A CONSTITUTIVE MODEL FOR CYCLIC VISCOPLASTICITY OF SOILS

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ABSTRACT

A constitutive model for soils subjected to monotonic and cyclic loading under general stress conditions is presented which has neither an elastic domain nor a yield condition. Two formulations are built in the model according to the sign (plus or minus) of the linear combination of the first and second invariants of the viscoplastic strain rate tensor. In each formulation, a new stress-ratio parameter $\gamma^{\#}$ is introduced, and the linear combination of these accumulated strain invariants is related to a function of effective stresses and time. Constitutive relations are derived by using the concept of potential theory, with the help of an elasto-viscoplastic model called the Sekiguchi-Ohta model. The model presented here agrees with the Sekiguchi-Ohta model in a special case, and with the original Cam clay model in a more special case. The model can describe anisotropic responses due to initial stress anisotropy, initial structural anisotropy and loading reversal, rate effect, phase transformation, etc.

The results of the simulations for several typical triaxial tests qualitatively indicate the applicability of the model to a wide extent of loading conditions.

Key words : anisotropy, compression, consolidation, constitutive equation of soil, dilatancy, stress path, stress-strain curve, time effect (IGC : D 5/D 6/D 7)

INTRODUCTION

Many of the constitutive models proposed for soils are based on the theory of plasticity. As the inevitable consequence of this approach, those models have to establish yield conditions, flow rules and hardening rules ; it is not easy, however, to establish models which can describe well the complex behaviour of soils.

Roscoe et al. (1963) derived an associative elasto-plastic model, what is called the

original Cam clay model, which explained well the isotropic behaviour of clays under static and monotonic loading. Since then, the efforts to represent more complex or more varied behavioural characteristics of soils have been made by many researchers, and many good models have been presented. Among them, models using the theory of critical state soil mechanics or using the concept of the critical state have the advantage of better comprehensibility and a smaller number of material parameters (Roscoe and

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Burland, 1968; Ohta, 1971; Pender, 1982; Carter et al., 1982; Mroz et al., 1982; etc.).

Models in which the non-associated flow rule is adopted (Poorooshasb et al., 1966, 1967) and models in which yield surfaces (or plastic potentials) are given to consolidation and shear respectively (Nishi and Esashi, 1978) are popular for sands.

Modelling for soils subjected to cyclic loading is complicated by the need for descriptions of non-linearity during unloading and for descriptions of induced anisotropy caused by reversal loading. A model by Pender (1982) for overconsolidated clays under cyclic triaxial loading is exceptionally simple. Ghaboussi and Momen (1982) and Poorooshasb et al. (1986) have proposed models for sands with a combination of the isotropic and kinematic hardening rules and with three anisotropic stress invariants. Models applying the concept of a field of hardening moduli (Mroz, 1967) have been proposed by Prevost (1977), Mroz and Norris (1982) and Aubry et al. (1982). In the above models for cyclic loading, the elastic domain during shear substantially vanishes or does not exist. Furthermore, there have been proposed models applying the concept of a mobilized plane (Matsuoka et al., 1985), endochronic models (Valanis and Read, 1982) and so forth.

A number of viscoplastic models for clays have been proposed, including those by Adachi and Okano (1974), Sekiguchi (1977), Sekiguchi and Ohta (1977), Matsui, T. and Abe (1982) and Adachi and Oka (1982). Cyclic viscoplasticity of granular materials has been discussed in some detail by Oka (1982). For details of the constitutive models available, the reader may refer to the review papers by Hashiguchi (1985) for elastoplastic models and by Sekiguchi (1985) for viscoplastic models.

Generally, an attempt to represent the behaviour of soils as realistically as possible will make the model complex and will make the constitutive parameters numerous and, as its consequence, will make practical application difficult. In this paper, a simple constitutive model will be presented. It is a viscoplastic

model with neither an elastic domain nor a yield condition. The model covers the behaviour of normally consolidated clays, lightly overconsolidated clays, quasi-overconsolidated clays and sands subjected to monotonic and cyclic loading under general stress conditions. The model is not directly based on the theory of critical state soil mechanics but indirectly recognizes the existence of the critical state, and covers only a certain range before the critical state. The model may be regarded as an extension of the elasto-viscoplastic models proposed by Sekiguchi (1977) and Sekiguchi and Ohta (1977) for normally consolidated clays, which will be referred to as 'the Sekiguchi-Ohta model' in this paper.

In what follows, the formulation will be derived. The performance characteristics of the model will be examined through several simulations of triaxial tests on imaginary soils. Quantitative assessment of the model will be made in comparison with experimental results on Drammen clay reported in research literature sources.

FORMULATION OF THE CONSTITUTIVE MODEL

On the basis of both typical stress-strain-time relations observed in two kinds of experimental tests—consolidation-swelling tests under an isotropic stress and strain condition and a triaxial test under a constant mean effective stress—and the dilatancy equation of the original Cam clay model, a linear combination of an accumulated inelastic (=viscoplastic) volumetric strain and an accumulated inelastic (=viscoplastic) deviatoric strain will be expressed as a function of effective stresses and time, introducing a new stress-ratio parameter η^* . Constitutive relations will be derived by using the concept of potential theory, with the help of the procedure of the Sekiguchi-Ohta model.

Throughout this paper, strains and stresses will be taken as positive in compression, and stresses will imply effective stresses.

