A CONSTITUTIVE MODEL FOR SANDS AND CLAYS EVALUATING PRINCIPAL STRESS ROTATION

HAJIME MATSUOKA¹⁾ and KAZUNARI SAKAKIBARA¹⁾

ABSTRACT

A constitutive model for sands and clays by which the general strain increments ($d\epsilon_x$, $d\epsilon_y$ and $d\gamma_{xy}$) are directly related to the general stress increments ($d\sigma_x$, $d\sigma_y$ and $d\tau_{xy}$) is proposed. This model is derived from the hyperbolic relationship between the general shear-normal stress ratio (τ_{xy}/σ_x or τ_{xy}/σ_y) and the general shear strain (γ_{xy}), and the "stress-dilatancy relation" expressed in general coordinates. In order to evaluate the influence of rotation of the principal stress axes on strains, "rotation tests", in which the stress path is the circumference of a Mohr's circle, have been carried out by a "two-dimensional general stress apparatus" using a stack of aluminium rods (mixture of $\phi 1.6$ mm and $\phi 3.0$ mm). The test results agree well with the analytical ones based on the proposed model. Simple shear tests and liquefaction tests with rotation of the principal stress axes are also analyzed by the proposed model. The model is extended to a three-dimensional one using the superposition of the "two-dimensional" principal strain increments. The three-dimensional model can analyze all the results of triaxial compression and extension tests under constant mean, major and minor principal stresses, and constant principal stress ratios, and "stress probe" tests in triaxial compression and extension conditions. The stress-strain parameters in the proposed model are determined from a conventional triaxial test and an oedometer test.

Key words : angle of internal friction, clay, constitutive equation of soil, dilatancy, granular material, liquefaction, sand, special shear test, stress path, stress-strain curve (IGC : D 6/D 5)

INTRODUCTION

It is natural to consider that the change in stresses (σ_x , σ_y and τ_{xy}) on an arbitrary plane causes strains (ϵ_x , ϵ_y and γ_{xy}) in every kind of materials. In the theory of plastic-

ity, however, such an idea is not satisfied because the constitutive relation is usually formulated by the stress invariants such as the principal stresses σ_1 , σ_2 and σ_3 or q and p ($q = \sigma_1 - \sigma_3$, $p = (\sigma_1 + \sigma_2 + \sigma_3)/3$) or τ_{oct} and σ_{oct} ($\tau_{oct} = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}/3$),

- i) Professor, Department of Civil Engineering, Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya 466.
- ii) Engineer, Japan Highway Public Corporation, 3-3-2, Kasumigaseki, Chiyoda-ku, Tokyo. (formerly Graduate Student at Nagoya Institute of Technology)

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$\sigma_{oct}=(\sigma_1+\sigma_2+\sigma_3)/3$) etc. For example, the strains caused by a "rotation test" along a Mohr's stress circle under constant principal stresses cannot be explained by the usual theory of plasticity.

Pande and Sharma (1983) have proposed the multi-laminate model of clays which has the same response as the Critical State model if no rotation of the principal stress axes takes place during plastic flow. By the model, they have numerically evaluated the influence of rotation of the principal stress axes on the volumetric and deviatoric strains.

In the present paper, a constitutive model for sands and clays by which the general strain increments ($d\epsilon_x, d\epsilon_y$ and $d\gamma_{xy}$) are directly related to the general stress increments ($d\sigma_x, d\sigma_y$ and $d\tau_{xy}$) is presented. The model predicts the strains not only due to "shear" and "consolidation", but also due to "rotation of the principal stress axes". In order to examine the proposed model, special tests involving stress paths along the radii of a Mohr's stress circle (called "radial tests") and involving stress paths along the circumference of Mohr's stress circles (called "rotation tests") have been carried out by a "two-dimensional general stress apparatus" using a stack of aluminium rods (mixture of $\phi 1.6$ mm and $\phi 3.0$ mm). The test results agree well with the analytical ones based on the proposed model in both cases of the radial and rotational stress paths. Simple shear tests and liquefaction tests with rotation of the principal stress axes are also analyzed by the proposed model. The model is extended to three dimensions using the concept of the superposition of the "two-dimensional" principal strain increments (Matsuoka, 1974, 1982). The triaxial compression and extension tests under various kinds of stress paths, including "stress probe" tests, are well analyzed by the proposed model.

HYPERBOLIC STRESS RATIO VS. SHEAR STRAIN RELATION EXPRESSED IN GENERAL COORDINATES

The hyperbolic relationship between the shear-normal stress ratio (τ_{xy}/σ_x or τ_{xy}/σ_y) and the shear strain (γ_{xy}) expressed in general coordinates is assumed as follows (see Figs. 1 and 2).

$$\gamma_{xy} = \frac{1}{G_0} \cdot \frac{(\tau_{xy}/\sigma_x)_f \cdot (\tau_{xy}/\sigma_x)}{(\tau_{xy}/\sigma_x)_f - (\tau_{xy}/\sigma_x)} \quad (1)$$

or

$$\gamma_{xy} = \frac{1}{G_0} \cdot \frac{(\tau_{xy}/\sigma_y)_f \cdot (\tau_{xy}/\sigma_y)}{(\tau_{xy}/\sigma_y)_f - (\tau_{xy}/\sigma_y)} \quad (2)$$

where $(\tau_{xy}/\sigma_x)_f$ and $(\tau_{xy}/\sigma_y)_f$ are the general shear-normal stress ratios at failure. G_0 is the initial tangent of the relation between τ_{xy}/σ_x (or τ_{xy}/σ_y) and γ_{xy} as shown in Fig. 1. Based on the idea that soil is governed by a frictional law, we may introduce the shear-

Fig. 1. Hyperbolic relation between shear-normal stress ratio and shear strain in general coordinates

Fig. 2. General shear and normal stresses expressed by Mohr's stress circle

normal stress ratio in general coordinates in Eqs. (1) and (2). The following equations are easily obtained from Fig. 2.

