TECHNICAL NOTE

CONSOLIDATION BY VERTICAL DRAINS TAKING WELL RESISTANCE AND SMEAR INTO CONSIDERATION

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ABSTRACT

A rigorous solution for the consolidation of soil by displacement-type vertical drains with finite permeability is presented taking into account the disturbance in a clay ring surrounding the drain pile caused by drain installation. The accuracy of both Barron’s and Hansbo’s approximate solutions regarding well resistance and smear is examined in detail by comparing with this rigorous solution.

The numerical results reveal that the influence of the disturbance upon the consolidation rate is notably large for the type of soil whose permeability decreases with the disturbance. In this case, existing approximate solutions are shown to be sufficiently accurate for practical application. For the type of soil whose compressibility increases with the disturbance, on the other hand, the influence of disturbance is relatively small.

For both soil types, simple equations are proposed to obtain an equivalent “ideal” well having an increased spacing ratio on the basis of analytical results derived from the rigorous solution.

Key words: anisotropy, (compressibility), consolidation, clay, design, permeability, (smear), vertical drain, (well resistance) (IGC: D 5/E 2)

INTRODUCTION

Alluvial clay deposit is generally anisotropic and its coefficient of consolidation for the horizontal flow, $c_h$, is larger than that for the vertical flow, $c_v$ (Rowe, 1959; Mckinlay, 1961; Aboshi and Monden, 1963). The value of $c_v$ is, however, used in the drain spacing design based on the experiential knowledge that the apparent horizontal coefficient in the whole ground, $c_h$, becomes nearly equivalent to the $c_v$ of undisturbed soil (Schmidt and Gould, 1968; Johnson, 1970; Aboshi and Inoue, 1986) as the result of a decrease in $c_v$ in the smeared zone due to the mandrel-type (i.e., displacement-type) installation of drains.

The observed in-situ consolidation settlements have most often been interpreted by comparing with the calculated results ob-

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Manuscript was received for review on March 18, 1988.

Written discussions on this note should be submitted before July 1, 1989, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4 F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.
tained from the existing approximate solutions on the basis of the assumption that the area of the smeared ring is equivalent to the cross-sectional area of the mandrel used (Casagrande and Poulos, 1969; Aboshi and Inoue, 1986; Mcdonald, 1985).

The solution for centripetal consolidation induced by vertical drains was initially presented by Barron (1948) considering the finite permeability of the drain material and smear. Since his solution was rather time-consuming to calculate, Hansbo (1981) offered a more comprehensive solution incorporating both factors. He compared the results derived from his own approximate solution with those obtained from Yoshikuni and Nakanodo’s (1974) rigorous solution considering the well resistance. Importantly, Hansbo demonstrated the validity of his solution with respect to the well resistance.

The accuracy of these approximate solutions when consideration is also given to the effects of disturbance, however, still remains uncertain. Moreover, although there exist several soil types whose compressibility changes predominantly due to the disturbance (Wakame et al., 1986), the change in smeared soil compressibility has never been taken into consideration in existing solutions.

Under these circumstances, this paper presents a rigorous solution for consolidation by vertical drains taking well resistance and smear into account in the case of equal strain consolidation, and scrutinizes the accuracy of existing approximate solutions.

Additionally, the influence of compressibility of smeared soil on the consolidation rate is elucidated to achieve a better understanding of the centripetal consolidation, and to identify important factors controlling disturbance effects. Though the measured data of smeared zone size and its actual consolidation characteristics remain as important subjects for future investigation, the results presented here should serve to motivate the development of a more reasonable drain spacing design method.

