univalued functions. This restriction exists as long as the angle defining the tangent to the slip surface at point (2), θ (see Fig. 2), is equal to, or less than 90° . Using the log-spiral geometry it can be verified that $\theta = \beta_2 + \phi_m$. Thus,

$$\theta = \beta_2 + \phi_m \leq (\pi/2) \quad (32)$$

If the required t [Eq. (24)] at $x = l$ (Fig. 2) is desired, then from Eq. (26) it follows that

$$X_c = L - A_1 \sin \beta_1 \quad (33)$$

where $L = l/H$.

If the maximum required value of t , occurring at some unknown critical distance l_{cr} , is desired, the argument presented in Ap-

Fig. 3. Computation scheme

pendix I is applicable. From Appendix I and Eq. (27) it follows that

$$X_c = \begin{cases} \frac{\cot(i)}{2} & \text{if } K \leq 0 \\ \frac{D}{\tan(\beta_1/2)} & \text{if } K > 0 \end{cases} \quad (34)$$

where

$$K = \frac{\cot(i)}{2} - \frac{D}{\tan(\beta_1/2)} \quad (35)$$

There are now 6 available equations [i. e., Eqs. (24), (27) through (29), (31), (33) or (34)]. The number of unknowns, however, is 7 (i. e., X_c , Y_c , A_1 , A_2 , β_1 , β_2 and t). For every assumed θ [i. e., Eq. (32)], therefore, one can solve this system of available equations until $\max(t)$ is at a specified location obtained.

Computation Scheme

The computation procedure required to determine t so that the prescribed safety factors F_ϕ , F_c and F_r are exceeded at any interface location l is schematically illustrated in Fig. 3. Notice that, in essence, the problem is solved for an assumed θ [see Eq. (32)]. The initial assumed value is the extreme angle $\pi/2$, i. e., vertical tangent at X_2 . This θ is changed until $\max(t)$ is obtained.

Observing Fig. 3 one sees that $\max(t)$ is determined also with respect to smaller values of d . In reality, d represents the maximum feasible depth of failure within the foundation. Depending on the problem, $\max(t)$ may not correspond to the original d but rather to a lower depth.

It should be pointed out that if $\max(t)$ corresponding to the critical case is desired [Eq. (34)] the mathematical value of l_{cr} might be larger than the embankment width. In this case, a feasible value of l [Eq. (33)] should be prescribed.

RESULTS

Only critical cases, corresponding to the suggested deep-seated rotational failure mechanism, are presented. Potential failures which may develop along the reinforcement or within the embankment were not investigated. It should be noted, however, that some of the presented results (i. e., Table 1, Figs. 4 and 6 through 10) include values of ϕ_m equal to 20° and 25° for an embankment with side slopes of 1:2 ($i = 26.6^\circ$). Since these selected ϕ_m are less than i , clearly the most critical surface is along the embankment's face. These values were included, however, because of design considerations. In reality ϕ is typically greater than 26.6° thus permitting side slopes of 1:2. Because of economical reasons, one may select a small safety factor, F , against superficial failure along the embankment's face ($F = \tan \phi / \tan i$). Conversely, deep-seated failure may be costly and its prevention is more involved in terms of analysis assump-

 Fig. 4. Typical traces of slip surfaces ($F_c = F_\phi$)

tions. Therefore, the designer may wish to increase the safety factor against such failures resulting in a design value of $\phi_m = \tan^{-1} [\tan \phi / F_\phi]$ which is smaller than i . Consequently, ϕ_m equals 20° and 25° represent meaningful values.

Comparison

The comparison is limited to unreinforced cases. The reinforcement effect in this work is formulated in an identical way to most other methods. It may be expected therefore that its effect is qualitatively the same.