$$\left\{ \frac{\tau_{xy}/\sigma_x}{\tau_{xy}/\sigma_y} \right\} = \frac{\sin \phi_{m0} \cdot \sin 2\alpha}{1 \pm \sin \phi_{m0} \cdot \cos 2\alpha} \quad (3)$$

$$\left\{ \frac{\tau_{xy}/\sigma_x}{\tau_{xy}/\sigma_y} \right\}_f = \frac{\sin \phi \cdot \sin 2\alpha}{1 \pm \sin \phi \cdot \cos 2\alpha} \quad (4)$$

where ϕ is the internal friction angle, ϕ_{m0} is the mobilized internal friction angle and α is the angle between an arbitrary plane and the principal stress plane. By substituting Eqs. (3) and (4) into Eqs. (1) and (2), both Eqs. (1) and (2) become the following equation.

$$\gamma_{xy} = \frac{1}{G_0} \cdot \frac{\sin \phi \cdot \sin \phi_{m0} \cdot \sin 2\alpha}{\sin \phi - \sin \phi_{m0}} \quad (5)$$

It is worth noting in Eq. (5) that γ_{xy} is multiplied by $\sin 2\alpha$ and $\gamma_{xy}/2$ lies on a Mohr's strain circle. The value of $1/G_0$ in Eq. (5) becomes large under the high confining pressure of σ_m as shown in Fig. 1, and the following equation holds as an experimental fact.

$$1/G_0 \propto \ln(\sigma_m/\sigma_{mi}) \quad (6)$$

where σ_{mi} is the basic value of the confining pressure σ_m . Therefore, the shear strain γ_{xy} is considered to be the function of ϕ_{m0} , α and σ_m . If Eq. (5) is totally differentiated with respect to ϕ_{m0} , α and σ_m , the following equation is obtained.

$$\begin{aligned} d\gamma_{xy} = & \frac{1}{G_0} \cdot \frac{\sin^2 \phi \cdot \cos \phi_{m0} \cdot \sin 2\alpha}{(\sin \phi - \sin \phi_{m0})^2} \cdot d\phi_{m0} \\ & + \frac{2}{G_0} \cdot \frac{\sin \phi \cdot \sin \phi_{m0} \cdot \cos 2\alpha}{\sin \phi - \sin \phi_{m0}} \cdot d\alpha \\ & + \frac{\sin \phi \cdot \sin \phi_{m0} \cdot \sin 2\alpha}{\sin \phi - \sin \phi_{m0}} \cdot \frac{\partial(1/G_0)}{\partial \sigma_m} \cdot d\sigma_m \end{aligned} \quad (7)$$

The second term on the right side of Eq. (7) expresses the shear strain increment caused by the change in α , i.e., rotation of the principal stress axes, and it is multiplied by $\cos 2\alpha$. As seen from Fig. 2, the shear stress τ_{xy} is changed by the function of $\sin 2\alpha$ with rotation of the principal stress axes. Therefore, as shown by the open circles in Fig. 3, the angle in the Mohr's stress circle is different by 90° from the

Fig. 3. Correlation between stress state and strain increment state during rotation of principal stress axes

angle in the Mohr's strain increment circle. In fact, as the direction of the principal stress σ_1 deviates by δ from that of the principal strain increment $d\epsilon_1$ under rotation of the principal stress axes, the angle in the Mohr's strain increment circle should be shifted by $(90^\circ - 2\delta)$ as shown in Fig. 3 (b). Then, $\cos 2\alpha$ in the second term of Eq. (7) should be changed to $\cos\{2\alpha - (90^\circ - 2\delta)\} = \sin 2(\alpha + \delta)$. Putting $1/G_0 \equiv k_s$, taking Eq. (6) into consideration and replacing the coefficient of the third term in the right side of Eq. (7) with k_c , the following equations are obtained.

$$d\gamma_{xy}^s = k_s \cdot \frac{\sin^2 \phi \cdot \cos \phi_{m0} \cdot \sin 2\alpha}{(\sin \phi - \sin \phi_{m0})^2} \cdot d\phi_{m0} \quad (8)$$

"shear"

$$d\gamma_{xy}^r = 2k_s \cdot \frac{\sin \phi \cdot \sin \phi_{m0} \cdot \sin 2(\alpha + \delta)}{\sin \phi - \sin \phi_{m0}} \cdot d\alpha \quad (9)$$

"principal stress rotation"

$$d\gamma_{xy}^{ac} = k_c \cdot \frac{\sin \phi \cdot \sin \phi_{m0} \cdot \sin 2\alpha}{\sin \phi - \sin \phi_{m0}} \cdot \frac{d\sigma_m}{\sigma_m} \quad (10)$$

"anisotropic consolidation"

where $d\gamma_{xy}^s$, $d\gamma_{xy}^r$ and $d\gamma_{xy}^{ac}$ mean the shear strain increments for the general coordinates due to "shear", "rotation of the principal stress axes" and "anisotropic consolidation" respectively. These three Eqs. (8), (9) and (10) correspond to the hardening functions in the theory of plasticity, which decide the magnitude of strains.

STRESS RATIO VS. STRAIN INCREMENT RATIO RELATION EXPRESSED IN GENERAL COORDINATES

From Rowe's Principal Stress Ratio-Principal Strain Increment Ratio Relation

The stress-dilatancy relationship proposed by Rowe (1962, 1971) is written as follows.

$$R \equiv \frac{\sigma_1}{\sigma_3} = K \cdot \left(-\frac{d\epsilon_3}{d\epsilon_1} \right), \quad K = \tan^2 \left(45^\circ + \frac{\phi_f}{2} \right) \quad (11)$$

where ϕ_f is the equivalent friction angle between particles ($\phi_\mu \leq \phi_f \leq \phi_{cv}$, ϕ_μ : friction angle between particle surfaces, ϕ_{cv} : internal friction angle in the critical state). The following equations are obtained from Eq. (11).

$$\begin{aligned} d\gamma_{xy} &= (d\epsilon_1 - d\epsilon_3) \cdot \sin 2\alpha \\ &= \{1 + (R/K)\} \cdot \sin 2\alpha \cdot d\epsilon_1 \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{d\epsilon_x}{d\epsilon_y} &= \frac{d\epsilon_1 + d\epsilon_3}{2} \pm \frac{d\epsilon_1 - d\epsilon_3}{2} \cdot \cos 2\alpha \\ &= \left\{ \frac{1 - (R/K)}{2} \pm \frac{1 + (R/K)}{2} \cdot \cos 2\alpha \right\} \cdot d\epsilon_1 \end{aligned} \quad (13)$$