**EXISTING SOLUTIONS CONSIDERING WELL RESISTANCE AND SMEAR**

A soil cylinder having a drain well is schematically pictured in Fig.1. Barron (1948) has developed the consolidation equation (1) for “equal strain consolidation” by eliminating the consideration of the vertical flow,

$$\frac{\partial \bar{u}_h}{\partial t} = c_h \left( \frac{\partial^2 u_h}{\partial z^2} + \frac{1}{r} \frac{\partial u_h}{\partial r} \right)$$

where \( u_h \) is the excess pore water pressure and \( \bar{u}_h \) is the radius direction average of \( u_h \) between \( r_e \) and \( r_v \). He assumed that the soil in the smeared zone was an incompressible solid having the coefficient of permeability, \( k_{st} \). The solution of Eq. (1) with a smeared zone is given by Eq. (2),

$$\bar{u}_h(T_h) = u_0 \exp(-8 T_h/\nu_s)$$

where

$$\nu_s = \frac{N^2}{N^2 - S^2} \ln\left(\frac{N}{S}\right) - \frac{3}{4} + \frac{S^2}{4N^2}$$

$$+ \frac{k}{k_{st}} \left( \frac{N^2 - S^2}{N^2} \right) \ln(S)$$

Here, \( T_h \) (\( = c_h t/d_s^2 \)) is the time factor, \( N \) (\( = r_d/r_w \)) is the spacing ratio, \( S \) (\( = r_d/r_w \)) is ratio of the smeared zone radius, and \( u_0 \) is
the initial excess pore water pressure. Barron presented also a solution which considers both the smear and well resistance.

Hansbo's (1981) approximate solution for the radial direction average consolidation degree, \( \bar{U}_r(z, T_h) \), involving both the disturbed and undisturbed zones is expressed as Eq. (4),
\[
\bar{U}_r(z, T_h) = 1 - \exp(-8 T_h/\mu_{sw}) \tag{4}
\]
where
\[
\mu_{sw} = \mu_s + \mu_w \tag{5}
\]
\[
\mu_s = \frac{N^2}{N^2 - 1} \left( \ln\frac{N}{S} + \gamma \ln S - \frac{3}{4} \right)
+ \frac{S^2}{4N^2} \left( 1 - \frac{S^2}{4N^2} \right) + \gamma \frac{S^2-1}{N^2-1} \left( \frac{S^2+1}{4N^2} - 1 \right) \tag{6}
\]
\[
\mu_w = \pi z(2H - z) \frac{k_h}{q_w} \left( 1 - \frac{1}{N^2} \right) \tag{7}
\]
and \( \gamma = k_h/k_{sw}, \quad q_w = \pi r_w^2 k_w \).

Although the model used by Hansbo is more realistic than that of Barron because the smeared zone consists of consolidable soil, the compressibility of smeared soil is still assumed to be equal to that of the undisturbed soil. Compressibility of clay is, however, generally increased by disturbance (Lo, 1972; Okumura, 1974). Furthermore, Hansbo also assumed as Barron did that the radial strain, \( \varepsilon_r \), and tangential strain, \( \varepsilon_\theta \), are equal to zero, and that the vertical strain, \( \varepsilon_z \), is independent of the radius, \( r \), in the cylindrical soil body. These assumptions are equivalent to the volumetric strain being also independent of \( r \).

As Yoshikuni (1973) explained, these assumptions are unrelated to the "equal strain consolidation", namely the uniform vertical settlement at the upper boundary of the clay cylinder.

Notably, these existing solutions are being widely used in analytical studies concerning actual in-situ measurements in spite of their inherent irrationalities.

ANALYSIS OF RIGOROUS SOLUTION

Basic Equations and Assumptions

According to Yoshikuni (1974), the generalized three-dimensional consolidation equations presented by Biot (1941) can be rewritten in terms of excess pore water pressure as
\[
\frac{\partial \bar{u}}{\partial t} = c_v P^2 u + \frac{\partial \phi}{\partial t} \tag{8}
\]
where \( \phi \) is the consolidation potential which satisfies the equation \( \bar{P}^2 \phi = 0 \). When the upper and lower surfaces of the clay cylinder are displaced in parallel with their original surfaces and when radial displacement is zero at both smooth boundaries of the impervious outer boundary and inner drain-side surface, as well as when the total applied load to the top and bottom surfaces of the cylindrical body is kept constant with respect to time, Eq. (8) can be reduced to
\[
\frac{\partial \bar{u}}{\partial t} = c_v \bar{P}^2 \bar{u} \tag{9}
\]

Accordingly, Barron's equation for free strain consolidation turned out to be the equation for the equal strain case.