Fig. 4 illustrates the critical slip surfaces for a 1:2 embankment having depth limits of $d/H = 0.1$ and 10. Notice that in this figure $F_c = F_\phi$. For the case $\phi_m = 40^\circ$ one sees that the slip surface through the embankment is rather steep. However, it is rather flat through the foundation. This failure closely resembles a bi-planar or a tri-planar surface commonly used in the

Table 1. Comparison of results for the unreinforced case

$D = \frac{d}{H}$	ϕ	$N = \frac{c_u}{\gamma H}$	$F_s = F_c = F_\phi$ based on :	
			The presented work	Bishop's method
0.1	20°	0.1453	1.00 (1.02)*	**1.05 (1.00)*
0.1	40°	0.0806	1.00 (1.00)*	1.36 (1.21)*
10	20°	0.1816	1.00 (1.02)*	**0.99 (1.03)*
10	40°	0.1657	1.00 (1.03)*	0.91 (0.93)*

* Numbers in parentheses represent F_s obtained from Spencer's method using the trace of the slip surface as determined from the presented method or from Bishop's method.

** The trace of Bishop's critical circle is very similar to the one determined from the presented method.

wedge method. For the case $\phi_m = 20^\circ$, however, the entire slip surface closely resembles a circle. Fig. 4 indicates that for deep failures the slip surface through the embankment is practically a vertical plane.

In many cases Bishop's method has been extended to deal with the reinforcement problem (e.g., Rowe and Soderman, 1984, 1985; Ingold and Miller, 1986). Thus it is of

particular interest to compare the results presented in Fig. 4 with Bishop's predictions. Table 1 summarizes this comparison. Note that the predicted safety factors, as well as the corresponding critical slip surfaces, are very close when $\phi=20^\circ$. As ϕ increases, however, Bishop's becomes significantly less conservative when $D=0.1$ and moderately more conservative when $D=10$. Application of Spencer's method, which is considered to be rigorous, using the trace of the critical slip surface obtained in each method, produced similar safety factors (see Table 1). This may indicate that the results predicted by Bishop's and the presented work approximately satisfy *all* limiting-equilibrium equations.

It should be pointed out that the likelihood of using reinforcement decreases as d increases. This is because of the following: (1) The required reinforcement tensile strength exponentially increases as the extent of failure increases up to an unfeasible value, and (2) The length l_{cr} at which the slip surface intersects the reinforcement increases with an increased d , up to a point where it exceeds the embankment width thus rendering an unrealistic problem (i.e., t is unattainable due to insufficient anchorage). Considering the solution performance for small values of d , it appears that the mechanism presented in this work is adequate to deal with the problem of reinforced embankment over a soft foundation.

Nondimensional Charts

To cover a wide range of possible problems a large number of nondimensional charts are needed. With the increased availability of personal computers it seems more practical to use a general computer code capable of dealing with infinite problems (Leshchinsky, 1986) rather than generate and use an excessive number of design charts. The limited number of charts presented, however, should provide insight into the numerical nature of the solution.

There are three steps of stability checks required in the analysis of the problem. The

first stability check must ensure that there is an adequate bearing-capacity of the foundation. If such stability is not available, reinforcement is useless since the embankment, through its entire width, will fail. The second step checks if the unreinforced structure is sound against deep-seated failure. If it is, no reinforcement is needed. Only if the structure is safe against bearing-capacity failure but is unsafe against deep-seated failure the third step is needed. In this step, the distribution of the required tensile strength of the reinforcement along the interface is determined so that all safety factors are exceeded everywhere. Once the distribution of required tensile strength has been determined, the feasibility of developing such strength must be investigated. This important exploration, however, is beyond the scope of the current work.

Fig. 5 is a bearing-capacity design chart for a 1:2 embankment having a width b (or $B=b/H$). It was computed based on an assumed circular slip surface extending from one toe and passing underneath the embankment. The mathematical formulation for this particular problem is presented in Appendix II. To use Fig. 5 first one has to calculate $D=d/H$ and $B=b/H$. Using D and B , N_m is then read from the chart and the factor of safety with respect to the shear strength of the foundation is determined; i.e., $F_c=c_u/(N_m\gamma H)$. Notice that for a given B , beyond a certain D the required N_m is

Fig. 5. Bearing-capacity design chart

Fig. 6. Design chart for the unreinforced case (N_m vs. D)

constant. This is because $\min(F_c)$ [or $\max(N_m)$] realizes at a depth which is less than the physical d , indicating that the critical slip circle then is independent of d (see Appendix II).