Dividing Eq. (13) by Eq. (12), the following equation is obtained as the stress ratio vs. strain increment ratio relation expressed in general coordinates.

$$\frac{d\epsilon_x/d\gamma_{xy}}{d\epsilon_y/d\gamma_{xy}} = \frac{(K-R)/(K+R) \pm \cos 2\alpha}{2 \cdot \sin 2\alpha} \quad (14)$$

From Matsuoka's Stress Ratio-Strain Increment Ratio Relation on Mobilized Plane

The stress ratio-strain increment ratio relationship on the mobilized plane ($(\tau/\sigma)_{\max}$ plane, i.e. $(45^\circ + \phi_{m0}/2)$ plane; see Fig. 2) has been proposed by Matsuoka (1974) as follows.

$$\frac{\tau}{\sigma_N} = \lambda \cdot \left(-\frac{d\epsilon_N}{d\gamma} \right) + \mu \quad (15)$$

where τ/σ_N is the shear-normal stress ratio on the mobilized plane ($\tau/\sigma_N = \tan \phi_{m0}$), $d\epsilon_N/d\gamma$ is the normal-shear strain increment ratio on the mobilized plane, and (λ and μ) are the material constants. The shear strain increment $d\gamma$ and the normal strain increment $d\epsilon_N$ on the mobilized plane are expressed as follows.

$$\begin{aligned} d\gamma &= (d\epsilon_1 - d\epsilon_3) \cdot \sin \{2(45^\circ + \phi_{m0}/2)\} \\ &= (d\epsilon_1 - d\epsilon_3) \cdot \cos \phi_{m0} \end{aligned} \quad (16)$$

$$\begin{aligned} d\epsilon_N &= \frac{d\epsilon_1 + d\epsilon_3}{2} + \frac{d\epsilon_1 - d\epsilon_3}{2} \cdot \cos \{2(45^\circ + \phi_{m0}/2)\} \\ &= \frac{d\epsilon_1 + d\epsilon_3}{2} - \frac{d\epsilon_1 - d\epsilon_3}{2} \cdot \sin \phi_{m0} \end{aligned} \quad (17)$$

Taking Eqs. (15), (16) and (17) into consideration, the following equations are obtained.

$$\begin{aligned} d\gamma_{xy} &= (d\epsilon_1 - d\epsilon_3) \cdot \sin 2\alpha = \frac{\sin 2\alpha}{\cos \phi_{m0}} \cdot d\gamma \quad (18) \\ \frac{d\epsilon_x}{d\epsilon_y} &= \frac{d\epsilon_1 + d\epsilon_3}{2} \pm \frac{d\epsilon_1 - d\epsilon_3}{2} \cdot \cos 2\alpha \\ &= \left\{ \frac{2(\mu - \tan \phi_{m0})}{\lambda} + \tan \phi_{m0} \right. \\ &\quad \left. \pm \frac{\cos 2\alpha}{\cos \phi_{m0}} \right\} \cdot \frac{d\gamma}{2} \end{aligned} \quad (19)$$

Dividing Eq. (19) by Eq. (18), the following equation is obtained in the same way as in Eq. (14).

$$\begin{aligned} \frac{d\epsilon_x/d\gamma_{xy}}{d\epsilon_y/d\gamma_{xy}} &= \frac{2 \cdot \frac{\mu - \tan \phi_{m0}}{\lambda} \cdot \cos \phi_{m0} + \sin \phi_{m0} \pm \cos 2\alpha}{2 \cdot \sin 2\alpha} \end{aligned} \quad (20)$$

It should be noted that 2α in Eqs. (14) and (20) is changed for $2(\alpha + \delta)$ when the strains due to "rotation of the principal stress axes" are calculated by using Eq. (9). These Eqs. (14) and (20) correspond to the flow rule in the theory of plasticity, which decide the direction of strain increments.

STRESS-STRAIN RELATIONSHIP EXPRESSED IN GENERAL COORDINATES

Combining Eqs. (8), (9) and (10) with Eq.

(14) or Eq. (20), the general strain increments ($d\varepsilon_x$, $d\varepsilon_y$ and $d\gamma_{xy}$) can be expressed by $d\phi_{m0}$, $d\alpha$ and $d\sigma_m$, which are represented by the following equations:

$$d\phi_{m0} = \cos^2 \phi_{m0} \cdot (A_1 \cdot d\sigma_x + A_2 \cdot d\sigma_y + A_3 \cdot d\tau_{xy}) \quad (21)$$

where

$$A_1 = (I_1/I_2) \{2 - (I_1/I_2) \cdot \sigma_y\} / (8 \tan \phi_{m0})$$

$$A_2 = (I_1/I_2) \{2 - (I_1/I_2) \cdot \sigma_x\} / (8 \tan \phi_{m0})$$

$$A_3 = (I_1/I_2) \{2(I_1/I_2) \cdot \tau_{xy}\} / (8 \tan \phi_{m0})$$

$$I_1 = \sigma_x + \sigma_y$$

$$I_2 = \sigma_x \sigma_y - \tau_{xy}^2$$

$$d\alpha = \cos^2 2\alpha \cdot \left\{ \frac{-\tau_{xy}}{(\sigma_x - \sigma_y)^2} \cdot d\sigma_x + \frac{\tau_{xy}}{(\sigma_x - \sigma_y)^2} \cdot d\sigma_y + \frac{1}{\sigma_x - \sigma_y} \cdot d\tau_{xy} \right\} \quad (22)$$

$$d\sigma_m = \frac{1}{2} (d\sigma_x + d\sigma_y) \quad (23)$$

In addition to the above-mentioned strain increments, the strain increments due to isotropic consolidation should be taken into account. The normal strain increments caused by isotropic consolidation are represented under two-dimensional conditions as follows:

$$\left. \begin{matrix} d\varepsilon_x^{ic} \\ d\varepsilon_y^{ic} \end{matrix} \right\} = \frac{1}{2} \cdot \frac{0.434 C_c}{1 + e_0} \cdot \frac{d\sigma_m}{\sigma_m} \text{ or } \frac{1}{2} \cdot \frac{0.434 C_s}{1 + e_0} \cdot \frac{d\sigma_m}{\sigma_m} \quad (24)$$

where C_c and C_s are the compression index and the swell index respectively. It should be noted that the coefficient of 1/2 in Eq. (24) is applicable only in a two-dimensional case.