Five assumptions are postulated when setting out to analyze the centripetal consolidation for the equal strain considering well resistance and smear:
1. The boundary surface between the disturbed and undisturbed zones is smooth and it does not move laterally.
2. The permeability of the drain well is finite.
3. Horizontal water flow in the drain well can be neglected.
4. Loads applied to the top surfaces of the disturbed and undisturbed soils are of the same values and are constant with respect to time.
5. The upper surfaces of the disturbed and undisturbed soils are pervious whereas the other surfaces of cylindrical clay body are impervious.

The consolidation of the clay in the smeared zone proceeds faster because the zone lies adjacent to the drain well. Additionally, if the compressibilities of both zones differ from each other, the boundary surface between them will neither maintain the initial position nor remain parallel to the initial surface. Incidentally, since sand piles
show the same vertical displacement as the clay ground in actual in-situ ground, the radius of the drain well is considered to vary with time. In this situation, however, the "constant \( r_w \)" assumption is adopted by all existing vertical drain consolidation theories. Because the physical approximation is the same, it is then acceptable to adopt assumption 1) used by both Barron and Hansbo.

Under assumption 1), the consolidation equations for the undisturbed and disturbed zones are

\[
\frac{\partial u}{\partial t} = c_h \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \right) \tag{10}_a
\]

and

\[
\frac{\partial u_s}{\partial t} = c_{hs} \left( \frac{\partial^2 u_s}{\partial r^2} + \frac{1}{r} \frac{\partial u_s}{\partial r} + \frac{\partial^2 u_s}{\partial z^2} \right) \tag{10}_b
\]

**Inducement of Analytical Solutions**

The boundary conditions are expressed as

BC1) \( u = u_t = 0 \) at \( z = 0 \) \( \tag{11} \)

BC2) \( \frac{\partial u}{\partial z} = u_t / \alpha_s = 0 \) at \( z = H \) \( \tag{12} \)

BC3) \( \frac{\partial u}{\partial r} = 0 \) at \( r = r_e \) \( \tag{13} \)

BC4) The continuity condition at the boundary between the drain well and the smeared soil is governed by the relation expressed as Eq. (14) (Yoshikuni and Nakando, 1974):

\[
\left( \frac{\partial u_t}{\partial r} \right)_{r=r_w} + \frac{r_w}{k_h} \left( \frac{\partial^2 u_t}{\partial z^2} \right)_{r=r_w} = 0 \tag{14}
\]

BC5) \( u = u_t \) at \( r = r_s \) \( \tag{15} \)

BC6) \( k_h (\partial u / \partial r) = k_s (\partial u_s / \partial r) \) at \( r = r_s \) \( \tag{16} \)

From BC1) and BC2), the solution to Eqs. (10)_a and (10)_b can be respectively expressed by the following general solutions:

\[
u = \begin{cases} C J_0(\alpha r) + D Y_0(\alpha r) \\ A J_0(\beta r) + B Y_0(\beta r) \end{cases} \tag{17}_a
\]

\[
u_s = \begin{cases} -c_h \left[ (\frac{m\pi}{2H})^2 + \alpha_s^2 \right] e^{i \omega t} \\ -c_{hs} \left[ (\frac{m\pi}{2H})^2 + \beta_s^2 \right] e^{i \omega t} \end{cases} \tag{17}_b
\]

where \( J_0 \) and \( Y_0 \) are Bessel functions of zero order of the first and second kind, respectively; \( A \), \( B \), \( C \) and \( D \) are integral constants; \( \alpha \) and \( \beta \) are eigenvalues; and \( l = mz/(2H) \) where \( m = 1, 3, 5, \ldots \).

