EFFECTS OF INITIAL ANISOTROPIC FABRIC AND σ_2 ON STRENGTH AND DEFORMATION CHARACTERISTICS OF SAND

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ABSTRACT

A laboratory investigation has been carried out into the effects of initial anisotropic fabric and $b = (\sigma_2' - \sigma_3') / (\sigma_1' - \sigma_3')$ on the strength and deformation characteristics of air-pluviated Toyoura sand. Drained tests in triaxial compression ($b = 0.0$), in plane strain compression ($b \doteq 0.2 \sim 0.5$) and in triaxial extension ($b = 1.0$) were performed at different directions of principal stresses with respect to the deposition direction. The strength in triaxial extension was found to be strongly influenced by failure modes. The inherent anisotropy in strength and the effects of both the parameter b and the failure mode on strength are portrayed in the form of a three dimensional surface having the axes of ϕ , b and the angles ω and ξ ; ω and ξ represent the deposition direction with respect to the directions of σ_1' , σ_2' and σ_3' . It was found that ϕ is rather insensitive to the change in b when the one of the planes of the maximum stress obliquity is in parallel with the bedding plane. The deformation characteristics including the stress-dilatancy relation are discussed in a similar way as for ϕ .

Key words : anisotropic fabric, angle of internal friction, dilatancy, drained shear, intermediate principal stress, plane strain compression, sand, triaxial compression, triaxial extension (IGC : D 6)

INTRODUCTION

An investigation has been made on the combined effects of the initial anisotropic fabric and the intermediate principal stress σ_2' on the strength and deformation properties of air-pluviated sand.

The effects of the initial anisotropic fabric and σ_2' on strength and deformation characteristics of sands have been investigated ra-

ther separately. The inherent anisotropy in strength was investigated first in triaxial compression (TC) (Arthur and Menzies, 1972; Oda, 1972), and then in plane strain compression (Arthur and Assadi, 1977; Oda et al., 1978; Oda, 1981; Tatsuoka et al., 1986 a) and further in triaxial extension (Shankariah and Ramamurthy, 1980). In these studies, specimens were cross-anisotropic, prepared by pluviating sand particles through

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Manuscript was received for review on July 9, 1987.

Written discussions on this paper should be submitted before October 1, 1988, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

Fig. 1. Definition of the directions of material axes with respect to the directions of σ_1' , σ_2' and σ_3'

air or water. In this case, the deposition direction, denoted as the n axis in Fig. 1, is the axis of cross-anisotropy. Then, the directions of the n axis can be defined by the two angles ω and ξ among others. In TC, the angle ξ is not needed to be defined since $\sigma_2' = \sigma_3'$, but only ω is needed. In triaxial extension (TE), the direction of the n axis can be defined by a single angle $\theta = \arcsin \{\sin \omega \cdot \sin \xi\}$ from the σ_1' - σ_2' plane since $\sigma_1' = \sigma_2'$. The angle θ is equal to ξ at $\omega = 90^\circ$ or ω at $\xi = 90^\circ$. When $0 < b < 1$, both ω and ξ are needed to define the direction of the n axis. So far, the strength anisotropy in plane strain compression has been investigated only at $\xi = 90^\circ$.

On the other hand, the effects of σ_2' have been studied in terms of the relationship between $\phi = \arcsin \{(\sigma_1' - \sigma_3') / (\sigma_1' + \sigma_3')\}_{\max}$ and $b = (\sigma_2' - \sigma_3') / (\sigma_1' - \sigma_3')$ by using cross-anisotropic cubic or prismatic specimens (Ko and Scott, 1968; Lade and Duncan, 1973; Reades and Green, 1976; Miyamori, 1976; Yamada and Ishihara, 1979; Goldsheider, 1982; Ochiai and Lade, 1983 among others). In these studies, the principal stress directions were fixed during shearing and aligned with the material axes; i.e., (1) $\omega = 0^\circ$, or (2) $\omega = 90^\circ$ and $\xi = 0^\circ$, or (3) $\omega = 90^\circ$ and $\xi = 90^\circ$. The results have been plotted on a normalized π -plane such as the one shown in Fig. 2.

The research works where the effects of b , ω and ξ were studied in a combined manner are very limited. Among them, Hight

Fig. 2. ϕ of air-pluviated Toyoura sand on the normalized π -plane

- σ_1' : VERTICAL STRESS IN TEST
- σ_c' : CONSTANT CONFINING STRESS IN TEST
- R: LUBRICATED RIGID BOUNDARY
- F: FLEXIBLE BOUNDARY
- TC: TRIAXIAL COMPRESSION
- TE: TRIAXIAL EXTENSION
- PSC: PLANE STRAIN COMPRESSION

Fig. 3. Descriptions of specimens

et al. (1987) obtained ϕ of dense Ham river sand as a function of both $b=0\sim 1.0$ and $\omega=0\sim 90^\circ$ at $\xi=90^\circ$ by means of a hollow cylinder specimen. In this study, due to too large stress non-uniformity across the specimen wall, ϕ was not obtained for area near the points ($b=0.0$, $\omega=90^\circ$) and ($b=1.0$, $\omega=0^\circ$).