Fig. 1. Characteristics assumed for isotropic compression

Modelling for Cyclic Isotropic Compression

Such a static compression response as a fully-drawn curve in Fig.1(a) will be observed in the isotropic compression (consolidation) test on clays. Let us here suppose that this response is given by

$$e = e_0 - \kappa \cdot \ln\left(\frac{p_c}{p_0}\right) - \lambda \cdot \ln\left(\frac{p}{p_c}\right) \quad (1)$$

where e is the void ratio, e_0 is the initial void ratio, p is the mean normal stress, p_0 is the initial mean stress, p_c is the yield pressure under consolidation, λ is the compression index ($=C_c/\ln 10$), $-\kappa$ is the initial slope of the e - $\ln p$ curve in the overconsolidated or swelling region, and $\ln$ denotes the natural logarithm.

If the increase of loading is stopped and p is kept constant, then compression creep such as that represented by the broken line in Fig. 1(a) will be observed. In order to express this characteristic of creep, let us refer to Fig.1(b) in which a dotted curve represents the primary consolidation process. As shown in Fig.1(b), the volumetric strain

will increase almost proportionally to $\ln t$ after a sufficiently long time. Accordingly, let us suppose that this is expressed by

$$dv = \alpha \cdot d(\ln t) \quad (2)$$

where v is the volumetric strain, α is the secondary compression index, t is the time elapsed from the initial time, and t_N is the value of t at the end of the primary consolidation.

Let us suppose that a series of points (e , p) at the end of the primary consolidation is the normal consolidation line. Then Eqs. (1) and (2) produce (see Appendix 1)

$$\left. \begin{aligned} v &= v^E + v^V, \quad v^E = \frac{\kappa}{1+e_0} \ln\left(\frac{p}{p_0}\right), \\ v^V &= \alpha \cdot \ln\left[1 + \frac{\dot{v}_N}{\alpha} \int_0^t \exp\left(\frac{f-f_c}{\alpha}\right) dt\right] \\ f &= \frac{\lambda-\kappa}{1+e_0} \ln\left(\frac{p}{p_0}\right), \quad f_c = \frac{\lambda-\kappa}{1+e_0} \ln\left(\frac{p_c}{p_0}\right) \end{aligned} \right\} \quad (3)$$

where v^E and v^V represent the elastic and inelastic (=viscoplastic) strain respectively, and $\dot{v}_N$ is the volumetric strain rate at a point on the normal consolidation line during compression creep under a constant p .

Thus, the viscoplastic potential function F for clays during monotonic isotropic compression is given as a function of f and t in the form :

$$F = \alpha \cdot \ln\left[1 + \frac{\dot{v}_N}{\alpha} \int_0^t \exp\left(\frac{f-f_c}{\alpha}\right) dt\right] = \int_0^t \dot{v}^V dt \quad (4)$$

where $\dot{v}^V$ is the inelastic (=viscoplastic) volu-

Fig. 2. Characteristics assumed for isotropic swelling

Fig. 3. (a) Typical e — $\ln p$ curve and (b) t_r — t relationship during cyclic compression ($t_0, t_1, t_2, \dots$ represent the values of time at points 0, 1, 2, ..., respectively)

metric strain rate. Eq. (4) is similar in form to the viscoplastic potential function of the Sekiguchi-Ohta model under an isotropic stress condition. In Eq. (4), overconsolidated cases are included and the initial state ($t=0, e=e_0, p=p_0$) is defined only once (see Appendix 1).

Let the static characteristics of isotropic swelling be as illustrated in Fig. 2, and assume that the existence of the swelling creep is similar to the compression creep. The parameter p_s is a hypothetical pressure for swelling, corresponding to p_c for compression. The equations derived for monotonic isotropic compression are then extended to those for cyclic isotropic compression. The viscoplastic potential function equations are defined respectively when $\dot{v}^V$ is positive and when negative :

$$\begin{aligned}
 F &= F^+(f, t) = \int_{t_r}^t \dot{v}^V dt \quad \text{when } \dot{v}^V > 0 \\
 F &= F^-(f, t) = - \int_{t_r}^t \dot{v}^V dt \quad \text{when } \dot{v}^V < 0
 \end{aligned}$$

where

$$\left. \begin{aligned}
 F^+ &= \alpha \cdot \ln \left[1 + \frac{\dot{v}_N}{\alpha} \int_{t_r}^t \exp \left(\frac{f - f^+}{\alpha} \right) dt \right] \\
 F^- &= \alpha \cdot \ln \left[1 + \frac{\dot{v}_N}{\alpha} \int_{t_r}^t \exp \left(\frac{f^- - f}{\alpha} \right) dt \right] \\
 f &= \frac{\lambda - \kappa}{1 + e_0} \ln \left(\frac{p}{p_0} \right) \\
 f^+ &= \frac{\lambda - \kappa}{1 + e_0} \ln \left(\frac{p_c}{p_0} \right) + \int_0^{t_r} \dot{v}^V dt
 \end{aligned} \right\} \quad (5)$$

$$f^- = \frac{\lambda - \kappa}{1 + e_0} \ln \left(\frac{p_s}{p_0} \right) + \int_0^{t_r} \dot{v}^V dt$$

t_r is the value of t at the last change of the sign of $\dot{v}^V$ in the case of considering neither p -constant creep nor v -constant relaxation, and $t_r=0$ when $t=0$ (see Fig. 3). Eq. (5) includes monotonic compression and monotonic swelling under isotropic conditions.