Observing that \( D = -CJ_0(\alpha r) / Y_0(\alpha r) \) from BC3), Eqs. (18) \sim (20) are obtained from BC5) and BC6) as

\[
c_h \left( \frac{m\pi}{2H} \right)^2 + \alpha^2 = c_{hs} \left( \frac{m\pi}{2H} \right)^2 + \beta^2 \tag{18}
\]

\[
A = C \cdot f \tag{19}
\]

\[
B = C \cdot g \tag{20}
\]

Here,

\[
f = -\frac{\pi \beta r_e}{\beta k_h} \left\{ w(\alpha r) Y_1(\beta r) - \frac{\alpha k_h}{\beta k_s} W_1(\alpha r) J_0(\beta r) \right\} \tag{21}
\]

\[
g = \frac{\pi \beta r_e}{2} \left\{ w(\alpha r) J_1(\beta r) - \frac{\alpha k_h}{\beta k_s} W_1(\alpha r) J_0(\beta r) \right\} \tag{22}
\]

and

\[
W_i(\alpha r) = J_i(\alpha r) - \frac{J_1(\alpha r)}{Y_1(\alpha r)} Y_i(\alpha r) \quad (i=0, 1) \tag{23}
\]

Eq. (24) is next derived from BC 4) :

\[
\left( \frac{m\pi}{2H} \right)^2 \left\{ fJ_0(\beta r_w) + gY_0(\beta r_w) \right\} + \frac{2\beta k_h}{r_w k_s} \left( fJ_1(\beta r_w) + gY_1(\beta r_w) \right) = 0 \tag{24}
\]

Since the number of eigenvalues is infinite for a value of \( m \), the eigenvalue \( \beta \) should be reexpressed as \( \beta_{mn} \) \( (m=1, 3, 5, \ldots; n=1, 2, 3, \ldots) \). In the same manner, \( C, f, g, \) and \( \alpha \) should be rewritten as \( C_{mn} \), \( f_{mn} \), \( g_{mn} \), and \( \alpha_{mn} \).

The general solutions, \( u, u_s \), are subsequently rewritten by Eqs. (25)_a and (25)_b by linearly combining all solutions with respect to each eigenvalue:

\[
u(r, z, T_h) = \sum_m \sum_n C_{mn} \left\{ \frac{W_0(\alpha_{mn} r)}{\alpha_{mn} \beta_{mn} r} \right\} \sin \left( \frac{m\pi z}{2H} \right) \times \exp \left[ -4N^2 T_h \left( \frac{m\pi r_w}{2H} \right)^2 + (\alpha_{mn} r_w)^2 \right] \tag{25}_a
\]

\[
u_s(r, z, T_h) = \sum_m \sum_n C_{mn} \left\{ \frac{W_0(\alpha_{mn} r)}{\alpha_{mn} \beta_{mn} r} \right\} \sin \left( \frac{m\pi z}{2H} \right) \times \exp \left[ -4N^2 T_h \left( \frac{m\pi r_w}{2H} \right)^2 + (\alpha_{mn} r_w)^2 \right] \tag{25}_b
\]

where

\[
V_0(\beta_{mn} r) = f_{mn} J_0(\beta_{mn} r) + g_{mn} Y_0(\beta_{mn} r) \tag{26}
\]

\[
N = r_t / r_w \quad \text{and} \quad T_h = c_{ht} d_t^2 \tag{26}
\]