In the case of clays (especially, normally consolidated clays), the volumetric strains caused by anisotropic consolidation (ex. $\sigma_1/\sigma_3 \equiv R=2$ and 3) are almost the same as those due to isotropic consolidation ($\sigma_1/\sigma_3 \equiv R=1$). Therefore, the normal strain increments due to anisotropic consolidation for clays are assumed as follows from the condition of no volumetric strain increment due to anisotropic consolidation ($d\varepsilon_v^{ac} = d\varepsilon_x^{ac} + d\varepsilon_y^{ac} = 0$).

$$\left. \begin{matrix} d\varepsilon_x^{ac} \\ d\varepsilon_y^{ac} \end{matrix} \right\} = \pm \frac{1}{2} \cdot k_c \cdot \frac{\sin \phi \cdot \sin \phi_{m0} \cdot \cos 2\alpha}{\sin \phi - \sin \phi_{m0}} \cdot \frac{d\sigma_m}{\sigma_m} \quad (25)$$

Taking Eqs. (21), (22) and (23) into consideration, the general strain increments

($d\varepsilon_x$, $d\varepsilon_y$ and $d\gamma_{xy}$) can be directly expressed by the general stress increments ($d\sigma_x$, $d\sigma_y$ and $d\tau_{xy}$), as shown by the following equation.

$$\begin{Bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\gamma_{xy} \end{Bmatrix} = [D]^{-1} \cdot \begin{Bmatrix} d\sigma_x \\ d\sigma_y \\ d\tau_{xy} \end{Bmatrix} \quad (26)$$

where $[D]$ is the stress-strain matrix. Eq. (26) is the constitutive equation for soils expressed in general coordinates, by which the general strain increments are directly related to the general stress increments.

EXPRESSION OF PRINCIPAL STRAIN INCREMENTS

For Sands

The principal strain increments are easily obtained from the normal strain increments ($d\varepsilon_x$ and $d\varepsilon_y$) by putting $\alpha=0$ (or $\alpha+\delta=0$ in the case of $d\varepsilon_x^r$ and $d\varepsilon_y^r$ due to "rotation of the principal stress axes"). The principal strain increments ($d\varepsilon_1$ and $d\varepsilon_3$) due to "shear", "rotation of the principal stress axes", and "anisotropic consolidation" and "isotropic consolidation" are expressed in a two-dimensional state as follows:

$$d\varepsilon_1^s = \frac{K}{K+R} \cdot k_s \cdot \frac{\sin^2 \phi \cdot \cos \phi_{m0}}{(\sin \phi - \sin \phi_{m0})^2} \cdot d\phi_{m0} \quad (27)$$

$$d\varepsilon_3^s = -\frac{R}{K+R} \cdot k_s \cdot \frac{\sin^2 \phi \cdot \cos \phi_{m0}}{(\sin \phi - \sin \phi_{m0})^2} \cdot d\phi_{m0} \quad (28)$$

$$d\varepsilon_1^r = \frac{K}{K+R} \cdot 2k_s \cdot \frac{\sin \phi \cdot \sin \phi_{m0}}{\sin \phi - \sin \phi_{m0}} \cdot d\alpha \quad (29)$$

$$d\varepsilon_3^r = -\frac{R}{K+R} \cdot 2k_s \cdot \frac{\sin \phi \cdot \sin \phi_{m0}}{\sin \phi - \sin \phi_{m0}} \cdot d\alpha \quad (30)$$

$$d\varepsilon_1^{ac} = \frac{K}{K+R} \cdot k_c \cdot \frac{\sin \phi \cdot \sin \phi_{m0}}{\sin \phi - \sin \phi_{m0}} \cdot \frac{d\sigma_m}{\sigma_m} \quad (31)$$

$$d\varepsilon_3^{ac} = -\frac{R}{K+R} \cdot k_c \cdot \frac{\sin \phi \cdot \sin \phi_{m0}}{\sin \phi - \sin \phi_{m0}} \cdot \frac{d\sigma_m}{\sigma_m} \quad (32)$$

$$d\varepsilon_1^{ic} = \frac{1}{2} \cdot \frac{0.434 C_c}{1 + e_0} \cdot \frac{d\sigma_m}{\sigma_m} \text{ or } \frac{1}{2} \cdot \frac{0.434 C_s}{1 + e_0} \cdot \frac{d\sigma_m}{\sigma_m} \quad (33)$$

Fig. 4. Directions of principal strain increments $d\epsilon_1^s$, $d\epsilon_1^{ac}$, $d\epsilon_1^{ic}$ and $d\epsilon_1^r$

$$d\epsilon_3^{ic} = \frac{1}{2} \cdot \frac{0.434 C_c}{1+e_0} \cdot \frac{d\sigma_m}{\sigma_m} \quad \text{or} \quad \frac{1}{2} \cdot \frac{0.434 C_s}{1+e_0} \cdot \frac{d\sigma_m}{\sigma_m} \quad (34)$$

where the superscript s , r , ac and ic mean the strain increments due to shear, rotation of the principal stress axes, anisotropic consolidation and isotropic consolidation. It should be noted that the direction of $d\epsilon_1^s$, $d\epsilon_1^{ac}$ and $d\epsilon_1^{ic}$ coincides with that of σ_1 , and the direction of $d\epsilon_1^r$ deviates by δ from that of σ_1 as shown in Fig. 4. Eqs. (27)~(34) are obtained from Eq. (14), but the principal strain increments can be also obtained by combining Eqs. (8), (9) and (10) with Eq. (20). They are expressed by λ and μ instead of K , though their equations are omitted here for simplicity. Eqs. (27) and (28) are easily integrated with respect to ϕ_{m0} as follows:

$$\epsilon_1^s = \frac{k_s \cdot \sin^2 \phi}{\{(K+1) - (K-1)\sin \phi\}} \cdot \left[\frac{K(1-\sin \phi)\sin \phi_{m0}}{\sin \phi(\sin \phi - \sin \phi_{m0})} + \frac{2K}{\{(K+1) - (K-1)\sin \phi\}} \cdot \log_e \frac{\sin \phi \{(K+1) - (K-1)\sin \phi_{m0}\}}{(K+1)(\sin \phi - \sin \phi_{m0})} \right] \quad (35)$$

$$\epsilon_3^s = - \frac{k_s \cdot \sin^2 \phi}{\{(K+1) - (K-1)\sin \phi\}} \cdot \left[\frac{(1+\sin \phi)\sin \phi_{m0}}{\sin \phi(\sin \phi - \sin \phi_{m0})} - \frac{2K}{\{(K+1) - (K-1)\sin \phi\}} \cdot \log_e \frac{\sin \phi \{(K+1) - (K-1)\sin \phi_{m0}\}}{(K+1)(\sin \phi - \sin \phi_{m0})} \right] \quad (36)$$