The meanings of f^+ and f^- will be clarified in the following paragraphs :

Consider the cases of recompressing after swelling or after creep, as shown in Fig. 4. Define $\tilde{p}_c$ as illustrated in Fig. 4. Then

$$\begin{aligned}
 f^+ &= \frac{\lambda - \kappa}{1 + e_0} \ln \left(\frac{p_c}{p_0} \right) + \int_0^{t_r} \dot{v}^V dt \\
 &= \frac{\lambda - \kappa}{1 + e_0} \ln \left(\frac{\tilde{p}_c}{p_0} \right) = \text{constant for } t \geq t_r
 \end{aligned}$$

Therefore f^+ represents the effect of the stepwise variable quasi-preconsolidation pressure. By considering when $t_r=0$, it will be found that p_c relates to the loading history prior to the defined initial time. Similarly for f^- , as illustrated in Fig. 5,

$$\begin{aligned}
 f^- &= \frac{\lambda - \kappa}{1 + e_0} \ln \left(\frac{p_s}{p_0} \right) + \int_0^{t_r} \dot{v}^V dt \\
 &= \frac{\lambda - \kappa}{1 + e_0} \ln \left(\frac{\tilde{p}_s}{p_0} \right) = \text{constant for } t \geq t_r
 \end{aligned}$$

Therefore f^- represents the effect of the stepwise variable quasi-preswelling pressure. Hence, from a viewpoint of plasticity theory,

Fig. 4. Illustrational relationship between $\tilde{p}_c$ and viscoplastic volumetric strain $\int \dot{v}^V dt$ ($\Delta e^V = (1+e_0) \int_0^{t_r} \dot{v}^V dt$) at recompressing after (a) swelling, (b) swelling (at a higher rate of compression, swelling and recompression) and (c) creep

Fig. 5. Illustration of $\tilde{p}_s$

f^+ and f^- play roles such as stepwise hardening parameters.

A set of constants p_c , p_s and $\dot{v}_N$ in Eq. (5) can be replaced by an arbitrary set of constants p_0^+ , p_0^- and $\dot{v}_0$ which satisfy the relation

$$\left(\frac{p_0^+}{p_c}\right)^{\frac{\lambda-\kappa}{(1+e_0)\alpha}} = \left(\frac{p_s}{p_0^-}\right)^{\frac{\lambda-\kappa}{(1+e_0)\alpha}} = \frac{\dot{v}_0}{\dot{v}_N} \quad (6)$$

(see Appendix 2). Consider the case of rapid loading shown in Fig. 6. It is possible to choose an apparent yield pressure under consolidation as p_0^+ and use p_0^- and $\dot{v}_0$ satisfying Eq. (6). In the process of creep $A \rightarrow B \rightarrow C$, the volumetric strain rate $\dot{v}(=\dot{v}^V)$ is equal to $\dot{v}_0$ at point A and to $\dot{v}_N$ at point B. The values of the parameters observed when $\dot{p}/p$

Fig. 6. e - $\ln p$ curve and illustration of p_0^+ and $\dot{v}_0$ for fast compression

(rate of $\ln p$) is of an arbitrary and constant magnitude are more convenient for experimental determination than those at the end of the primary consolidation. Instead of using the value symbols p_c , p_s and $\dot{v}_N$, hereafter these values will be referred to using the terms 'initial quasi-preconsolidation pressure', 'initial quasi-preswelling pressure' and 'reference volumetric strain rate', respectively.

Viscoplastic Potential Function and Hardening Parameter for Monotonic Shear

To begin with, the following relations are given :

Fig. 7. Stress-ratio-strain curves assumed for p -constant triaxial compression : (a) η - γ curve ; (b) η - v curve

$$\left. \begin{aligned} \dot{\sigma}_{ij} &= \dot{s}_{ij} + \dot{p}\delta_{ij}, & \dot{p} &= \dot{\sigma}_{ij}\delta_{ij}/3 \\ \dot{\epsilon}_{ij} &= \dot{\gamma}_{ij} + \dot{v}\delta_{ij}/3, & \dot{v} &= \dot{\epsilon}_{ij}\delta_{ij} \\ \dot{\gamma}_{ij} &= \dot{\gamma}_{ij}^E + \dot{\gamma}_{ij}^V, & \dot{v} &= \dot{v}^E + \dot{v}^V \\ \dot{\gamma}_{ij}^E &= \dot{s}_{ij}/(2G), & \dot{v}^E &= \dot{p}/K \end{aligned} \right\} \quad (7)$$

where $\dot{\sigma}_{ij}$ is the stress rate tensor, $\dot{s}_{ij}$ is the deviatoric stress rate tensor, $\dot{\epsilon}_{ij}$ is the strain rate tensor, $\dot{\gamma}_{ij}$ is the deviatoric strain rate tensor, $\dot{\gamma}_{ij}^E$ and $\dot{\gamma}_{ij}^V$ are respectively the elastic and inelastic (=viscoplastic) components of the deviatoric strain rate tensor, $\dot{v}^E$ and $\dot{v}^V$ are respectively the elastic and inelastic (=viscoplastic) components of the volumetric strain rate, G and K are respectively the shear and bulk moduli, and δ_{ij} denotes the Kronecker delta, and the Einstein summation convention applies. In Eq. (7), the upper four relations are self-evident, and the lower four relations are based on the assumption that the strain rate is composed of the elastic and inelastic (=viscoplastic) components.