Substituting Eqs. (25)_a and (25)_b into the initial condition of \( u = u_t = u_0 \) (constant) when \( t = 0 \), and using Fourier's sine expansion of \( u_0 \), the following equations are derived:

\[
\sum_n \sum_m C_{mn} \left\{ \frac{W_0(\alpha_{mn} r)}{\alpha_{mn} \beta_{mn} r} \right\} \sin \left( \frac{m\pi z}{2H} \right) \times \exp \left[ -4N^2 T_h \left( \frac{m\pi r_w}{2H} \right)^2 + (\alpha_{mn} r_w)^2 \right] \tag{27}_a
\]

and
\[ \sum_m C_{mn} V_0(\beta_{mn}r) = \frac{4u_0}{m\pi} \text{ at } r_w < r < r_e. \quad (27)_b \]

To solve for \( C_{mn} \) after multiplying both sides of Eq. (27)_a by \( (k_b/k_{ht}) r W_0(\alpha_{mn}r) \) followed by integration from \( r_s \) to \( r_e \), and multiplying both sides of Eq. (27)_b by \( (c_h/c_{ht}) r V_0(\beta_{mn}r) \) followed by integration from \( r_w \) to \( r_e \), the following equation can be obtained by adding both left sides and both right sides of the two integrated equations:

\[
\sum_m C_{mn} \left( \frac{k_b}{k_{ht}} \right) \int_{r_s}^{r_e} r W_0(\alpha_{mn}r) W_0(\alpha_{mn}r) \, dr \]

\[
C_{mn} = \frac{8u_0m}{\pi^2 \beta_{mn}^2} \left( \frac{m^2}{4 \alpha_{mn}^2 H^2} \right) \left[ W_1(\alpha_{mn}r_s) + \frac{\eta^2}{L} V_0(\beta_{mn}r_w) \right] \]

\[
- \frac{r_s^2 W_1(\alpha_{mn}r_s)}{\eta \sigma^2 W_0(\alpha_{mn}r_e) - (\eta - \mu^2) r_s^2 W_0(\alpha_{mn}r_s) - \left( \eta - \mu^2 \eta \frac{\beta_{mn}}{\beta_{mn}} \right) r_s^2 W_1(\alpha_{mn}r_s) - \mu^2 r_w \left( \frac{m^2}{r_w L \beta_{mn}^2} \right) V_0(\beta_{mn}r_w) \right] \]

where \( \eta = k_b/k_{ht}, \mu = c_h/c_{ht}, S = r_s/r_w \) and \( L \) represents the coefficient of well resistance, i.e.,

\[
L = \frac{32}{\pi^2} \frac{k_b}{k_{ht}} \left( \frac{H}{d_w} \right)^2 \]

The radial average pore pressure involving both disturbed and undisturbed zones is given by

\[
\bar{u}(z, T_h) = \frac{2}{N^2 - 1} \sum_m \sum_n \frac{C_{mn}}{\beta_{mn}r_w} \left[ \frac{\eta(\alpha_{mn}r_w)^2 - (\beta_{mn}r_w)^2}{\alpha_{mn} \beta_{mn}r_w^2} S W_1(\alpha_{mn}r_w S) - V_1(\beta_{mn}r_w) \right] \sin \left( \frac{m\pi}{2H} \right) \exp \left[ -4N^2T_h \left( \frac{m^2}{2H} + (\alpha_{mn}r_w)^2 \right) \right] \]

The overall average excess pore water pressure is then

\[
\bar{u}(T_h) = \frac{4}{\pi(N^2 - 1)} \sum_m \sum_n \frac{m^2 \beta_{mn}r_w^2}{\beta_{mn}r_w^2} \left[ \frac{\eta(\alpha_{mn}r_w)^2 - (\beta_{mn}r_w)^2}{\alpha_{mn} \beta_{mn}r_w^2} S W_1(\alpha_{mn}r_w S) - V_1(\beta_{mn}r_w) \right] \exp \left[ -4N^2T_h \left( \frac{m^2}{2H} + (\alpha_{mn}r_w)^2 \right) \right] \]