TEST PROGRAM

In view of the above, a series of drained shear tests was performed to obtain a better picture of the strength and deformation char-

acteristics of air-pluviated Toyoura sand as a function of (b, ω, ξ). In the present study, only rather conventional testing methods were employed; i.e., the triaxial compression test (TC), the plane strain compression test (PSC), and the triaxial extension test (TE), all by means of a conventional triaxial cell. The reasons are as follows:

(1) It was considered that an overall picture concerning the ϕ - b relation for $b=0\sim 1.0$ for a fixed combination of (ω, ξ) may be obtained from the values of ϕ measured only at $b=0.0$ (TC), $b\approx 0.3$ (PSC) and $b=$

Table 1. List of tests performed by the present study

	$b^{1)}$	$\omega^{2)}$ (deg)	$\xi^{2)}$ (deg)	$\sigma_e'^{3)}$ (kgf/cm ²)	Sample size $H \times W \times L^{4)}$ (mm)	H/W	End Lubri- cation ⁵⁾
Triaxial Compression (TC-3) ⁶⁾	0.0	0, 90	— ⁷⁾	$=\sigma_3'$ $=1.0$	160×78×78	2.0	Type 4
		0, 30, 60, 90			78×78×78	1.0	
		90			40×78×78	0.5	
		0, 30, 60, 90			20×78×78	0.25	
Plane Strain Compression (PSC-3) ⁶⁾	0.2 ~ 0.55	0, 30, 60, 90	90 ⁸⁾	$=\sigma_3'$ $=1.0$	105×55×105	1.9	Type 4
		0, 90			55×55×105	1.0	
		0, 30, 60, 90			27.5×55×105	0.5	
		0, 30, 60, 90			13.8×55×105	0.25	
		0, 30, 60, 90	0		105×55×105	1.9	
		0, 90			55×55×105	1.0	
		0, 30, 60, 90			27.5×55×105	0.5	
		0, 30, 60, 90			13.8×55×105	0.25	
		90	0, 30, 60, 90		105×55×105	1.9	
			0, 90		55×55×105	1.0	
			0, 30, 60, 90		27.5×55×105	0.5	
			0, 30, 60, 90		13.8×55×105	0.25	
Triaxial Extension (TE)	1.0	$\theta=0^\circ, 30^\circ,$ 60° and 90° ⁹⁾		$=\sigma_1'$ $=4.0$	160×78×78	2.0	Type 1
					78×78×78	1.0	
					20×78×78	0.25	
		$\theta=90^\circ$ ⁹⁾			150H×70D ¹⁰⁾	2.1	

NOTE: 1) $=(\sigma_2'-\sigma_3')/(\sigma_1'-\sigma_3')$. 2) see Fig. 1.

3) effective cell pressure (1.0 kgf/cm²=98 kN/m²). 4) see Fig. 3;

5) latex disks smeared with Dow grease were used; (Type 1) at each end, a grease layer is placed between a rigid end surface and a latex disk in contact with sand. (Type 4) at each end, a grease layer is placed between two latex disks of which one is in contact with a rigid end surface and the other is in contact with sand (Tatsuoka et al., 1984),

6) numbered by the sequence of tests performed at the authors' laboratory.

7) not needed to be defined.

8) at $\xi=90^\circ$, $90^\circ-\omega=\delta$ defined in Tatsuoka et al. (1986 a and b).

9) $\theta=\arcsin \{\sin \omega \cdot \sin \xi\} = " \omega "$ at $\xi=90^\circ = " \xi "$ at $\omega=90^\circ$.

10) cylindrical only in this case; D =the diameter of specimen.

1.0 (TE).

(2) From the simplicity of the conventional triaxial apparatus, rather reliable stresses can be measured more easily when compared to much more sophisticated apparatuses which have been used to obtain continuous ϕ - b relations for $b=0\sim 1.0$.

Consequently, the boundary condition of specimen was mixed, consisting of lubricated rigid planes in one direction and flexible planes of membranes in the other two directions in TC and TE, or lubricated rigid planes in two directions and flexible planes of membrane in the remaining one direction in PSC (Fig. 3). As discussed later, this factor should be taken into account when the results are compared with those obtained by using other boundary conditions; i. e., all planes are either rigid or flexible, or different combinations of rigid and flexible boundaries are used.

Specimens at different combinations of ω and ξ were prepared by pluviating air-dried sand particles at prescribed values of ω and ξ (θ in TE) (see Table 1). Specimens were prismatic having square cross-sections except one kind of TE using cylindrical specimens. The combination of the two angles employed were only (1) $\xi=0\sim 90^\circ$ at $\omega=90^\circ$, (2) $\omega=0\sim 90^\circ$ at $\xi=90^\circ$, and (3) $\omega=0\sim 90^\circ$ at $\xi=0^\circ$ (see Fig. 3). The cases when both ω and ξ were neither 0° nor 90° were not investigated.

TESTING METHODS

Toyoura sand is a uniform sand with a high content of feldspar, consisting of subangular particles with $D_{50}=0.16$ mm, $U_c=1.46$, $G_s=2.64$ and $e_{\max}=0.977$ and $e_{\min}=0.605$.

The stress conditions of specimens when ω and ξ are either 0° or 90° can be represented by the angle $\alpha=\arctan [\sqrt{3}(\sigma_x-\sigma_z)/\{(\sigma_y-\sigma_x)+(\sigma_y-\sigma_z)\}]$ on a normalized π -plane of constant $(\sigma_x'+\sigma_y'+\sigma_z')$ in the $\sigma_x'-\sigma_y'-\sigma_z'$ space as shown in Fig. 2. In this figure, σ_y' is the principal stress in the deposition direction and σ_x' and σ_z' are the other two principal stresses normal to the deposition direction. Each of σ_x' , σ_y' and σ_z' may be one of the major, intermediate and minor

principal stresses, σ_1' , σ_2' and σ_3' .

When either ω or ξ or both are neither 0° nor 90° , stress conditions of specimen cannot be represented on the normalized π -plane shown in Fig. 2. Fig. 3 illustrates specimens in this case, which is the case examined in the present study. In TC, the deposition direction was in parallel with one of the two planes on which σ_3' was applied (σ_3' -plane) whereas it had an angle ω from the other σ_3' -plane. In PSC, only three different cases were examined. In each case, the deposition direction was in parallel with one principal stress plane. In TE, the deposition direction was in parallel with one of two planes on which σ_1' was applied (σ_1' -plane) whereas it had an angle θ from the other σ_1' -plane (see also Fig. 1).