For Clays

The principal strain increments due to

“shear”, “rotation of the principal stress axes” and “isotropic consolidation” are the same as in the case of sands. However, the principal strain increments due to “anisotropic consolidation” are different from those in the case of sands, and are obtained from Eq. (25) as follows:

$$d\epsilon_1^{ac} = \frac{1}{2} \cdot k_c \cdot \frac{\sin \phi \cdot \sin \phi_{m0}}{\sin \phi - \sin \phi_{m0}} \cdot \frac{d\sigma_m}{\sigma_m} \quad (37)$$

$$d\epsilon_3^{ac} = -\frac{1}{2} \cdot k_c \cdot \frac{\sin \phi \cdot \sin \phi_{m0}}{\sin \phi - \sin \phi_{m0}} \cdot \frac{d\sigma_m}{\sigma_m} \quad (38)$$

It is interesting to see that if $R=K$, Eqs. (31) and (32) for sands coincide with Eqs. (37) and (38) for clays respectively.

STRAIN INCREMENTS UNDER VARIOUS STRESS CHANGES

It is important to determine whether the strain increments given by Eqs. (8), (9), (10) and (24) are caused or not under such various stress changes as shown in Fig. 5. The results of the determination are shown by the following equations with respect to the shear strain increment due to “shear” $d\gamma_{xy}^s$, the shear strain increment due to “anisotropic consolidation” $d\gamma_{xy}^{ac}$, the volumetric strain increment due to “isotropic consolidation” $d\epsilon_v^{ic}$ and the shear strain increment due to “principal stress rotation” $d\gamma_{xy}^r$.

$$\begin{aligned} & a) \quad d\gamma_{xy}^s (\text{Shear}) \\ & d\phi_{m0} \geq 0 (\text{I and II}) : d\gamma_{xy}^s > 0 (\text{Loading}) \end{aligned} \quad (39)$$

Fig. 5. Strain increments under various stress changes

$$d\phi_{m0} < 0 (\text{III and IV}) : d\gamma_{xy}^s < 0 (\text{Unloading}) \quad (40)$$

b) $d\gamma_{xy}^{ac}$ (Anisotropic Consolidation)

$$d\sigma_m \geq 0 \text{ and } d\phi_{m0} \geq 0 (\text{I}) : d\gamma_{xy}^{ac} > 0 \quad (41)$$

$$d\sigma_m \geq 0 \text{ and } d\phi_{m0} < 0 (\text{IV}) : d\gamma_{xy}^{ac} = 0 \quad (42)$$

$$d\sigma_m < 0 (\text{II and III}) : d\gamma_{xy}^{ac} = 0 \quad (43)$$

c) $d\varepsilon_v^{ic}$ (Isotropic Consolidation)

$$d\sigma_m \geq 0 (\text{I and IV}) : d\varepsilon_v^{ic} = \frac{0.434 C_c}{1+e_0} \cdot \frac{d\sigma_m}{\sigma_m} \quad (44)$$

$$d\sigma_m < 0 (\text{II and III}) : d\varepsilon_v^{ic} = \frac{0.434 C_s}{1+e_0} \cdot \frac{d\sigma_m}{\sigma_m} \quad (45)$$

d) $d\gamma_{xy}^r$ (Principal Stress Rotation)

$$d\alpha \neq 0 : d\gamma_{xy}^r \text{ is existent} \quad (46)$$

As the formulation of the constitutive model described here is not based on the theory of elasto-plasticity, the stress-strain relation under unloading conditions in Eq. (40) is freely determined (Matsuoka, Koyama and Yamazaki, 1985). If the dilatancy mechanism between particles is considered, $d\gamma_{xy}^{ac} = 0$ in Eq. (42) can be assumed in the region of $d\phi_{m0} < 0$. Sometimes the undrained effective stress paths for clays are calculated by using Eq. (44), so the regions applicable to Eqs. (44) and (45) may be flexible. Eq. (46) means that the strain increment due to rotation of the principal stress axes is also caused by the reverse rotation.

EXTENSION TO THREE DIMENSIONS

The constitutive relation mentioned here is a two-dimensional one. In order to extend it to three dimensions, the superposition of the "two-dimensional" principal strain increments is assumed as follows (Matsuoka, 1974, 1982).

$$\left. \begin{aligned} d\varepsilon_1 &= d\varepsilon_{1(12)} + d\varepsilon_{1(13)} \\ d\varepsilon_2 &= d\varepsilon_{2(12)} + d\varepsilon_{2(23)} \\ d\varepsilon_3 &= d\varepsilon_{3(23)} + d\varepsilon_{3(13)} \end{aligned} \right\} \quad (47)$$

where $d\varepsilon_{1(12)}$, for example, denotes the "two-dimensional" major principal strain increment caused under the two principal stresses σ_1 and σ_2 (see Fig. 6). The terms in the right

Fig. 6. Superposition of "two-dimensional" principal strain increments under respective pairs of principal stresses

Fig. 7. Angles between principal strain increments and x-axis

sides of Eq. (47) mean such "two-dimensional" principal strain increments as expressed by Eqs. (27)~(38). It should be noted here that the direction of $d\varepsilon_1^r$ deviates by δ from that of σ_1 as shown in Fig. 7. Therefore, after the superposition of the principal strain increments, $d\varepsilon_1^r$ should be transformed by the angle of $(\alpha + \delta)$ to the general coordinates (x-axis). Under triaxial compression and extension conditions, Eq. (47) is expressed by the following equations.