The radial average consolidation degree involving both zones is

\[
\bar{U}(z, T_h) = 1 - \bar{u}(z, T_h) / u_0 \quad (33) \]

and the overall average consolidation degree is expressed by

\[
\bar{U}(T_h) = 1 - \bar{u}(T_h) / u_0 \quad (34) \]

When the coefficient of consolidation for the horizontal flow differs from that for the vertical flow, or when only the consolidation caused by the effect of the drains is required, Carrillo's procedure (1942) can be utilized as Barron used. Specifically, the excess pore water pressure, \( u_h(r, z, T_h) \), at an arbitrary point due to the radial flow only can be derived from Terzaghi's solution of one-dimensional consolidation, \( u_0(z, T_h) \):

\[
u_h(r, z, T_h) = u(r, z, T_h) - u_0(z, T_h) \quad (35) \]

Additionally, the radial average consolidation degree and overall average consolidation degree are obtained by Eqs. (36) and (37) respectively using the one-dimensional consolidation degree, \( U_0(z, T_h) \), at any depth along with its overall average consolidation degree, \( \bar{U}_0(T_h) \):

\[
\bar{u}_h(z, T_h) = 1 - \{ 1 - \bar{U}(z, T_h) \} / \{ 1 - \bar{U}_0(T_h) \} \quad (36) \]

\[
\bar{u}(T_h) = 1 - \{ 1 - \bar{U}(T_h) \} / \{ 1 - \bar{U}_0(T_h) \} \quad (37) \]

The effect of consolidation due to the vertical flow is hereafter excluded in all numerical calculation results described below.
NUMERICAL CALCULATION RESULTS AND DISCUSSION

Influence of Disturbance on Pore Water Pressure Distribution

When the effect of well resistance is negligible, i.e., when \( L = 0 \), and the vertical flow effect is excluded, the radial distribution of the excess pore water pressure is found to be independent of the depth. Fig. 2 presents the pore pressure distribution for a radius ratio of the smeared zone, \( S \), being equal to 1.5 and both the permeability ratio, \( \eta \), and coefficient ratio of consolidation, \( \mu^2 \), being equal to 5, namely, the compressibility, \( m_{\text{eff}} \), in the smeared zone being equal to that of undisturbed soil.

The calculated results arising from Hansbo's solution are also entered for comparison in the figure, and coincide well with the results given by the rigorous solution.

The difference in the pore pressure distribution between cases with or without a smeared zone is exemplified in Fig. 3 along with the effect caused by the change in the smeared soil compressibility. As is evident, for the soil type whose \( c_{\text{ph}} \) decreases from \( c_b \) in accordance with the increase in \( m_{\text{eff}} \) due to disturbance, the magnitude and the distribution pattern are similar to the case where no smear is present except for the early stage of consolidation. It is thus clear that the influence of the smear-induced change in the \( m_{\text{eff}} \) value gradually lessens as the consolidation proceeds. Conversely, for the soil type whose \( c_{\text{ph}} \) decreases with the decrease in \( k_{\text{sat}} \), the small permeability of the smeared zone having a thickness of only \( r_w \) remarkably hinders the drop in the pore pressure inside the undisturbed zone during the entire consolidation process.

Accordingly, although the coefficient of consolidation in the smeared zone decreases equally to one-fifth that in the undisturbed zone, different causes for the decrease, such as \( m_{\text{eff}} \) or \( k_{\text{sat}} \), result in differing pore water pressure distributions and in varied types of consolidation curves as will be mentioned in greater detail later.

Equivalent Spacing Ratio

Let \( \varepsilon \) represent the compressibility ratio, i.e., \( m_{\text{eff}}/m_{\text{pr}} \). Focusing now on the soil whose \( k_{\text{sat}} \) decreases with no change in the \( m_{\text{eff}} \) value due to disturbance, i.e., \( \varepsilon = 1 \), the consolidation degree curves are indicated in Fig. 4(a) when \( N = 5 \) with parameters \( \eta \) and \( S \), and in Fig. 4(b) when \( N = 15 \).