As shown in Fig. 3, the top and bottom ends were in contact with lubricated, rigid planes and all the lateral faces were covered with a latex membrane (thickness=0.3 mm). σ_v' and σ_c' are, respectively, the stress working on the rigid end boundaries and the cell pressure working on the flexible lateral surfaces during loading a specimen in the triaxial cell. In PSC, the σ_2' -planes were in contact with lubricated, rigid planes of confining platens. The height of specimen H is defined as the distance between the top and bottom rigid end planes and the width W is defined as the distance between the flexible lateral faces in contact with the cell water.

A wide range of H/W was examined by the following reasons. First, it was intended to examine whether a large degree of strength anisotropy can be observed still in very short (i. e., very small H/W) specimens as well as in long specimens in both TC and PSC. When the present study was started, it was considered that in a short specimen of Toyoura sand failure may occur at larger strains and therefore, the strength may be affected only slightly by the initial cross-anisotropic fabric as reported by Ochiai and Lade (1983). Secondly, it has been shown by Barden and Khayatt (1966) and Barden and Procter (1971) and Lade (1982) that the triaxial extension strength can be largely

affected by the dimensions of specimen. Therefore, it was intended to examine whether a large degree of strength anisotropy can be observed also in triaxial extension for a wide range of H/W , although the strength may be largely affected by H/W .

The technique described by Tatsuoka et al. (1986 a, b), Lam (1986) and Lam and Tatsuoka (1988) was used for preparing such specimens as illustrated in Fig. 3. After the pluviation of sand particles, a sequence of moistening and freezing was applied to the specimen, and then it was thawed under a partial vacuum after having been set in a triaxial cell as illustrated in Fig. 3. Effects of this sequence on the strength and deformation characteristics of Toyoura sand have been found negligible (Tatsuoka et al., 1986 b).

Lubrication of rigid planes were achieved by means of latex disk(s) with a thickness of 0.3 mm smeared with a layer of Dow grease with a thickness of 50 μm (refer to Tatsuoka et al., 1984 for further details).

For TC tests and PSC tests, specimens were isotropically consolidated to 1.0 kgf/cm² (98 kN/m²) and then σ_1' was increased at an axial strain rate of 0.25%/min. at a constant $\sigma_e' = \sigma_3'$. For TE tests, specimens were isotropically consolidated to 4.0 kgf/cm² (392 kN/m²) and then σ_3' was decreased at an axial strain rate of -0.25%/min. at a constant $\sigma_e' = \sigma_1'$. The value of σ_1' of 4.0 kgf/cm² was selected so that σ_3' at failure be close to 1.0 kgf/cm².

In PSC, confining plates equipped with a load cell to measure σ_2' were set to a specimen isotropically vacuumed at -0.8 kgf/cm² with an appropriate seating stress in order to compensate a slight length reduction of specimen during isotropic compression from $\sigma_3' = 0.8$ to 1.0 kgf/cm². In TE, an inner load cell and a differential pressure transducer were utilized to measure the axial load and the effective confining pressure $\sigma_e' = \sigma_1'$ in order to obtain very accurate values of ϕ (refer to Lam, 1986; Tatsuoka, 1988 b for further details). The other testing methods are the same as the previous studies performed at the authors' laboratory, which are listed in Table 2.

For all the tests, corrections for the end restraint were not applied to stress values. (note that presented in Fig. 8 (a) of Tatsuoka (1987) are the values of ϕ by TC-3 for $H/W = 1.0$ after the correction for the end friction, which are about one degree smaller than those presented in this paper). However, it is very likely that for results for $H/W = 1.0 \sim 2.0$ in TC and PSC, which were mainly discussed in this paper, and for all the data in TE, these corrections are not large. Corrections of stresses for membrane forces were found negligible for all the data.

TRIAXIAL COMPRESSION TESTS

The values of ϕ have been summarized in Fig. 4. For $H/W = 2.0$, only average curves

Table 2. Other similar tests on air-pluviated Toyoura sand, quoted in this paper

	b	ω (deg)	ξ (deg)	$\sigma_e' = \sigma_3' =$ (kgf/cm ²)	Sample size $H \times D$ or $H \times W \times L$ (mm)	References
TC-1	0	0	—	0.029~4.0	Cylindrical; 150 $H \times 70 D$	Fukushima and Tatsuoka (1984)
TC-2	0	90	—	1.0	80 $\times$ 43 $\times$ 80	Tatsuoka et al. (1986 a)
PSC-1	0.16~0.4 ¹⁾	0, 23, 45, 56, 67, 79, 90	90	0.05~4.0	105 $\times$ 40 $\times$ 80	Tatsuoka et al. (1986 a)
PSC-2 ²⁾	0.21~0.26	0	—	$K_0 \times 2.0$	80 $\times$ 80 $\times$ 43	Tatsuoka et al. (1986 a)
PSC-4 ³⁾	0.25~0.3	0	—	0.5	a. 105 $\times$ 40 $\times$ 80 b. 200 $\times$ 80 $\times$ 160	Nakamura (1987)

1) measured only at $\sigma_3' = 0.05$ kgf/cm² (4.9 kN/m²).

2) K_0 -consolidated only in this test.

3) air-dried specimens only in this test.

are presented for clarity (refer to Lam and Tatsuoka, 1988). It may be seen that even in very short specimens of $H/W=0.25$ the strength has been strongly affected by the initial anisotropic fabric. This point may be seen also in Fig. 5. Note that very accurate comparisons cannot be made among the data shown in Fig. 5 since the void ratios are slightly different.