a) Under triaxial compression conditions

$$(\sigma_1 \geq \sigma_2 = \sigma_3, d\varepsilon_1 \geq d\varepsilon_2 = d\varepsilon_3) : \quad (48)$$

$$d\varepsilon_1 = 2 \cdot d\varepsilon_{1(13)} \therefore d\varepsilon_{1(13)} = d\varepsilon_1/2$$

$$d\varepsilon_2 = d\varepsilon_3 = d\varepsilon_{3(13)} + d\varepsilon_{3(33)} \quad (49)$$

b) Under triaxial extension conditions

$$(\sigma_1 = \sigma_2 \geq \sigma_3, d\varepsilon_1 = d\varepsilon_2 \geq d\varepsilon_3) : \quad (50)$$

$$d\varepsilon_1 = d\varepsilon_2 = d\varepsilon_{1(13)} + d\varepsilon_{1(11)}$$

$$d\varepsilon_3 = 2 \cdot d\varepsilon_{3(13)} \therefore d\varepsilon_{3(13)} = d\varepsilon_3/2 \quad (51)$$

It should be noted that $d\varepsilon_{3(33)}$ in Eq. (49) and $d\varepsilon_{1(11)}$ in Eq. (50) correspond to the case of $\phi_{m0} = 0$ under the same two principal stresses. Therefore, the following equation

for isotropic consolidation is obtained by considering Eqs. (27)~(38).

$$d\varepsilon_{3(33)} = d\varepsilon_{1(11)} = \frac{1}{6} \cdot \frac{0.434C_c}{1+e_0} \cdot \frac{d\sigma_m}{\sigma_m} \quad (52)$$

where the coefficient of $1/6$ in Eq. (52) is understood by taking the six terms in the right sides of Eq. (47) into account.

To obtain the "two-dimensional" principal strains from the result of a triaxial compression test, it is understood by Eq. (48) that the measured ε_1 should be divided by 2.

STRESS-STRAIN PARAMETERS

The stress-strain parameters in the proposed constitutive model are $k_s, k_c, K, \phi, C_c/(1+e_0)$ and $C_s/(1+e_0)$. λ and μ are used for K (see Eqs. (11) and (15)). ϕ denotes the internal friction angle in terms of effective stresses, C_c the compression index, C_s the swell index and e_0 the initial void ratio. The meaning and the method of determination of k_s, k_c and K are explained as follows.

a) k_s

k_s is a parameter to be found in Eqs. (8) and (9) or in Eq. (5) ($1/G_0 \equiv k_s$), which decides the magnitude of the shear strains due to shear and rotation of the principal stress axes. It is seen from Eq. (5) that k_s is the initial tangent of the relation between $(\varepsilon_1 - \varepsilon_3)$ and $(\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3) = \sin \phi_{m0}$ (in the case of $\alpha = 45^\circ$). k_s can be estimated from a conventional (drained or undrained) triaxial test on sands and clays.

b) k_c

k_c is a parameter in Eq. (10), which decides the magnitude of the shear strain due to anisotropic consolidation. It is seen from Eq. (10) that k_c is determined from the relation between $(\varepsilon_1 - \varepsilon_3)$ and $\log_{10} \sigma_m$ under anisotropic consolidation. However, k_c is easily estimated by the following equation which is derived from a microscopic study of the compression mechanism of granular materials (Matsuoka and Fujii, 1985).

$$k_c = 0.44 \frac{C_c}{1+e_0} \quad (53)$$

Eq. (53) is useful for the estimation of k_c

and it also gives the coefficient of earth pressure at rest K_0 as follows:

$$K_0 = \frac{1}{1+2\sin\phi} \quad (54)$$

Eq. (54) is similar to the following Jaky's equation.

$$K_0 = 1 - \sin\phi \quad (55)$$

Both Eqs. (54) and (55) indicate that $K_0 = 1$ at $\phi = 0^\circ$ and $K_0 = 0.5$ at $\phi = 30^\circ$.

c) K

K is a parameter in Eqs. (11) and (14), which determines the strain increment ratio. The following equation for K is given by Rowe (1962, 1971) as shown in Eq. (11).

$$K = \tan^2\left(45^\circ + \frac{\phi_f}{2}\right) \quad (56)$$

It is also seen from Eq. (11) that K is determined from the gradient of the linear relation between σ_1/σ_3 and $-d\varepsilon_3/d\varepsilon_1$ rearranged to a two-dimensional state (see Eq. (48)). We can assume $\phi_f = \phi$ for normally consolidated clays.

COMPARISON WITH VARIOUS KINDS OF TEST RESULTS

Two-Dimensional General Stress Apparatus

A general stress apparatus which can apply the general stresses (σ_x, σ_y and τ_{xy}) independently has been made for a stack of aluminium rods (mixture of $\phi 1.6$ mm and $\phi 3.0$ mm, 50 mm in length) as a two-dimensional model of granular materials. Photo. 1

Photo 1. Two-dimensional general stress apparatus for rod mass

Fig. 8. Main part of two-dimensional general stress apparatus for rod mass

and Fig. 8 show the main part of the two-dimensional general stress apparatus. The sample size is about $18\text{ cm} \times 18\text{ cm}$. The normal stress σ_x in the vertical direction and the normal stress σ_y in the horizontal direction are applied by the wires through pulleys, and the shear stress τ_{xy} is applied by the loading plate on the sample. These three stresses σ_x , σ_y and τ_{xy} can be measured by the load cells independently (see Fig. 7). The general strains ϵ_x , ϵ_y and γ_{xy} can be also measured by dial gauges independently. The mean principal stress $\sigma_m = (\sigma_1 + \sigma_3)/2$ is usually kept constant ($\sigma_m = 0.4\text{ kgf/cm}^2$; $1\text{ kgf/cm}^2 = 98\text{ kN/m}^2$) except for the simple shear tests. The initial void ratio of the sample of aluminium rod mass is about 0.23 for all tests.