Fig. 6 is a design chart for the case of an unreinforced 1:2 embankment. It relates the geometry of the problem to the materials comprising it. This chart is assembled based on the procedure presented in Fig. 3, using, however, reinforcement with zero strength (i. e., unreinforced problem). It represents the critical results corresponding to a surface which extends to depth D . To use this chart, first one needs to determine $D = d/H$ and to compute $\phi_m = \tan^{-1} [\tan \phi / F_\phi]$ based on an adequately selected safety factor with respect to friction, F_ϕ . Using this D and ϕ_m , N_m is read from the chart and $F_c = c_u / (N_m \gamma H)$ is then computed. If F_c exceeds the design value then the embankment may be considered safe against deep-seated failure. Otherwise, reinforcement may help.

Since an embankment has a finite width b , one needs to check if the critical case determined from Fig. 6 can indeed be realized; i. e., does the embankment bracket the critical slip surface. Such information is provided in Fig. 7; for the D and ϕ_m used in Fig. 6, the corresponding value of $l_{cr} = L_{cr} H$ can be read. It is a conservative approximation to state that if $l_{cr} < b$ then F_c determined via

Fig. 7. Design chart for the unreinforced case (L_{cr} vs. D)

Fig. 8. Design chart for the reinforced case ($D=0.1$)

Fig. 6 is valid. Otherwise, this critical result does not correspond to the actual dimensions of the problem and it is likely, therefore, that F_c is larger. In fact, for $l_{cr} \geq b$ only the bearing-capacity check is valid, and as stated before, if F_c then is less than the design value, reinforcement is useless. One should therefore proceed with a stability check only if F_c , corresponding to $l_{cr} < b$, is smaller than prescribed.

Once it has been determined that there is sufficient bearing-capacity but inadequate stability against deep-seated failure developing through the embankment, an estimate if reinforcement might help is needed. Figs. 8 through 10 enable such an estimate for a 1:2 embankment and $D=0.1$, 1 and 10:

Fig. 9. Design chart for the reinforced case ($D=1$)

Fig. 10. Design chart for the reinforced case ($D=10$)

It was numerically observed that l_{cr} is affected very little by the reinforcement tensile force. Consequently, the analysis philosophy with regard to l_{cr} and b , discussed in the preceding paragraph, still holds.

Utilizing these charts is rather straightforward. For a given D the appropriate chart is selected. Using ϕ_m and N_m , computed based on the design safety factors F_ϕ and F_c , one can read the required nondimensional tensile force T_m and thus compute $t_{max} = F_r \cdot T_m \cdot \gamma H^2$. It should be emphasized that t_{max} corresponds to the critical case (i.e., $l_{cr} < b$).

Notice from Figs. 8 through 10 that there is a significant increase in the required tensile force as D increases, possibly approaching impractical values. Also observe that

for the sake of completeness there are values of T_m corresponding to $N_m = 0$ (i.e., foundation with zero strength; e.g., water). Obviously this is unrealistic. A bearing-capacity check, however, will indicate that this structure is unfeasible for any $D > 0$ (see Fig. 5).

Note that t_{max} at l_{cr} does not necessarily guarantee a stable structure; i.e., failure can develop through other sections of the structure where insufficient reinforcement exists. In addition to t_{max} , therefore, one needs to assess the required distribution of the reinforcement tensile force (or resistance) so that the design safety factors (i.e., F_ϕ , F_c and F_r) are exceeded for all potential failure surfaces. This assessment is extremely important from a design viewpoint since the force $t(l)$ may develop only if sufficient anchorage is available at $x=l$; i.e., one must check the feasibility of developing $t(l)$ as a result, for example, of reinforcement/soil frictional and adhesion interaction. Furthermore, in certain cases it might be more economical to use several layers of reinforcement so that the demand for the strength $t(l)$ is matched but not wastefully exceeded. It is worthwhile mentioning an inherent assumption of any limit-equilibrium analysis; i.e., a single well defined slip surface has the potential to develop. Therefore, if $t(l)$ corresponds exactly to F_ϕ , F_c and F_r , only at one random location will the reinforcement tensile resistance actually be required; at all other locations only the potential will be needed. This phenomenon points out the weakness of limit-equilibrium analysis where true interaction is ignored; however, considering short-term stability and deep-seated failures (i.e., the scope of this paper) its application appears reasonable.