Now, let the stress-ratio-strain curves obtained through the p -constant triaxial slow compression test be as illustrated in Fig. 7. Here

$$\begin{aligned} \eta &= q/p, \quad q = \sigma_a - \sigma_r, \quad p = (\sigma_a + 2\sigma_r)/3, \\ \gamma &= (\epsilon_a - \epsilon_r)2/3, \quad v = \epsilon_a + 2\epsilon_r, \end{aligned}$$

where σ_a and σ_r denote respectively the axial and radial stresses, ϵ_a and ϵ_r denote respectively the axial and radial strains, and M and μ are positive material constants which are termed the critical state parameter and the dilatancy parameter, respectively. Let the η - γ curve shown in Fig. 7(a) be expressed by

$$\gamma = c \cdot \ln\left(\frac{M}{M-\eta}\right) \quad (8)$$

where c is a constant. Moreover, assume

$$dv/d\gamma = \mu - \eta \quad (9)$$

after the stress-dilatancy relationship of the original Cam clay model. Then, from Eqs. (8) and (9), the equation

$$v = c \cdot \eta - c(M-\mu) \cdot \ln\left(\frac{M}{M-\eta}\right) \quad (10)$$

is derived. If $\mu = M$, then $v = c \cdot \eta$ and c corresponds to the coefficient of dilatancy D introduced by Shibata (1963) for clays. The relation $\mu = M$ will be observed in clays and loose sands and $\mu < M$ in dense sands.

Thus the strains during p -constant triaxial slow compression have been expressed by the static functions, and in the same way as in the case of isotropic compression, these expressions can be developed to viscoplastic potential function equations with time. At this time, the viscoplastic potential functions can be separately defined for volumetric strain due to isotropic compression (consolidation), for volumetric strain due to shear (dilatancy), and for shear strain. In this paper, however, a combination of these strains is related to the viscoplastic potential function.

Observing the fact that the right-hand side of Eq. (8) multiplied by $-(M-\mu)$ is equal to the second term in the right-hand side of Eq. (10), then

$$v + (M-\mu)\gamma = c \cdot \eta$$

Because this v is the volumetric strain due to shear, the addition of the strain due to isotropic compression produces the following :

$$f = \frac{\lambda - \kappa}{1 + e_0} \ln\left(\frac{p}{p_0}\right) + c \cdot \eta, \quad f^+ = \frac{\lambda - \kappa}{1 + e_0} \ln\left(\frac{p_0^+}{p_0}\right)$$

As in the case of isotropic compression, the function $F(f, t)$ is derived in the form:

$$F = \alpha \cdot \ln\left[1 + \frac{\xi_0}{\alpha} \int_0^t \exp\left(\frac{f - f^+}{\alpha}\right) dt\right] = \int_0^t \xi dt \quad (11)$$

where

$$\xi = \dot{v}^V + (M - \mu) |\dot{\gamma}^V| \quad (> 0, \text{ say})$$

$$|\dot{\gamma}^V| = \sqrt{\frac{2}{3} \dot{\gamma}_{ij}^V \dot{\gamma}_{ij}^V}$$

ξ_0 corresponds to $\dot{v}_0$ and is named the reference strain rate, and $|\dot{\gamma}^V|$ stands for the length of the generalized inelastic (=viscoplastic) shear strain rate. $\int_0^t \xi dt$ plays a role such as hardening parameter in plasticity theory. ξ is equivalent to the rate of the combined hardening parameter applied by Nova and Wood (1979) and discussed by Mroz and Norris (1982).

Note that, as shown in the next subsection, the relation

$$\dot{v}^V / |\dot{\gamma}^V| = \frac{\partial F}{\partial p} / \left(\frac{1}{p} \frac{\partial F}{\partial \eta} \right) = \frac{\partial f}{\partial p} / \left(\frac{1}{p} \frac{\partial f}{\partial \eta} \right)$$

$$= \frac{\lambda - \kappa}{(1 + e_0)c} - \eta$$

holds in the present model. The constant c can be determined as $c = (\lambda - \kappa) / [(1 + e_0)\mu]$ in view of the fact of the phase transformation line $\eta = \mu$ (Ishihara et al., 1975), and then

$$f = \frac{\lambda - \kappa}{(1 + e_0)\mu} \left[\mu \cdot \ln\left(\frac{p}{p_0}\right) + \eta \right] \quad (12)$$

Introduction of New Parameters for Cyclic Shear

In order to extend to cyclic loading under general stress conditions, a new parameter η^* is introduced instead of η in Eq. (12); this parameter is termed the relative stress ratio. Associated with the introduction of this parameter, the new parameters q^* , $\hat{q}$ and s_{ij}^R are also introduced, where $(q^* - \hat{q})$ is termed the relative shear stress, and s_{ij}^R is the (piecewise constant) reversal deviatoric stress tensor whose value is changed at the

reversal of shear. At the same time the anisotropy of the initial structure is incorporated by using a new constant parameter $\bar{s}_{ij}$.

Let η^* , q^* and $\hat{q}$ be defined as follows:

$$\left. \begin{aligned} \eta^* &= (q^* - \hat{q}) / p < M \\ q^* &= \sqrt{q_{ij}^* q_{ij}^*}, \quad q_{ij}^* = \sqrt{\frac{3}{2}} (s_{ij} - s_{ij}^R) \\ \hat{q} &= \sqrt{\hat{q}_{ij} \hat{q}_{ij}}, \quad \hat{q}_{ij} = \sqrt{\frac{3}{2}} (\bar{s}_{ij} - s_{ij}^R) \end{aligned} \right\} \quad (13)_1$$

in which s_{ij} is the (current) deviatoric stress tensor, and $\bar{s}_{ij}$ is the material constant tensor representing the deviatoric stress tensor due to the initial structure. The reversal deviatoric stress tensor s_{ij}^R is defined as