Stress Ratio-Strain Increment Ratio Relations

Shear tests on planes which are at angles of α from the principal stress plane ($\alpha = 22.5^\circ$, 45° (pure shear), 67.5° and $45^\circ + \phi_{m0}/2$ (mobilized plane)), "rotation tests" along the Mohr's stress circles under constant principal stresses, and simple shear tests have been carried out by using the two-dimensional general stress apparatus. Fig. 9 shows the relationship between the principal stress ratio (σ_1/σ_3) and the principal strain increment ratio ($-d\epsilon_3/d\epsilon_1$) for all the test results mentioned above. It is interesting that all the test results arranged in the σ_1/σ_3 vs. $-d\epsilon_3/d\epsilon_1$

Fig. 9. Principal stress ratio vs. principal strain increment ratio relation for all tests on rod mass

$d\epsilon_1$ relation are plotted nearly on the same line, even if the direction of the principal stress deviates considerably from that of the principal strain increment. In fact, as shown in Fig. 12, the direction of the principal stress deviates by $\delta = 30^\circ$ from that of the principal strain increment in three kinds of the "rotation tests" along the Mohr's stress circles, indicated by three double circles in Fig. 9. Fig. 10 shows the relation between the shear-normal stress ratio (τ/σ_N) and the normal-shear strain increment ratio ($-d\epsilon_N/d\gamma$) on the mobilized plane for the same test results as in Fig. 9. The strain increments $d\epsilon_N$ and $d\gamma$ on the mobilized plane

Fig. 10. Stress ratio vs. strain increment ratio relation on mobilized plane for all tests on rod mass

are calculated by Eqs. (16) and (17). The shear-normal stress ratio τ/σ_N on the mobilized plane is expressed by the following equation.

$$\frac{\tau}{\sigma_N} = \tan \phi_{m0} = \frac{\sigma_1 - \sigma_3}{2\sqrt{\sigma_1 \sigma_3}} \quad (57)$$

As seen from Fig. 10, the τ/σ_N vs. $-\epsilon_N/d\gamma$

Fig. 11. Comparison between measured values of "radial tests" along the radii of a Mohr's stress circle on rod mass and calculated values

Fig. 12. Relation between angle α from principal stress direction and angle β from principal strain increment direction during "rotation tests" along the circumference of Mohr's stress circles on rod mass ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

relation on the mobilized plane also holds uniquely for all the test results. It can be understood from Figs. 9 and 10 that Eqs. (11) and (15) hold good for various kinds of test results. The solid line in Fig. 9 is obtained by transforming Eq. (15) on the mobilized plane to the relation on the principal stress plane ($\lambda=0.8$ and $\mu=0.19$).

Radial Tests of a Mohr's Stress Circle

Fig. 11 shows a comparison between the measured values from the radial tests in which the stress paths are the radii of a Mohr's stress circle ($\alpha=22.5^\circ$, 45° and 67.5°) and the calculated values by the proposed model. It is interesting that the stress-strain curves for the same sample are quite different for planes inclined at α to the principal plane.

Fig. 13. Comparison between measured values of "rotation tests" along the circumference of Mohr's stress circles on rod mass and calculated values ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

Rotation Tests along Mohr's Stress Circles

Fig. 12 shows the measured relationship between the angle α from the principal stress direction and the angle β from the principal strain increment direction during three kinds of rotation tests along the Mohr's stress circles. It is seen from this figure that the angle δ between the principal stress direction and the principal strain incre-

ment direction is nearly 30° for all the rotation tests.

Fig. 13(a) and (b) shows the comparison between the measured values of the rotation tests along the Mohr's stress circles with the fixed principal stresses and the calculated values. It is worth noting that the compressive volumetric strain ϵ_v is directly proportional to the rotation angle 2α .

Simple Shear Tests

Fig. 14(a), (b) and (c) shows the comparison between the measured values of a simple shear test on a stack of aluminium

Fig. 14. Comparison between measured values of simple shear test on rod mass and calculated values ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

Fig. 15. Comparison between measured values of simple shear test on Toyoura sand and calculated values ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

rods and the calculated values. The calculated values are obtained from the following conditions of no vertical stress increment and no lateral strain increment.

$$d\sigma_x = 0 \quad (\sigma_x = \text{const.}) \quad (58)$$

$$d\varepsilon_y = d\varepsilon_y^s + d\varepsilon_y^r + d\varepsilon_y^{ac} + d\varepsilon_y^{ic} = 0 \quad (59)$$

where $d\varepsilon_y^s$, $d\varepsilon_y^r$, $d\varepsilon_y^{ac}$ and $d\varepsilon_y^{ic}$ are the lateral strain increments due to shear, principal stress rotation, anisotropic consolidation and isotropic consolidation respectively. The broken lines in Fig. 14(a) mean the calculated values neglecting the lateral strain increment due to principal stress rotation ($d\varepsilon_y^r = 0$). Fig. 14(b) shows the measured and calculated values of the changes in stresses (σ_x , σ_y , σ_1 and σ_3) during simple shear. Fig. 14(c) also represents the measured and calculated values of the changes in angles of the principal axes (σ_1 , $d\sigma_1$ and $d\varepsilon_1$) during simple shear. Fig. 15(a), (b) and (c) shows the similar comparison between the measured values by a simple shear test on the Toyoura sand ($D_{50} = 0.2$ mm, $U_c = 1.3$, $G_s = 2.65$ and $e_0 = 0.66$) and the analytical results based on the proposed model. These analytical results are similar to the measured values by Roscoe et al. (1967).

Excess Pore Pressure Caused by Principal Stress Rotation

As compressive volumetric strains are produced during rotation of the principal stress axes as shown in Fig. 13, a positive excess pore pressure will be generated if the undrained condition is maintained during the rotation. We can analyze such excess pore pressure by the following undrained condition.

$$d\varepsilon_v = d\varepsilon_v^s + d\varepsilon_v^r + d\varepsilon_v^{ic} = 0 \quad (60)$$

where $d\varepsilon_v^s$, $d\varepsilon_v^r$ and $d\varepsilon_v^{ic}$ are the volumetric strain increments due to shear, principal stress rotation and isotropic consolidation (by Eq. (45)) respectively. Fig. 16 shows a comparison between the measured values and the calculated values. The measured values were obtained from a torsional shear test on a hollow-cylindrical Toyoura sand specimen (Ishihara and Towhata, 1983). Al-

Fig. 16. Comparison between measured values and calculated values of excess pore pressure caused by rotation of principal stress axes ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

though the torsional shear test does not satisfy the plane strain condition in the strict sense, there is a good agreement between the measured and calculated values.