Furthermore, it may be seen in Fig. 6 that axial strains at failure $(\epsilon_1)_f$ also depend on the initial anisotropic fabric. It is to be noted that strain values presented in Figs. 5 and 6 have not been corrected for bedding error at the lubrication layers. It is also to be noted that in Fig. 5 $\epsilon_1 - \epsilon_3$ has been obtained as $\epsilon_1 - (\epsilon_v - \epsilon_1)/2$ (ϵ_v ; volumetric strain).

Fig. 4. ϕ in TC for $H/W=0.25, 1.0$ and 2.0

Fig. 5. Stress-strain characteristics of dense specimens in TC for (a) $H/W=1.0$ and (b) $H/W=0.25$

Fig. 6. Axial strains at failure in TC

This calculation method is correct only when two principal strains ϵ_2 and ϵ_3 are the same, whereas it is not the case for ω other than 0° . In spite of the above two factors, it can be considered that in each of Figs. 5 and 6 the results reflect the comparative variation which have been created by the initial cross-anisotropic fabric.

On the other hand, Ochiai and Lade (1983) observed a large degree of initial anisotropy in the fabric by directly measuring grain shape distributions. However, they observed a difference in ϕ of only about 1.5° in TC between $\omega=0^\circ$ and $\omega=90^\circ$ for cubic dense ($D_r=90\sim93\%$) specimens of $H/W=1.0$ having a mixed boundary as used in the present study (rigid σ'_1 -planes and flexible σ'_3 -planes). This difference is smaller when compared to about 4° which was observed in the present study for dense specimens of $H/W=1.0$. This smaller degree of strength anisotropy as they have observed may be related partly to a larger particle size ranging between 0.84 and 2.0 mm as compared to the specimen size; $H=76$ mm (i.e., $D_{50}/H=0.011\sim0.026$). On the other hand, in the present study D_{50}/H was only 0.008 even for $H/W=0.25$. It may be conceivable that for a larger D_{50}/H with rigid top and bottom ends, strain localization is restrained to a larger extent, thereby failure occurs at large-

er strains associating larger changes in the fabric.

It may be seen in Fig. 4 that while ϕ is similar between $H/W=1.0$ and 2.0 , ϕ for $H/W=0.25$ is larger than those for $H/W=1.0$ and 2.0 especially for looser specimens. This tendency has been observed also for cylindrical specimens, but to a lesser degree (Goto and Tatsuoka, 1988). This behavior may be due partly to a larger effect of end restraint for smaller H/W and partly to unknown mechanism associated with the changes in failure mechanism as H/W decreases. Further research works are needed to clarify this point.

Fig. 7 shows plots of the stress-dilatancy relation at failure $R_{\max}=K \cdot D_f$ where $R_{\max}=(\sigma'_1/\sigma'_3)_{\max}$ and $D_f=(1-d\varepsilon_v/d\varepsilon_1)_f=-(2 d\varepsilon_3/d\varepsilon_1)_f$ at $R_{\max}$. In Fig. 7, two hatched zones show the ranges of data by the tests PSC-1 at $\omega=0^\circ$ and TC-1 at $\omega=0^\circ$, respectively (see Table 2). The broken line is the lower bound in PSC-1 found at $\omega=56^\circ\sim 67^\circ$ (Tatsuoka et al., 1986 a; the angle δ used in this previous paper is equal to $90^\circ-\omega$ at $\xi=90^\circ$). It may be seen that the results by the present study as denoted by TC-3 are consistent with the previous results as denoted by TC-1 and TC-2. It has been found that in PSC at $\xi=90^\circ$, K is largely dependent on the angle ω (Tatsuoka et al., 1986 a). It may be seen

from Fig. 7 that also in TC, the ratio K is dependent on the initial cross-anisotropic fabric as in PSC, but to a much lesser extent. It is very interesting to note that at $\omega=0^\circ$ the relation is very different between TC and PSC, whereas it is much more similar at $\omega\approx 60^\circ$.

PLANE STRAIN COMPRESSION TESTS

It may be seen by comparing Figs. 4(b) and 8 that the effects of H/W on ϕ are larger in PSC than in TC. This is due partly to that b at failure increased as H/W decreased especially for $H/W\leq 0.5$. As seen from Fig. 9, the overall stress-strain relations are also influenced to a large extent by H/W ; i. e., as H/W decreases, strains and b at failure increase (see also Fig. 10).

It has been shown by Nakamura (1987) that difference in stress-strain relations among different H/W as shown in Figs. 9 and 10 is due only partly to the bedding error. It was also reported by Nakamura that the bedding error for Type 1 at failure in similar PSC tests as the present study was of the order of $0.3\sim 0.4$ mm. The bedding error for Type 4 can be considered to be similar to that for Type 1. In fact, if the differences in $(\varepsilon_1)_f$ seen in Fig. 10 are due totally to the bedding error, it amounts to of the order of $1\sim 2$ mm. This value is too large as compared to both the values estimated by Nakamura and the initial total thickness of lubrication layers of Type 4 at the two ends; i. e., 1.3 mm.

On the other hand, it is reasonable to consider that the difference in b among different H/W cannot be explained by the bedding error. This difference is probably due to different degrees of uniformity of deformation among different H/W . It is likely that at a larger H/W a shear band(s) in parallel with the σ'_2 -direction is formed more easily and consequently the deformation becomes more two-dimensional, resulting in a smaller value of σ'_2 (or b). The values of ϕ and b at failure for $e_{0.3}=0.70$ in each case were read from the average curves presented in

Fig. 7. Stress-dilatancy plots at failure in TC as compared with those in PSC

Fig. 8. Effects of H/W on ϕ in PSC

Fig. 9. Stress-strain characteristics of dense specimens in PSC at $\omega=0^\circ$ for different values of H/W

Fig. 8 and were plotted in Fig. 11. It may be seen that the increase in ϕ with the decrease in H/W from 1.9 to 0.25 for an identical value of b between 0.2 and 0.5 is of the order of 1~2 degrees, which is comparable with those in TC.