$$\left. \begin{aligned} s_{ij}^R(t+0^+) &= \begin{cases} s_{ij}^R(t) & \text{when } \tilde{\eta}^*_{(t+0^+)} > 0 \\ \bar{s}_{ij} & \text{when } \tilde{\eta}^*_{(t+0^+)} < 0, \eta^{**}_{(t+0^+)} > 0 \\ s_{ij}(t) & \text{when } \tilde{\eta}^*_{(t+0^+)} < 0, \eta^{**}_{(t+0^+)} < 0 \end{cases} \end{aligned} \right\} \quad (13)_2$$

where

$$\tilde{\eta}^*_{(t+0^+)} = \frac{\partial \eta^*}{\partial s_{ij}(t)} \dot{s}_{ij}(t+0^+)$$

$$\eta^{**} = \frac{1}{p} \sqrt{\frac{3}{2}} (s_{ij} - \bar{s}_{ij})(s_{ij} - \bar{s}_{ij})$$

and η^{**} is the rate of η^* .

According to the above definitions, s_{ij}^R is updated and η^* undergoes a sudden change of value at the moment $\tilde{\eta}^* < 0$ when the reversal of shear occurs, and subsequently η^* again increases monotonically until the next reversal of shear, so that η^* has discontinuous points. Except at such points, $\tilde{\eta}^*$ coincides with the rate of η^* (i.e. $\tilde{\eta}^* = \dot{\eta}^*$). As η^* can be reduced to a simple form in a particular case (see Appendix 3), to consider such a case here will make it easy to understand the above definitions. Those definitions are concretely explained by Fig. 8. Fig. 8(a) illustrates the typical $q/p - v$ curve in p -constant cyclic triaxial tests on soils which satisfy Eqs. (A.7) in Appendix 3. Here, if $s_{ij}^R(t=0) = \bar{s}_{ij}$, then

$$\tilde{\eta}^* < 0, \eta^{**} > 0 \text{ only at point 2,}$$

$$\tilde{\eta}^* < 0, \eta^{**} < 0 \text{ only at points 0, 1, 3, 4}$$

and if $s_{ij}^R(t=0) = s_{ij}(t=0)$, then

$$\tilde{\eta}^* < 0, \eta^{**} > 0 \text{ only at point 2,}$$

Fig. 8. (a) Typical q/p_0-v curve and (b) t_r-t relationship during p -constant cyclic triaxial loading ($t_0, t_1, t_2, \dots$ represent the values of time at points 0, 1, 2, ..., respectively)

$\tilde{\eta}^* < 0$, $\eta^{**} < 0$ only at points 1, 3, 4.

Hence, for example,

$$s_{ij}^R(t+0^+) = \bar{s}_{ij} = \text{const. for } t_2 \leq t < t_3, \\ s_{ij}^R(t+0^+) = s_{ij}(t_3) = \text{const. for } t_3 \leq t < t_4.$$

Viscoplastic Potential Function for General Stress Conditions

Eq. (11) is extended in the same way as in the case of isotropic compression, and the viscoplastic potential function F for monotonic and cyclic loading under general stress conditions is induced in the form :

$$F = F^+(f, t) = \int_{t_r}^t \dot{\xi}^+ dt \text{ when } \dot{\xi} > 0$$

$$F = F^-(f, t) = \int_{t_r}^t \dot{\xi}^- dt \text{ when } \dot{\xi} < 0$$

where

$$\left. \begin{aligned} F^+ &= \alpha \cdot \ln \left[1 + \frac{\dot{\xi}_0}{\alpha} \int_{t_r}^t \exp \left(\frac{f - f^+}{\alpha} \right) dt \right] \\ F^- &= \alpha \cdot \ln \left[1 + \frac{\dot{\xi}_0}{\alpha} \int_{t_r}^t \exp \left(\frac{f^- - f}{\alpha} \right) dt \right] \\ f &= \frac{\lambda - \kappa}{(1 + e_0)\mu} \left[\mu \cdot \ln \left(\frac{p}{p_0} \right) + \eta^* \right] \\ f^+ &= \frac{\lambda - \kappa}{(1 + e_0)\mu} \left[\mu \cdot \ln \left(\frac{p_0^+}{p_0} \right) + \eta_0^* \right] + \int_0^{t_r} \dot{\xi}^+ dt \\ f^- &= \frac{\lambda - \kappa}{(1 + e_0)\mu} \left[\mu \cdot \ln \left(\frac{p_0^-}{p_0} \right) + \eta_0^* \right] + \int_0^{t_r} \dot{\xi}^+ dt \\ \dot{\xi}^+ &= \dot{\xi} = \dot{v}^V + (M - \mu) |\dot{\gamma}^V| \\ \dot{\xi}^- &= -\dot{v}^V + (M - \mu) |\dot{\gamma}^V| \\ \dot{\xi}^\pm &= \begin{cases} \dot{\xi}^+ & \text{when } \dot{\xi} > 0 \\ -\dot{\xi}^- & \text{when } \dot{\xi} < 0 \end{cases} \end{aligned} \right\} \quad (14)$$

η_0^* is the initial value of η^* , i.e. $\eta_0^* = \eta^*(t=0^+)$, and $\dot{\xi}^+$, $\dot{\xi}^-$ and $\dot{\xi}$ play a role like the rate of hardening parameter in plasticity theory. Because the present model assumes that no elastic domain exists, the inelastic (=viscoplastic) strain rate always occurs, and therefore $\dot{\xi} \neq 0$ at any time. And t_r is the time nearer the current time t between the last time when $\tilde{\eta}^*$ becomes negative and the last time when $\dot{\xi}$ undergoes a change of sign, and initially $t_r = 0$. In an example shown in Fig. 8(a), as $\dot{\xi} > 0$ at any time, t_r is updated only at points (0,) 1, 2, 3 and 4 (Fig. 8(b)).