Figs. 11 and 12 show $t(l)$ at various interface locations $x=l$ for a 1:2 embankment, $\phi_m = 30^\circ$, and $D=1$ or 10. Once t_{max} has been determined one can proceed and select the proper chart (say Fig. 11 or 12) to estimate $t(l)$ based on N_m . Notice from these charts that if N_m is rather small, the rate of decrease of $t(l)$ is slow indicating that,

Fig. 11. Distribution of required tensile force ($D=1$, $\phi_m=30^\circ$)

Fig. 12. Distribution of required tensile force ($D=10$, $\phi_m=30^\circ$)

potentially, at the embankment ends $t(l)$ is physically unattainable. Such cases, however, would be excluded from reinforcement considerations already in the bearing-capacity check (Fig. 5). It is important to note that failure may develop to the other side of the embankment as well. Since the problem is symmetrical with respect to the center-line of the embankment, t at $x=(b-l)$ must be equal to t at $x=l$. At every point $x=l$ therefore there are two values of t , each needed to prevent failure in opposing direction. Since failure can occur only in one direction at a time, the most critical value of t (i.e., the greater value) should be considered.

Figs. 11 and 12 are for failures developing to the full depth D . It is possible, however,

that at some $x=l$ the critical surface is shallower requiring larger $t(l)$. Thus, many additional such charts, covering the whole range of possible D 's, are needed to determine $t(l)$ with confidence. This makes the generation and utilization of design charts rather impractical. The insight into the solution performance provided by the various charts, however, is valuable in understanding the reinforcement effects.

Example Problem

The example problem is fully defined in Fig. 13. Using the computer program SORE (Stability of Reinforced Embankment; Leshchinsky, 1986), which is based on the computational scheme shown in Fig. 3, this problem was analyzed. For bearing-capacity purposes, the factor of safety with respect to c_u is $F_c=5.43(>3.0)$. F_c for the unreinforced structure (taking $F_\phi=1.3$) is 1.53 (<3.0) with the critical surface passing at $l_{cr}=34$ ft. (10.4 m) and extending 10 ft. (3 m) down to the hard layer. Since the design

Design Safety Factors: $F_\phi=1.3$, $F_c=3.0$, $F_r=2.5$

Fig. 13. Example problem (1 ft=0.305 m, 1 lb/ft²=47.85 kPa, 1 lb/ft³=0.157 kN/m³)

Fig. 14. Distribution of required tensile resistance in example (1 ft=0.305 m, 1 lb/ft=14.6 N/m)

F_c is not exceeded in the unreinforced structure, the potential of using reinforcement was checked. It was found that the maximum tensile resistance needed is 18,324 lb/ft (267 kN/m) at $l_{cr}=34$ ft. (10.4 m) and a depth limit of 10 ft. (3 m). The required distribution of t , so that F_ϕ , F_c and F_r are exceeded everywhere is shown in Fig. 14. It should be noted that, for example, at $x=20$ ft. (6.1 m), the depth limit rendering $\max(t)$ is 7 ft. (2.1 m) (i.e., less than $d=10$ ft. (3 m)). Notice that due to the problem's symmetry, a mirror image of the computed t is presented in Fig. 14. This reflected image ensures that F_ϕ , F_c and F_r are exceeded also for potential failures to the other side of the symmetrical problem. For design, only the envelope defined by the two curves should be used (see discussion in last section).

CONCLUSION

A rotational failure mechanism for a reinforced free-draining embankment over soft clayey foundation, which corresponds to the critical stage of end-of-construction, has been presented. This mechanism is kinematically admissible from the strict viewpoint of limit-analysis. Physically, the mechanism allows the embankment to break rather steeply and enables the foundation to be squeezed out. It should be pointed out that this mechanism is suitable for deep-seated failures only. Failure, however, may develop at the interface (i.e., immediately above or below the reinforcement plan, depending on interface properties). Such potential failures and the subsequently induced tensile force in the reinforcement can be estimated using the conventional wedge method.