Fig. 10. Axial strains at failure in PSC at $\omega=0^\circ$ for different values of H/W

Since, in each of TC and PSC, ϕ is similar between $H/W=1.0$ and $1.9\sim 2.0$, ϕ for $H/W=1.0$ in TC and ϕ for $H/W=1.9$ in PSC will be compared in the later part.

Fig. 8(d) shows the values of b at failure for $\omega=0^\circ$ at various values of σ_3' ranging between 0.05 and 1.0 kgf/cm² for $H/W=1.9\sim 2.6$. This result suggests that the values of b at failure is rather independent of σ_3' .

Figs. 12(a) and (b) show the values of ϕ and b at failure for various combinations of the angles ω and ξ for $H/W=1.9$ and 0.5, respectively. Note that the stress condition is the same when $\omega=0^\circ$ and $\xi=0^\circ$ and when $\omega=0^\circ$ and $\xi=90^\circ$. Large effects of the initial anisotropic fabric on both ϕ and b at failure may be seen. This point may also be seen by comparing the data shown in Figs. 8(a), (b) and (c). It is to be noted that even in short specimens of as small as $H/W=0.5$ a large degree of strength anisotropy as for long specimens of $H/W=1.9$ can be appreciated. It may be seen also from Fig. 8 that even for $H/W=0.25$, ϕ for dense specimens at $\omega=0^\circ$ is much larger than that at $\omega=90^\circ$ and $\xi=90^\circ$. As compared to such a large degree of strength anisotropy as described above, Ochiai and Lade (1983) reported that for dense specimens in PSC the difference in ϕ was only 2.4° between the case of $\omega=0^\circ$ and the case of $\omega=90^\circ$ and $\xi=90^\circ$. They used a cubic sample having a mixed boundary as used in the present study (rigid σ_1' - and σ_2' -planes

Fig. 11. ϕ and b relations for different values of H/W in TC and PSC

Fig. 12. ϕ and b at failure in PSC for (a) $H/W=1.9$ and (b) $H/W=0.5$

and flexible σ_3' -planes). As discussed before, further research works are required to clarify this inconsistency.

On the other hand, the results presented in Figs. 8 and 12 show that the strength anisotropy in loose specimens becomes less clear as H/W decreases.

As shown in Figs. 13 and 14, the effects of the initial anisotropic fabric on the stress-strain relations were considerable. The largest variation in the behavior can be seen when ω changes from 0° to 90° at $\xi=90^\circ$. At $\omega=0^\circ$, ϕ and the initial rigidity are the largest and strains at failure are the smallest, whereas, when $\omega=60^\circ$ and $\xi=90^\circ$, ϕ is the smallest among the data obtained by the present study. However, this value of ϕ

Fig. 13. Stress-strain characteristics of dense specimens in PSC for $H/W=1.9$ at (a) $\xi=90^\circ$, (b) $\xi=0^\circ$ and (c) $\omega=90^\circ$

may not be a possible lower bound, since smaller values have been observed when $\omega=56^\circ$ and $\xi=90^\circ$ by Tatsuoka et al. (1986 a) (also refer to Oda et al., 1978, 1981). On the other hand, it may be seen from Fig. 14 that largest strains at failure were not observed when $\omega=60^\circ$ and $\xi=90^\circ$, but observed when $\omega=90^\circ$ and $\xi=90^\circ$. Similar behavior has been observed also by Tatsuoka et al. (1986 a).

It may be seen from Fig. 12 that the value of b at failure is the largest when ξ is $0^\circ \sim 30^\circ$ at $\omega=90^\circ$ (i.e., when the bedding plane is normal to, or almost normal to, the σ_2' -axis), and it is the smallest when ξ is 90° (i.e., when the bedding plane is in parallel with the σ_2' -axis). This behavior may be related to the anisotropic deformation properties as has been theoretically discussed by Tatsuoka (1980).

Fig. 14. Axial strains at failure in PSC for $H/W=1.9$ for different combinations of ω and ξ

The initial anisotropic fabric influences also the stress-dilatancy relation at failure (Fig. 15). The largest variation in the ratio $K = R_{\max}/D_f$ ($R_{\max} = (\sigma_1'/\sigma_3')_{\max}$ and $D_f = (1 - d\varepsilon_v/d\varepsilon_1)_f$ at $R_{\max}$) was observed for $\omega=0^\circ \sim 90^\circ$ at $\xi=90^\circ$. The smallest K was observed when $\omega=60^\circ$ and $\xi=90^\circ$. This result, as denoted by PSC-3 in Fig. 15(a), is consistent with that by Tatsuoka et al. (1986a) which are denoted as PSC-1. It may be seen from Fig. 15 that the variation in K for $\omega=0^\circ \sim 90^\circ$ at $\xi=0^\circ$ and for $\xi=0^\circ \sim 90^\circ$ at $\omega=90^\circ$ is much less than that for $\omega=0^\circ \sim 90^\circ$ at $\xi=90^\circ$.

TRIAXIAL EXTENSION TESTS

In TE with $H/W=0.25 \sim 2.0$, the following three different failure modes were observed (see Fig. 16(d)):

(a) A shear band develops through two opposite flexible σ_1' -planes without intersecting with the rigid σ_3' -planes. This was observed only for $H/W=2.0$, and will be called 'No intersection' failure mode.

(b) A shear band intersects with either the top or bottom rigid σ_3' -plane. This was observed for $H/W=1.0$ and 2.0 , and will be called 'Single intersection' failure mode.