In the case of considering creep and stress relaxation, a slight modification is necessary for the definition of s_{ij}^R and t_r . However, a detailed expression of such a definition is beyond the scope of this discussion.

The viscoplastic potential function presented by Eq. (14) is very similar to that of the Sekiguchi-Ohta model. They agree with each other in a special case where the soil characterized as $s_{ij}s_{ij} = 0$ when $t = 0$, as $\mu = M$, $p_0^+ = p_0$ and $\bar{s}_{ij}\bar{s}_{ij} = 0$ in the present model and as $D = (\lambda - \kappa)/[(1 + e_0)M]$ in the Sekiguchi-Ohta model, is subjected to monotonic loading with $\dot{v}^V > 0$.

Constitutive Equations for General Stress Conditions

Constitutive equations are derived in the

same way as that of the Sekiguchi-Ohta model.

Since F given by Eq. (14) is the function of f and t , and since f is the function of p and s_{ij} , then it follows that

$$\begin{aligned}\dot{F} &= \frac{\partial F}{\partial f} \dot{f} + \frac{\partial F}{\partial t} \\ &= \frac{\partial F}{\partial f} \left(\frac{\partial f}{\partial p} \dot{p} + \frac{\partial f}{\partial s_{ij}} \dot{s}_{ij} \right) + \frac{\partial F}{\partial t} \\ &= \frac{\partial F}{\partial f} \left(\frac{\partial f}{\partial p} \dot{p} + \frac{1}{p} \frac{\partial f}{\partial \eta^*} \sqrt{\frac{3}{2}} \frac{q_{ij}^*}{q^*} \dot{s}_{ij} \right) + \frac{\partial F}{\partial t}\end{aligned}\quad (15)$$

(see Appendix 3). Suppose that

$$\begin{aligned}\dot{F} &= \dot{F}^+ = \dot{v}^V + (M - \mu) |\dot{\gamma}^V| \quad \text{when } \dot{\xi} > 0 \\ \dot{F} &= \dot{F}^- = -\dot{v}^V + (M - \mu) |\dot{\gamma}^V| \quad \text{when } \dot{\xi} < 0\end{aligned}\quad (16)$$

From the potential theory, assume that

$$\begin{aligned}\dot{\gamma}_{ij}^V &= A^+ \frac{\partial F^+}{\partial f} \frac{\partial f}{\partial s_{ij}} = A^+ \frac{\partial F^+}{\partial f} \frac{1}{p} \frac{\partial f}{\partial \eta^*} \sqrt{\frac{3}{2}} \frac{q_{ij}^*}{q^*} \\ \dot{v}^V &= A^+ \frac{\partial F^+}{\partial f} \frac{\partial f}{\partial p} \quad \text{when } \dot{\xi} > 0, \\ \dot{\gamma}_{ij}^V &= A^- \frac{\partial F^-}{\partial f} \frac{\partial f}{\partial s_{ij}} = A^- \frac{\partial F^-}{\partial f} \frac{1}{p} \frac{\partial f}{\partial \eta^*} \sqrt{\frac{3}{2}} \frac{q_{ij}^*}{q^*} \\ \dot{v}^V &= A^- \frac{\partial F^-}{\partial f} \frac{\partial f}{\partial p} \quad \text{when } \dot{\xi} < 0\end{aligned}\quad (17)_1$$

where A^+ and A^- are respectively the positive and negative scalar quantities (viscoplastic multipliers), and neither $A^+ \partial F^+ / \partial f$ nor $A^- \partial F^- / \partial f$ is negative. Then

$$\begin{aligned}|\dot{\gamma}^V| &= A^+ \frac{\partial F^+}{\partial f} \frac{1}{p} \frac{\partial f}{\partial \eta^*} \quad \text{when } \dot{\xi} > 0 \\ |\dot{\gamma}^V| &= A^- \frac{\partial F^-}{\partial f} \frac{1}{p} \frac{\partial f}{\partial \eta^*} \quad \text{when } \dot{\xi} < 0\end{aligned}\quad (17)_2$$

Hence

$$\dot{\gamma}_{ij}^V = |\dot{\gamma}^V| \sqrt{\frac{3}{2}} \frac{q_{ij}^*}{q^*} \quad (17)_3$$

From Eqs. (16) and (17), A^+ and A^- are immediately obtained in the explicit expressions:

$$\begin{aligned}A^+ &= \dot{F}^+ \left/ \left[\frac{\partial F^+}{\partial f} \left\{ \frac{\partial f}{\partial p} + (M - \mu) \frac{1}{p} \frac{\partial f}{\partial \eta^*} \right\} \right] \right. \\ A^- &= \dot{F}^- \left/ \left[\frac{\partial F^-}{\partial f} \left\{ \frac{\partial f}{\partial p} + (M - \mu) \frac{1}{p} \frac{\partial f}{\partial \eta^*} \right\} \right] \right.\end{aligned}\quad (18)$$

The stress-rate-strain-rate-time relations are derived from Eqs. (7), (15), (17) and (18).

