TECHNICAL NOTE

PORE PRESSURE DISSIPATION RATIO FOR A NONLINEAR CONSOLIDATION PROBLEM

YOSHIKI YOSHIMI¹⁾

ABSTRACT

A special case of nonlinear stress-strain relationship in one-dimensional consolidation can be treated by the Mikasa theory in which the dependent variable is the strain rather than the excess pore water pressure. This note shows that the pore pressure dissipation ratio during nonlinear consolidation can be obtained from the Mikasa theory without the need of numerical analysis that is necessary for the Terzaghi theory of consolidation. For the case of constant compression index, a simple expression is derived that shows the pore pressure dissipation ratio as a function of the load increment ratio and the degree of consolidation in terms of strain.

Key words : clay, consolidation, pore pressure, strain, stress-strain curve (IGC : D 5)

INTRODUCTION

Davis and Raymond (1965) obtained a numerical solution of the Terzaghi theory of consolidation for the case in which the coefficient of permeability, k , and the coefficient of volume compressibility, m_v , were allowed to vary in such a way that the coefficient of consolidation, $c_v = k/m_v \gamma_w$, was constant. The paper is entitled "A Non-Linear Theory of Consolidation" because m_v may vary during consolidation.

Davis and Raymond concluded that the settlement-time relationship was identical to the linear Terzaghi theory regardless of the load increment ratio. Barden (1966) pointed out that the same conclusion could be drawn

based on the theory by Mikasa (1965) in which the dependent variable was the strain rather than the excess pore water pressure. Based on their solution, Davis and Raymond also showed that the time history of the pore pressure dissipation ratio depended on the load increment ratio.

The object of this note is to show that the same result concerning the pore pressure can directly be derived from the Mikasa theory without the need for numerical calculation, and that the result can be expressed in a simple explicit form.

PORE PRESSURE DISSIPATION RATIO

In the Mikasa theory (Mikasa, 1963, 1965),

i) Professor, Tokyo Institute of Technology, Meguro-ku, Tokyo.

Manuscript was received for review on December 25, 1986.

Written discussions on this note should be submitted before April 1, 1988, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4 F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

Fig. 1. Void ratio vs. effective stress relation with constant compression index

the degree of consolidation, U_z , is defined in terms of strain as follows :

$$U_z = \frac{\varepsilon}{\varepsilon_f} = \frac{e_0 - e}{e_0 - e_f} \quad (1)$$

in which ε is the strain, e is the void ratio, and the subscripts o and f denote the initial and the final states, respectively. As long as a void ratio vs. effective stress curve, however nonlinear, is uniquely defined, i.e., if structural viscosity effects are ignored, the effective stress for any depth and time can be determined by computing e from Eq. (1) and then by reading the value of effective stress off the curve. In reference to Fig. 1, the degree of consolidation for constant compression index, C_c , can be expressed by

$$U_z = \frac{\log(\sigma'/\sigma'_0)}{\log(\sigma'_f/\sigma'_0)} \quad (2)$$

in which σ' is the effective stress, and the subscripts o and f denote the initial and the final states, respectively. From Eq. (2), one obtains

$$\frac{\sigma'}{\sigma'_0} = \left(\frac{\sigma'_f}{\sigma'_0} \right)^{U_z} \quad (3)$$

When a load increment is applied instantaneously, the sum of the excess pore water pressure and the effective stress, i.e., the total stress, remains constant, as follows :

Fig. 2. Time history of pore pressure dissipation ratio computed from Eq. (6)

$$u + \sigma' = u_0 + \sigma'_0 \quad (4)$$

Because u_0 is equal to load increment, $\Delta\sigma$, the pore pressure dissipation ratio, or the degree of consolidation in terms of pore pressure, is given by

$$\begin{aligned} \frac{u_0 - u}{u_0} &= \frac{\sigma' - \sigma'_0}{\Delta\sigma} = \frac{\sigma'_0}{\Delta\sigma} \cdot \frac{\sigma'}{\sigma'_0} - \frac{\sigma'_0}{\Delta\sigma} \\ &= \frac{\sigma'_0}{\Delta\sigma} \left(\frac{\sigma'}{\sigma'_0} - 1 \right) \end{aligned} \quad (5)$$

Substituting Eq. (3) into Eq. (5),

$$\begin{aligned} \frac{u_0 - u}{u_0} &= \frac{\sigma'_0}{\Delta\sigma} \left\{ \left(\frac{\sigma'_f}{\sigma'_0} \right)^{U_z} - 1 \right\} \\ &= \left\{ \left(1 + \frac{\Delta\sigma}{\sigma'_0} \right)^{U_z} - 1 \right\} / \left(\frac{\Delta\sigma}{\sigma'_0} \right) \end{aligned} \quad (6)$$

The expression has been published in Japanese (JSSMFE, 1965 ; JSSMFE, 1985). But it has not, to the author's knowledge, been brought to the attention of the profession outside Japan.

Eq. (6) does not require any restriction on the coefficient of consolidation, c_v . When one wishes to evaluate U_z , however, the assumption of constant c_v enables one to use the series solution of U_z of the same form as the Terzaghi solution for the pore pressure dissipation ratio. The value of U_z at the impermeable boundary, for example, is shown in broken line in Fig. 2.

Fig. 2 shows the time history of pore pressure dissipation ratio at the impermeable

Fig. 3. Comparison with laboratory test data by Barden and Berry (1965) for a laminated, lacustrine silty clay (a), and a slightly organic estuarine clay with silt inclusions (b) (Yoshimi, 1967)

boundary for four values of $\Delta\sigma/\sigma'_0$ computed from Eq. (6). The result is in perfect agreement with that given by Davis and Raymond (1965). The closed-form expression of Eq. (6) has an obvious advantage over any numerical solution. It can also be shown from Eq. (6) that $(u_0 - u)/u_0$ approaches U_z as $\Delta\sigma/\sigma'_0$ approaches zero.

As shown in Fig. 3, the pore pressure dissipation ratio computed from Eq. (6) agrees with the experimental result by Barden and Berry (1965) better than the linear Terzaghi solution.

CONCLUSION

For a nonlinear consolidation problem in which the compression index is constant, it is possible to obtain from the Mikasa theory of consolidation a simple expression for the pore pressure dissipation ratio as a function of the load increment ratio and the degree of consolidation in terms of strain.

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