The presented analysis is based on limit-equilibrium. For the chosen failure mechanism, the moment limiting-equilibrium equation is assembled rigorously. This equation is extremized to determine the required tensile resistance of the reinforcement so that the specified safety factors with respect to the shear strengths of the embankment and foundation soils are exceeded everywhere.

Results for the unreinforced problem show that if the foundation layer is thin relative to the height of the embankment, the proposed mechanism produces conservative results as compared to Bishop's method predictions. Since reinforcement has the potential to be effective only when the extent of the soft foundation is limited relative to the embankment dimensions, it appears that the presented mechanism is, at least, as adequate to deal with reinforced structures as Bishop's.

Some nondimensional design charts are presented. The charts shed some light on the reinforcement effects. For design, however, where the distribution of the required tensile resistance is needed, many more such charts must be utilized. Therefore, a computer program which is based on the presented computation scheme is recommended.

It should be noted that the proposed computation procedure can be expanded to deal with external loads (e.g., Leschinsky, 1986) which might be significant in applications such as reinforced highway embankments.

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APPENDIX I

THE MIDPOINT ARGUMENT

The following is an extension of the well known midpoint argument (e.g., Taylor, 1937; Baker, 1981) to the problem at hand. It follows the sequence of arguments as presented by Baker (1981). Based on equilibrium considerations, it is shown that the center of the critical slip surface lies on a vertical line passing through the middle of the slope face.

Consider the situation shown in Fig. I.1. The pole P of the slip surface is located on a vertical line passing through the midpoint

Fig. I. 1. Locating the center of the critical slip surface (Rearranged from Baker, 1981)

M of the slope face $\overline{BC}$. If the slope $\overline{BC}$ is cut away (parallel to itself) to the line $\overline{B_1C_1}$ one can then construct the resulting incremental force diagram. The change in weight, ΔW_1 , is an upward force acting through the center of gravity of the area removed, CG_1 . The cohesive force developed along arc AH remains unchanged. The line of action of the reinforcing force T_m , which acts in the direction of line AH , must be the same as before the removal of the area BCC_1B_1 ; thus, ΔT_{m1} and ΔW_1 , must intersect at point O_1 . The characteristic property of the log-spiral family of slip surfaces requires that the resultant of normal and frictional forces acting over the slip surface must pass through the pole. This property together with equilibrium require that the force ΔP_1 , which is the change in the resultant of both the normal and frictional forces along the arc HD and normal force along the arc AH , must also pass through point O_1 and through the pole P . Thus the line-of-action of ΔP_1 is also known and the force component ΔT_{m1} , ΔW_1 and ΔP_1 may be drawn at point O_1 as shown in Figure I.1. From this force diagram, it is seen that ΔT_{m1} acts to the left which indicates a decrease in the required reinforcement tensile force T_m . It follows therefore that removal of the area BCC_1B_1 has increased stability.

If the surface $\overline{BC}$ is built up to the line $\overline{B_2C_2}$, the change in weight ΔW_2 is a downward force and an analysis identical to the above indicates that in this case ΔT_{m2} also acts to the left. It follows therefore that moving the midpoint of the slope to either side of the vertical line through the pole leads to greater stability. Thus, the critical condition is realized when the center of rotation is located at

$$X_c = \frac{\cot(i)}{2} \quad (I.1)$$

This condition, however, is not always possible since it may result in X_1 being positive; i.e., a slip surface emerging above the toe. This is contrary to the basic assumption of the analysis [see derivation of Eq.

varying θ in Eq. (II.9), one can determine $\min(F_c)$ and the corresponding critical surface.

It should be pointed out, however, that $\min(F_c)$ corresponds to a circular arc extending, at most, to depth d . If the critical slip circle tends to extend to a depth greater than d , then d should be prescribed as a limit to the circle. Thus, instead of Eq. (II.5),

for each θ the following equations should be used in conjunction with Eqs. (II.2) through (II.4)

$$A_1 = \frac{d}{\tan(\beta_1/2) \sin \beta_1} \quad (\text{II. 10})$$

$$x_c = b - A_1 \sin \beta_1 \quad (\text{II. 11})$$

Eq. (II.10) and (II.11) originate from Eqs. (27) and (II.6), respectively.