Isotropic and Anisotropic Consolidation

Fig. 17 shows the measured values of isotropic and anisotropic consolidation tests on the Toyoura sand by triaxial apparatus (Nakai and Matsuoka, 1983), and the calculated values using Eqs. (31)~(34) and (53). It is seen from this figure that the dilatancy characteristics under anisotropic consolidation are evaluated by the proposed model. Fig. 18 represents the measured values of isotropic and anisotropic consolidation tests on the

Fig. 17. Comparison between measured values of isotropic and anisotropic consolidation tests on Toyoura sand and calculated values ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

Fig. 18. Comparison between measured values of isotropic and anisotropic consolidation tests on Fujinomori clay and calculated values

Fujinomori clay ($LL=41\%$, $PL=23\%$, $G_s=2.67$), normalized by the consolidation pressure σ_{m0} (Nakai and Matsuoka, 1986), and the calculated value using Eqs. (33), (34), (37) and (38).

Triaxial Compression and Extension Tests under Constant Mean, Major and Minor Principal Stresses

Fig. 19(a) and (b) shows the comparisons between the measured values from triaxial compression and extension tests on the Toyoura sand under a constant mean principal stress σ_m (Nakai and Matsuoka, 1983), and the calculated values by the proposed models. Fig. 20(a) and (b) shows the similar comparisons between the measured values under a constant major principal stress σ_1 (Nakai

Fig. 19. Comparison between measured values from triaxial compression and extension tests on Toyoura sand under a constant mean principal stress σ_m and calculated values ($1\text{ kgf/cm}^2=98\text{ kN/m}^2$)

Fig. 20. Comparison between measured values from triaxial compression and extension tests on Toyoura sand under a constant major principal stress σ_1 and calculated values ($1\text{ kgf/cm}^2=98\text{ kN/m}^2$)

Fig. 21. Comparison between measured values from triaxial compression and extension tests on Toyoura sand under a constant minor principal stress σ_3 and calculated values ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

and Matsuoka, 1983), and the calculated values. Fig. 21 (a) and (b) indicates the similar comparisons between the measured values under a constant minor principal stress σ_3 (Nakai and Matsuoka, 1983), and the calculated values. The experimental results from triaxial compression and extension tests under constant mean, major and minor principal stresses agree well with the calculated ones by the proposed model.

"Stress Probe" Tests in Triaxial Compression and Extension Conditions

Such "stress probe" tests on the Toyoura sand as shown in Fig. 22 have also been carried out in triaxial compression and extension conditions, using an automatically controlled loading and measuring system for

Fig. 22. Stress paths in "stress probe" tests under triaxial compression and extension conditions ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

Fig. 23. Comparison between measured strains from "stress probe" tests in triaxial compression conditions and calculated strains

triaxial apparatus (Horii, Nakai and Fujiwara, 1985). In this figure, $q=|\sigma_a-\sigma_r|$ and $p=(\sigma_a+2\sigma_r)/3$, in which σ_a denotes the axial principal stress and σ_r the radial principal stress. Fig. 23(a) and (b) shows the comparison between the measured strains from the "stress probe" tests in triaxial compres-

sion, and the calculated strains by the proposed model. Fig. 24(a) and (b) shows a similar comparison in triaxial extension conditions. In these figures, $\varepsilon_d=2(\varepsilon_a-\varepsilon_r)/3$ and $\varepsilon_v=\varepsilon_a+2\varepsilon_r$, in which ε_a denotes the axial principal strain and ε_r the radial principal strain. It is interesting to see that the calculated strains along paths OE and OF in which q/p decreases and path OH which would be inside a usual yield surface explain well the measured ones.

The stress-strain parameters used for these analyses are as follows. For the stack of aluminium rods: $k_s=0.31\%$, $K=1.86$, $\phi=24^\circ$, $C_c/(1+e_0)=1.2\%$ and $\delta=30^\circ$. For the Toyoura sand: $k_s=0.23\%$, $K=2.62$, $\phi=40^\circ$, $C_c/(1+e_0)=0.928\%$, $C_s/(1+e_0)=0.578\%$ and $\delta=30^\circ$. For the Fujinomori clay: $C_c/(1+e_0)=10.5\%$. For the analysis of the simple shear test on the Toyoura sand, $C_c/(1+e_0)=1.3\%$ is used because its value is obtained from the compression test using the same simple shear apparatus. We can use $\lambda=0.8$ and $\mu=0.19$ instead of $K=1.86$ for the stack of aluminium rods, and $\lambda=1.2$ and $\mu=0.20$ instead of $K=2.62$ for the Toyoura sand.

Fig. 24. Comparison between measured strains from "stress probe" tests in triaxial extension conditions and calculated strains

CONCLUSIONS

The main results of this paper are summarized as follows:

1) A constitutive model for sands and clays by which the general strain increments ($d\varepsilon_x$, $d\varepsilon_y$ and $d\gamma_{xy}$) are directly related to the general stress increments ($d\sigma_x$, $d\sigma_y$ and $d\tau_{xy}$) is proposed by combining the hyperbolic relationship between the general shear-normal stress ratio and the general shear strain with the "stress-dilatancy relation" expressed in general coordinates, which corresponds to the flow rule in the theory of plasticity. The shear strain increment $d\gamma_{xy}$ of the proposed model consists of those due to "shear", "principal stress rotation" and "anisotropic consolidation". In addition to these strain increments, the strain increments due to "isotropic consolidation" are also taken into account.

2) The constitutive model explains well

the results of shear tests on such planes as inclined at α to the principal stress plane, i.e. "radial tests" along the radii of a Mohr's stress circle.

3) The constitutive model also predicts the results of "rotation tests" along the circumference of Mohr's stress circles. The influence of rotation of the principal stress axes on strains is evaluated by the proposed model.

4) The stress-strain behaviour under simple shear and the excess pore pressure caused by rotation of the principal stress axes are also analyzed by the proposed model.

5) The isotropic and anisotropic consolidation tests on a sand and a clay using the triaxial apparatus are explained by the proposed model. The dilatancy characteristics of the sand during anisotropic consolidation are also evaluated.

6) The triaxial compression and extension tests on a sand under various kinds of stress paths, including "stress probe" tests, are also explained by the proposed model.

7) The stress-strain parameters in the proposed model are determined from a conventional triaxial test and an oedometer test.

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