As shown in Fig. 4, the curves for different \( \eta \) are practically the same as those under the absence of smear (\( \gamma = 1 \)) being displaced in the horizontal direction. Thus, the change in the \( \eta \) value performs approximately the same role as the change in the spacing ratio, \( N \). Considering this point, Richart (1957) presented a diagram for converting the effect of disturbance to a modified value of the spacing ratio, \( N' \), on the basis of Barron's approximate solution, Eq. (2). In the present paper a similar diagram (Fig. 5) is presented to obtain the equivalent spacing ratio, \( N' \), on the basis of the author's rigorous solution.
Richart's paper of 1957. It can accordingly be said that Barron's approximate solution also yields a sufficiently high degree of accuracy for practical use.

Incidentally, the relationship between the variables and parameters in Fig. 5 is closely expressed by

\[ N' = N \cdot S^{n-1} \]  \hspace{1cm} (38)

Specifically, the \( N' \sim \eta \) relationships obtained from Eq. (38) are also demonstrated in Fig. 5 using broken lines. These broken lines exactly match the solid lines when \( N = 15 \), and for small values of \( S \), such as 1.2, even when \( N = 5 \), although the \( N' \) values read from the broken lines are slightly larger, i.e., safer results, than those taken from the solid lines.

**Influence of Smear Soil Compressibility**

In order to investigate the consolidation of the soil type whose \( m_{es} \) differs from \( m_{e} \), three kinds of changes in the consolidation constants due to disturbance are compared:

1. \( k_{hs} \) and \( m_{es} \) in the smeared soil exhibit the same degree of decrease (i.e., \( c_{hs} \) is constant);
2. \( c_{hs} \) decreases in proportion to the decrease in \( k_{hs} \) (i.e., \( m_{es} \) is constant);
3. \( c_{hs} \) decreases in proportion to the increase in \( m_{es} \) (i.e., \( k_{hs} \) is constant).

Fig. 6 compares cases 1 and 2 for \( N = 5 \). As indicated in Fig. 6, though the consolidation rate is significantly retarded with the decrease in \( k_{hs} \), the rate is little affected regardless of the decrease in \( m_{es} \). The consolidation curve for case 1 precisely matches that for case 2 when \( N = 15 \), al-

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**Fig. 4.** Consolidation degree curves influenced by decrease in \( k_{hs} \)

**Fig. 5.** Relationship between equivalent spacing ratio and permeability ratio in the case where \( k_{hs} \) decreases

First, the time factor of \( T_{hs 80} \) for the consolidation degree reaching 80% is read from the curves corresponding to each value of \( \eta \) in Fig. 4. The \( N' \) value is next obtained from the \( \bar{U}_{hs} \sim T_{hs} \) curves (which is not indicated in the present paper) having parameter \( N \) for the no smear case. Here, this \( N' \) value gives the same time factor value as the above read \( T_{hs 80} \) to reach the 80% consolidation degree in the case when no disturbance is present.

Fig. 5 indicates the values of \( N' \) plotted against \( \eta \) in the above manner. The figure coincides rather well with Fig. 4 given in
though not presented in the figure. Consequently, the changes in the $c_{d5}$ and $m_{n5}$ values due to disturbance are not significant in comparison with the decrease in the $k_{d5}$ value, which is the predominant factor in consolidation retardation.

Consolidation curves for case 3 are given in Fig. 7. According to Fig. 7, when $\mu = \varepsilon = 15$ with $N = 5$ and $S = 1.5$, the time necessary to reach 10% consolidation is 2.5 times longer compared with the case in which smear is absent. This consolidation delay time, however, becomes less as the consolidation proceeds. Namely, it is only 1.2 times with respect to $T_{h b0}$. As seen in Fig. 4(a), this result markedly differs from that in case 2, e.g., in the case when $c_{d5}$ decreases to 1/15 times $c_{d5}$ due to the decrease of $k_{d5}$, wherein the time necessary to reach any degree of consolidation increases to approximately 7 times that in the no smear case.