(c) A shear band intersects simultaneous-

Fig. 15. Stress-dilatancy plots at failure in PSC as compared with those in TC

ly with the top and bottom rigid σ_3' -planes. This was observed only for $H/W=0.25$, and will be called 'Double intersection' failure mode.

The values of ϕ for the three failure modes were plotted separately in Fig. 16. For $H/W=1.0$ and 2.0 , the tendency of necking was observed before the peak stress condition in most tests. Therefore, the values of ϕ were calculated from the deviator stress defined for the smallest cross-sections of specimen at failure, which was measured by the photogrammetric method (Lam, 1986). For some tests this correction of ϕ was found negligible. Furthermore, the value of σ_3' for each triaxial extension test deviated at a different degree from 1.0 kgf/cm^2 (98 kN/m^2). Therefore, the values of ϕ have been corrected to those $\sigma_3'=1.0 \text{ kgf/cm}^2$ in such that the correction is $\Delta\phi = 10.0 (1-e) \log \sigma_3'/1.0$ (in degree) in which e is the void ratio and σ_3' is in kgf/cm^2 . This equation was obtained from the dependency of ϕ on σ_3' observed in the plane strain compression tests at $\omega=0^\circ$, PSC-1. The largest correction $\Delta\phi$ was only -1.1° for a dense specimen at $\theta=0^\circ$. In the previous paper (Lam and Tatsuoka, 1988), the values ϕ for $H/W=0.25$ at $\theta=0^\circ$ and 90° have been reported, which had been corrected for the local area reduction as well as for $H/W=1.0$ and 2.0 . However,

Fig. 16. (a), (b) and (c) ϕ in TE for three failure modes, (d) schematic diagrams of three failure modes (two figures in each case show two σ'_2 -planes at a right angle; not in scale)

in this paper the values of ϕ for $H/W=0.25$ have not been corrected for the local area reduction, since in some tests at $\theta=30^\circ$ and 60° , the local cross-sectional area could not be well defined. Furthermore, in the paper by Lam and Tatsuoka (1988) the values of ϕ for all the values of H/W had not been corrected to those at $\sigma'_3=1.0 \text{ kgf/cm}^2$.

The effects of failure mode on ϕ was found very large. It is to be noted that for Single intersection failure mode ϕ was very similar between $H/W=1.0$ and 2.0 (Fig. 16 (b)). On the other hand, strains at failure seem to be controlled by H/W (as seen in Figs. 17 and 18). It may be seen from these figures that strains at failure increase as H/W decreases, since the deformation is more uniform as H/W decreases.

Since the stress-dilatancy relations at failure were found to be controlled by failure

Fig. 17. Stress-strain characteristics of dense specimens in TE for different failure modes

mode, these were plotted separately for each failure mode as shown in Fig. 19. In Fig. 19, both $R_{\max}$ and $D_f = (1 - d\epsilon_v/2 d\epsilon_1)_f = -(d\epsilon_3/2 d\epsilon_1)_f$ at $R_{\max}$ have not been corrected to those at $\sigma'_3=1.0 \text{ kgf/cm}^2$ (98 kN/m²). In the later part, after correcting $R_{\max}$ and D_f to those at $\sigma'_3=1.0 \text{ kgf/cm}^2$, the stress-dilatancy plots for TE will be compared with those for TC and PSC at $\sigma'_3=1.0 \text{ kgf/cm}^2$ (see Fig. 25). It seems that such plots as shown in Fig. 19 should be treated with caution, since local values of $R_{\max}$ and D_f within a specimen may be largely different from the averaged values shown in Fig. 19, owing to a large degree of nonuniform deformation as discussed below.

Smaller strains at failure associating a lower strength value for a larger H/W as observed in these TE tests seem to suggest that a triaxial extension specimen having a larger H/W , especially $H/W=2.0$, exhibits a prema-

Fig. 18. Strains at failure in TE for different values of H/W

ture failure (Barden and Khayatt, 1966; Barden and Proctor, 1971; Lade, 1982). Fig. 20 shows strain fields observed on the σ_1' -plane of a TE specimen obtained by the photogrammetric method. Strain value at the center of each $5\text{ mm} \times 5\text{ mm}$ rectangular element was calculated from the displacements measured at the four nodes assuming linear change of strain in each element. The length of the side of each square seen in Fig. 20 is proportional to the magnitude of ε_1 - ε_3 . Each square is located at the center of each element at either (a) $\varepsilon_1 = 0\%$ or (b) $\varepsilon_1 = -4\%$. It may be seen that the deformation has already been remarkably localized at $\varepsilon_3 = -4\%$, which was before the peak stress condition. This strain localization has been called "necking". This strain localization was accelerated between two stages where $\varepsilon_3 = -4$ and -5% , between which the peak average stress ratio σ_1'/σ_3' was attained.

It has not been well understood yet whether such large effects of H/W on failure mode,

Fig. 19. Stress-dilatancy plots at failure in TE for different failure modes as compared to those in TC and PSC

ϕ and stress-strain behavior in TE as described above are due to that the σ_1' -planes are flexible or that the stress condition is such as $b = 1.0$ or both. Nevertheless, it seems that a large variation near and at $b = 1.0$ in the $\phi \sim b$ relations which can be found in the literature is due largely to that in different laboratories such different failure modes as observed in this study were caused by different specimen dimensions or different bound-

Fig. 20. Distribution of $\epsilon_1 - \epsilon_3$ ($e_{0.3} = 0.668$, $\theta = 90^\circ$, $H/W = 2.0$, see Fig. 17 (d))

ary conditions (i. e., boundaries are all rigid or all flexible or mixed) or both. This point will be discussed again later.