In this paper, the constitutive equations which apply only when $\dot{\xi} > 0$ are presented in the following.

$$\begin{aligned}\dot{\epsilon}_{ij} &= \left(K - \frac{2}{3} G \right) \dot{v} \delta_{ij} + 2G \dot{\epsilon}_{ij} \\ &\quad - \left(K \frac{\partial f}{\partial p} \delta_{ij} + 3G \frac{1}{p} \frac{\partial f}{\partial \eta^*} \sqrt{\frac{2}{3}} \frac{q_{ij}^*}{q^*} \right) A\end{aligned}\quad (19)$$

where

$$\begin{aligned}A &= \left[\left(K \frac{\partial f}{\partial p} \dot{v} + 3G \frac{1}{p} \frac{\partial f}{\partial \eta^*} \sqrt{\frac{2}{3}} \frac{q_{kl}^*}{q^*} \dot{\epsilon}_{kl} \right) \right. \\ &\quad \times \frac{\partial F^+}{\partial f} + \frac{\partial F^+}{\partial t} \left. \right] \left/ \left[K \left(\frac{\partial f}{\partial p} \right)^2 + 3G \right. \right. \\ &\quad \times \left(\frac{1}{p} \frac{\partial f}{\partial \eta^*} \right)^2 \left. \right] \frac{\partial F^+}{\partial f} + \frac{\partial F^+}{\partial p} + (M - \mu) \\ &\quad \times \frac{1}{p} \frac{\partial f}{\partial \eta^*} \left. \right]\end{aligned}$$

$$\begin{aligned}\dot{\epsilon}_{ij} &= \left(\frac{1}{3K} - \frac{1}{2G} \right) \dot{p} \delta_{ij} + \frac{1}{2G} \dot{v} \delta_{ij} \\ &\quad + \left(\frac{1}{3} \frac{\partial f}{\partial p} \delta_{ij} + \frac{1}{p} \frac{\partial f}{\partial \eta^*} \sqrt{\frac{3}{2}} \frac{q_{ij}^*}{q^*} \right) B\end{aligned}\quad (20)$$

where

$$\begin{aligned}B &= \left[\left(\frac{\partial f}{\partial p} \dot{p} + \frac{1}{p} \frac{\partial f}{\partial \eta^*} \sqrt{\frac{3}{2}} \frac{q_{kl}^*}{q^*} \dot{v} \right) \frac{\partial F^+}{\partial f} \right. \\ &\quad \left. + \frac{\partial F^+}{\partial t} \right] \left/ \left[\frac{\partial f}{\partial p} + (M - \mu) \frac{1}{p} \frac{\partial f}{\partial \eta^*} \right] \right.\end{aligned}$$

Hereupon, from Eq. (14), the following relations hold:

$$\begin{aligned}\frac{\partial F^+}{\partial f} &= 1 - \exp \left(-\frac{1}{\alpha} \int_{t_r}^t \dot{\xi}^+ dt \right) \\ \frac{\partial F^+}{\partial t} &= \dot{\xi}_0 \cdot \exp \left\{ \frac{1}{\alpha} \left(f - f^+ - \int_{t_r}^t \dot{\xi}^+ dt \right) \right\} \\ \frac{\partial F^-}{\partial f} &= \exp \left(-\frac{1}{\alpha} \int_{t_r}^t \dot{\xi}^- dt \right) - 1 \\ \frac{\partial F^-}{\partial t} &= \dot{\xi}_0 \cdot \exp \left\{ \frac{1}{\alpha} \left(f^- - f - \int_{t_r}^t \dot{\xi}^- dt \right) \right\} \\ \frac{\partial f}{\partial p} &= \frac{1}{p} \frac{\lambda - \kappa}{(1 + e_0) \mu} (\mu - \eta^*) = \frac{1}{p} \frac{\partial f}{\partial \eta^*} (\mu - \eta^*) \\ \frac{\partial f}{\partial p} + (M - \mu) \frac{1}{p} \frac{\partial f}{\partial \eta^*} &= \frac{1}{p} \frac{\lambda - \kappa}{(1 + e_0) \mu} (M - \eta^*)\end{aligned}\quad (21)$$

From Eqs. (20) and (21), it will be seen that $\dot{\epsilon}_{ij} \dot{\epsilon}_{ij} \rightarrow \infty$ as $\eta^* \rightarrow M$. This holds true even in the case $\dot{\xi} < 0$. The following relations (dilatancy equations) hold:

$$\begin{aligned}\dot{v}^V / |\dot{\gamma}^V| &= \mu - \eta^* \quad \text{when } \dot{\xi} > 0 \\ -\dot{v}^V / |\dot{\gamma}^V| &= \mu - \eta^* \quad \text{when } \dot{\xi} < 0\end{aligned}\quad (22)$$

The bulk and shear moduli are assumed to

be given by

$$K = \frac{\kappa}{1+e_0} p, \quad G = \frac{G_0}{p_0} p \quad (23)$$

Two particular cases are presented in the following. First, let stress, strain and the initial structure be isotropic, and substitute the relations

$$\left. \begin{aligned} s_{ij}s_{ij} &= \dot{s}_{ij}\dot{s}_{ij} = \bar{s}_{ij}\bar{s}_{ij} = q^* = \eta^* = 0 \\ r_{ij}r_{ij} &= \dot{r}_{ij}\dot{r}_{ij} = |\dot{r}^V| = 0 \end{aligned} \right\} \quad (24)$$

into Eqs. (15) and (16). Then

$$\left. \begin{aligned} \dot{v} &= \dot{v}^E + \dot{v}^V, \quad \dot{v}^E = \frac{\dot{p}}{K} = \frac{\kappa}{1+e_0} \frac{\dot{p}}{p} \\ \dot{v}^V &= \begin{cases} \frac{\partial F^+}{\partial f} \frac{\lambda - \kappa}{1+e_0} \frac{\dot{p}}{p} + \frac{\partial F^+}{\partial t} & \text{when } \dot{v}^V > 0 \\ -\frac{\partial F^-}{\partial f} \frac{\lambda - \kappa}{1+e_0} \frac{\dot{p}}{p} - \frac{\partial F^-}{\partial t} & \text{when } \dot{v}^V < 0 \end{cases} \end{aligned} \right\} \quad (25)$$