Although the effect of disturbance on the consolidation rate in the constant $k_{d5}$ case is complex owing to the influence varying with the consolidation degree as mentioned previously, it is important to focus on the precise influence in the final stage of consolidation for practical application.

Fig. 8 provides the relationship between $\varepsilon$ and $N'$ with respect to $T_{h b0}$ for the purpose of converting the influence of $\varepsilon$ on the spacing ratio to the no-smear case. The relationship is approximately expressed by

$$N' = N \cdot 10^{0.85(\varepsilon - 1)^2 + (\varepsilon - 1)/N^2}$$  \hspace{1cm} (39)

The broken lines in Fig. 8 calculated from Eq. (39) exactly coincide with the solid lines.

**Effects of Well Resistance and Smear**

Fig. 9(a) and 9(b) exemplify the consolidation degree curves calculated from the rigorous solution including the effects of both well resistance and smear in the decreasing $k_{d5}$ value case. The broken lines represent the integrated results calculated from Hansbo's approximate solution indicating fairly good approximation accuracy except for the early stage of consolidation.

Hansbo's solution must be integrated to obtain the overall average consolidation accuracy.
degree. To avoid this, the approximate equation proposed by Yoshikuni (1979) involving well resistance, Eq. (40), can be utilized:

$$U_h(T_w) = 1 - \exp\left(-8 T_w(F(N) + 0.8 L)\right)$$

\hspace{1cm} (40)

where

$$F(N) = \frac{N^2}{N^2 - 1} \ln N - \frac{3N^2 - 1}{4N^2}$$

\hspace{1cm} (41)

Taking the effect of smear into consideration requires that the following \(F(N')\) be substituted into \(F(N)\) using the value of \(N'\) indicated in Eq. (38):

$$F(N') = \frac{(N')^2}{(N')^2 - 1} \ln (N') - \frac{3(N')^2 - 1}{4(N')^2}$$

\hspace{1cm} (42)

The consolidation curves calculated by combining Eqs. (40) and (42) are also indicated in Fig. 9. The combined use of the both equations gives slightly lower values than those from the rigorous solution when the value of \(L\) is as large as 5, or when \(z\) is as large as 10. As is clear from the figure, however, the results from the combined equations coincide well with those from the rigorous solution in the later stage of consolidation.

On the other hand, in the increasing \(m_{e1}\) value case the combined use of Eqs. (40) and (42) using Eq. (39) lessens accuracy when the value of \(L\) exceeds 0.5. Therefore, consolidation degree should be calculated by the rigorous solution when well resistance cannot be disregarded in the increasing \(m_{e1}\) value case.

CONCLUSIONS

A rigorous solution of consolidation by drain wells taking both well resistance and smear into consideration has been introduced. Three principal conclusion were drawn from the present work.

1. For the type of soil whose compressibility increases due to disturbance, the influence of smear caused by installation of a vertical drain on the consolidation rate is small. In this case, the smear effect can be converted to the equivalent spacing ratio, \(N'\), given by Eq. (39) without the need for considering disturbance.

2. For the type of soil whose permeability decreases due to disturbance, the decreased permeability of the smeared soil significantly prevents the drop in pore pressure in the undisturbed zone. In this case, both Barron’s and Hansbo’s approximate solutions provide a sufficiently high degree of accuracy for practical application. Also, the equivalent spacing ratio for this case can be given by Eq. (38).

3. When the overall average consolidation degree for the latter type of soil is required to obtain ground surface settlement considering both well resistance and smear, it is easily obtained with good accuracy by substituting the equivalent spacing ratio into Yoshikuni’s approximate equation.

REFERENCES