While there is a large degree of uncertainty about the mechanism which controls the strength and deformation characteristics of sand in TE, the effects of the initial anisotropic fabric are obvious. It may be seen from Fig. 16 that in each failure mode ϕ is the largest at $\theta = 0^\circ$ and the smallest at $\theta = 60^\circ$ (i. e., $\omega = 60^\circ$ at $\xi = 90^\circ$) not at $\theta = 90^\circ$ (i. e., $\omega = 90^\circ$ at $\xi = 90^\circ$). This is a very much similar tendency as in PSC. It may be seen

further from Figs. 17 and 18 that strains at failure are the smallest at $\theta = 0^\circ$ as observed at $\omega = 0^\circ$ in TC and PSC. In addition, it may be seen from Fig. 19 that for each failure mode the ratio $K = R_{\max}/D_f$ is largest at $\theta = 0^\circ$ and the smallest at $\theta = 60^\circ$ as in PSC.

SUMMARY AND DISCUSSION

The values of ϕ for $e_{0.3} = 0.70$ at $\sigma_3' = 1.0$ kgf/cm² (98 kN/m²) have been summarized as a function of b , ω and ξ (or θ for TE) as shown in Figs. 2, 21 and 22. Note that the values of ϕ plotted in Fig. 2 are limited to those where ω and ξ are either 0° or 90° . The numbers in circles presented in Figs. 21 and 22 correspond to those in Fig. 2. Only ϕ for $H/W = 1.0$ in TC and ϕ for $H/W = 1.9$ in PSC were used to portray the strength since the effects of end restraint on ϕ could not be accurately evaluated for very short specimens (i. e., $H/W = 0.25$ and 0.50). In Figs. 21(b) and 22, the curves were drawn only between the data points for the tests PSC-3 and those for Single intersection mode in TE for clarity.

It may be seen from Figs. 21 and 22 that for $e_{0.3} = 0.70$ the degree of strength anisotropy is slightly larger in TE than in PSC (at

Fig. 21. ϕ for $e_{0.3} = 0.70$ at $\sigma_3' = 1.0$ kgf/cm² (98 kN/m²) as a function of b , ω and ξ (θ for TE)

Fig. 22. Three dimensional plots of ϕ

least PSC-3), whereas it is much smaller in TC. It may be seen further by comparing Figs. 4, 12 and 16 that such a tendency as described above becomes more obvious for looser specimens. It is seen from Figs. 5, 13 and 17 that in general the strain at failure is the smallest in TE, intermediate in PSC and the largest in TC. Therefore, as suggested by Oda et al. (1978) and Ochiai and Lade (1983), it seems that in the testing method where strains at failure are larger the effects of the initial anisotropic fabric on ϕ become less significant due to a larger amount of change in the fabric which had occurred at larger strains. A lesser degree of strength anisotropy for a smaller H/W which was found in each testing method may be explained in a similar way, since strain at failure increases as H/W decreases.

The values of ϕ for the three testing methods (TC, PSC and TE) shown in Figs. 21 and 22 should be compared with caution, since failure modes may not be consistent among these testing methods. However, it may be seen that the relationship between ϕ and b for fixed values of ω and ξ (or θ) is significantly different for different values of

ω and ξ (or θ). For the data obtained by the present study, the smallest change in ϕ for $b=0.0\sim 1.0$ was found when $\omega=60^\circ$ and $\xi=90^\circ$ ($\theta=60^\circ$ for TE), and the largest one was found when $\omega=90^\circ$ and $\xi=0^\circ$.

For the data obtained by the present study, as shown in Fig. 21, only when $\xi=60^\circ$ and $\omega=90^\circ$ and when $\xi=90^\circ$ and $\omega=60^\circ$, the values of ϕ for No intersection failure mode in TE are smaller than those in TC. However, in the other cases, ϕ is always larger in TE than in TC. Furthermore, when the values of ϕ for No intersection failure mode in TE are excluded because of possible premature failure, there is only one case where ϕ is smaller in TE than in PSC: i.e., ϕ at $\theta=60^\circ$ for Single intersection failure mode in TE as compared with ϕ at $\xi=60^\circ$, $\omega=90^\circ$ in PSC.

It is very interesting to note that, when ϕ for Single or Double intersection failure mode in TE is employed, the relationship between ϕ and b at $\omega=0^\circ$ ($\theta=0^\circ$) is very similar to that obtained by Read and Green (1976) by means of a true triaxial apparatus using cubic specimens with mixed boundaries (rigid σ_1' - and σ_2' -planes and flexible σ_3' -

planes). Also in their results, ϕ increases slightly as b increases from 0.2~0.3 to 1.0. This is probably due to that also in the tests by Reades and Green (1976) premature failure as observed for No intersection failure mode in the present study may not have occurred, since both the σ_1' - and σ_2' -planes were rigid. Furthermore, Goldsheider (1982) reported a relationship between ϕ and b when $\omega=90^\circ$ and $\xi=0^\circ$ of Karlsruhe sand, obtained by means of a true triaxial apparatus having all rigid boundaries. In this relation, ϕ increases gradually as b increases from around 0.3 to 1.0. This tendency is similar to the relationship for $\omega=90^\circ$ and $\xi=0^\circ$ when ϕ for Single-or Double-intersection pattern in TE is used by the present study.

It is also very interesting to note that, when ϕ for No intersection failure mode is employed, the relationship between ϕ and b at $\omega=0^\circ$ ($\theta=0^\circ$) is similar to some data obtained by means of true triaxial apparatuses with all flexible boundaries (Ko and Scott, 1968, Miyamori, 1976, Yamada and Ishihara, 1979). Also in these results ϕ decreases slightly as b is approaching to 1.0. This may be due partly to that also in these true triaxial tests, at and near $b=1.0$ a sort of premature failure has occurred.

In summary, any discussion about the relationship between ϕ and b without taking into account the effects of the initial anisotropic fabric and the failure mode (especially for ϕ at $b=1.0$ or close to 1.0) may be very misleading.

It may also be seen from Figs. 21 and 22 that among these data ϕ is the largest for Double intersection failure mode in TE at $\theta=0^\circ$ and the smallest for No intersection failure mode in TE at $\theta=60^\circ$, the difference being as large as about 12° .

To estimate ϕ in PSC when both ω and ξ are neither 0° nor 90° , the values of ϕ for $e_{0.3}=0.70$ were plotted as a function of ω and ξ in a three dimensional space (Fig. 23). The value of ϕ at the point e , for example, was obtained by interpolation such as $\phi_e = (2\phi_a + 2\phi_c + \phi_b + \phi_d)/6$ where $\phi_a, \dots, \phi_e$ are respectively the values of ϕ at the points

$a, \dots$, and e .

As shown below, effects of the initial anisotropic fabric and σ_3' on the dilatancy rate at failure D_f are smaller than those on ϕ , but it may be seen from Fig. 24 that these cannot be neglected. In Fig. 24, D_f is $(-2 d\varepsilon_3/d\varepsilon_1)_f$ for TC, $(-d\varepsilon_3/d\varepsilon_1)_f$ for PSC and $(-d\varepsilon_3/2 d\varepsilon_1)_f$ for TE. In Fig. 24, only the largest and the smallest values of D_f for various combinations of ω and ξ in each of TC, PSC and TE are shown. In Fig. 24 (b), the values of D_f for TE have been corrected to those at $\sigma_3'=1.0 \text{ kgf/cm}^2$, which are $(D_f \text{ as measured}) \times (R_{\max} \text{ corrected for local area reduction and corrected to } \sigma_3'=1.0 \text{ kgf/cm}^2) / (R_{\max} \text{ as measured})$. This procedure was used so that D_f for TE can be compared with those for TC and PSC obtained at $\sigma_3'=1.0 \text{ kgf/cm}^2$. The reason why in TE D_f decreases as H/W increases seems to be that for a larger H/W the deformation is very much non-uniform at failure, thus D_f averaged for the whole of specimen becomes less than the possibly largest value D_f in the failing zone.

To get better insight into the stress-dilatancy characteristics as a function of the initial anisotropic fabric and b , the relationships in TC, PSC and TE for $\omega=0\sim90^\circ$ at $\xi=90^\circ$ (or $\theta=0\sim90^\circ$ for TE) for $e_{0.3}=0.70$ are compared in Fig. 25. In this figure, the values of $R_{\max}$ and D_f for TE have been corrected to those at $\sigma_3'=1.0 \text{ kgf/cm}^2$. It may be seen that the effects of the initial anisotropic

Fig. 23. ϕ in PSC as a function of ω and ξ

Fig. 24 Dilatancy rate at failure D_f in (a) TC and PSC and (b) TE

fabric on the ratio $R_{\max}/D_f$ are due mainly to its effect on $R_{\max}$ and partly to its effects on D_f . It may be seen that at $\omega=0^\circ$ ($\theta=0^\circ$) plots for TC, PSC and TE are located separately, whereas at $\omega=60^\circ$, $\xi=90^\circ$ ($\theta=60^\circ$ in TE) they are located very closely. Consequently, as for ϕ , the ratio K should be considered to be a function of the angles ω and ξ (and θ), b and failure mode as illustrated in Fig. 26. The general tendency seen in Fig. 26 is similar to that seen in Fig. 21.

CONCLUSIONS

On the basis of the limited number of drained test in triaxial compression (TC), plane strain compression (PSC) and triaxial extension (TE) on saturated Toyoura sand prepared by the air-pluviation method, the

Fig. 25. Comparison of stress-dilatancy plots for $\omega=0\sim90^\circ$, $\xi=90^\circ$ ($\theta=0\sim90^\circ$ in TE) among TC, PSC and TE

Fig. 26. Ratio $K=R_{\max}/D_f$ for $e_{0.3}=0.70$ at $\sigma'_3=1.0 \text{ kgf/cm}^2$ (98 kN/m^2) as a function of b, ω and ξ (θ for TE)

following was found :

(1) In each of TC, PSC and TE, the effects of the initial anisotropic fabric on the strength and deformation properties of sand were significant not only for long specimens but also for very short specimens. It seems that in the present study even for short specimens sufficient changes in the fabric have not occurred even at large failure strains for erasing the effects of the initial anisotropy.

pic fabric.

(2) In TE, the strength and deformation characteristics were strongly controlled by different failure modes, which were primarily controlled by the specimen aspect ratio in terms of H/W .

(3) The angle ϕ of air-pluviated Toyoura sand was found to be a function of the directions of principal stresses with respect to the material axes (as represented the bedding plane direction), the value of $b = (\sigma_2' - \sigma_3') / (\sigma_1' - \sigma_3')$ and failure mode (especially in TE). Consequently, the relationship between ϕ and b is a function of the directions of principal stresses with respect to the material axes and failure mode.

(4) The smallest value of ϕ in each of TC, PSC and TE was found when one of two sets of the maximum stress obliquity planes (i.e., shear stress/normal stress = max) is rather in parallel with the bedding plane. In this case, the effect of b on ϕ and the stress-dilatancy relation at failure become a minimum.

ACKNOWLEDGEMENTS

The first author wishes to express his appreciation and gratitude to the Japanese Government for providing the Monbusho scholarship grant for his doctoral study at Institute of Industrial Science, University of Tokyo. The authors would like to acknowledge the cooperation provided by Messrs. K. Kondo, Y. Ogawa, T. Sato and T. B. S. Pradhan and Miss M. Torimitsu. Financial support provided by the Ministry of Education, Science and Culture of the Japanese Government is also greatly acknowledged.

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