Second, let $\alpha, \dot{\xi}_0 \rightarrow 0$, and suppose

$$\frac{\partial F^+}{\partial f} = -\frac{\partial F^-}{\partial f} = 1, \quad \frac{\partial F^+}{\partial t} = \frac{\partial F^-}{\partial t} = 0 \quad (26)$$

Then the constitutive equations are reduced to those in which time effects are disregarded (Shibata and Sekiguchi, 1980).

The material parameters are composed of four kinds of initial parameters (e_0 , p_0 , $s_{ij}(t=0)$ and G_0) and nine kinds of constant parameters (M , μ , λ , κ , α , $\dot{\xi}_0$, p_0^+ , p_0^- and $\bar{s}_{ij}$). However there exist in essence only eight kinds of constant parameters, because the three parameters $\dot{\xi}_0$, p_0^+ and p_0^- act only in two forms of lumped parameters (see Appendix 2). The parameter η_0^* is determined by the model itself from the material parameters and the initial ($t=0^+$) loading condition.

MODEL SIMULATIONS

In the following, three typical types of triaxial tests on imaginary soils are simulated by the present model in order to examine the model's performance, and comparisons between predictions and reported measurements are made for undrained triaxial tests on Drammen clay.

Isotropic Consolidation Tests ($\dot{\xi} > 0$ or $\dot{\xi} < 0$)

The first type of simulation deals with

Table 1. Material parameters of soil A for the simulations whose results are shown in Figs. 9–10

$\lambda=0.28$	$\kappa=0.03$	$e_0=1.5$	$p_0^+/p_0=2.0$
$\alpha=0.01$	$\dot{\xi}_0=10^{-4}$ per unit time		$p_0^-/p_0=0.3$

Fig. 9. Simulation for an isotropic consolidation test : cyclic effect

Fig. 10. Simulation for isotropic consolidation tests : rate effect

Table 2. Material parameters of soil B for the simulations whose results are shown in Figs. 11–16

$M=1.5$	$\mu=1.0$	$p_0^*/p_0=1.0$	$\frac{\lambda}{1+e_0}=0.035$
$\alpha=0.01$	$\dot{\xi}_0=10^{-6}$ per unit time		$\frac{\kappa}{1+e_0}=0.005$
$\bar{q}=0$	initial $q=0$	$G_0/p_0=100$	

Note that μ , $\bar{q}$ and initial q have different values in some simulations for Figs. 13–16

isotropic consolidation tests on an imaginary soil A whose material parameters are listed in Table 1, where Eq. (24) is supposed to hold. This soil A might be regarded as a clay.

The numerical result for $|\dot{p}|/p$ -constant cyclic loading is shown in Fig. 9. The void ratio decreases at a progressively decreasing rate with an increasing number of cycles.

Fig. 11. Simulation for p -constant monotonic triaxial compression tests

Fig. 12. Simulation for p -constant cyclic triaxial tests ($|\dot{\gamma}|=10^{-3}$ per unit time)

The quasi-overconsolidation effect is observed.

Fig. 10 presents the numerical results for different $|\dot{p}|/p$ -constant loading. Different curves are obtained according to the rate of loading.

p-constant Triaxial Tests ($p=p_0$, $\dot{p}=0$, $\dot{\xi}>0$)

The second type of simulation deals with *p*-constant triaxial tests on an imaginary soil B in which Eqs. (A.7) shown in Appendix 3 hold and whose material parameters are listed in Table 2. Here $\dot{\xi}>0$ all the time. This soil B might be regarded as a dense sand, but the value of the creep parameter α is so large that it is a quite imaginary soil. It is also important to note

that the 'drained' condition requires a sufficiently small value of $\dot{\eta}$ in practice.

Fig. 11 presents the numerical results for different $\dot{\eta}$ -constant loading. The symbols in the diagrams (and also in the following) are defined by

$$\left. \begin{aligned} \eta &= q/p, & \dot{\eta} &= (\dot{q} - \eta \dot{p})/p \\ q &= \sigma_a - \sigma_r, & \dot{q} &= \dot{\sigma}_a - \dot{\sigma}_r \\ \gamma &= (\epsilon_a - \epsilon_r)2/3, & \dot{\gamma} &= (\dot{\epsilon}_a - \dot{\epsilon}_r)2/3 \end{aligned} \right\} \quad (27)$$

where σ_a and σ_r are the normal stresses, $\dot{\sigma}_a$ and $\dot{\sigma}_r$ are the normal stress rates, ϵ_a and ϵ_r are the normal strains, $\dot{\epsilon}_a$ and $\dot{\epsilon}_r$ are the normal strain rates, and the subscripts 'a' and 'r' refer to the axial and radial components respectively. In Fig. 11, it is seen that the responses vary with the rate of loading, but it is found that $\eta - \dot{p}/\dot{\gamma}$ curves ap-

Fig. 13. Simulation for undrained triaxial tests with one-way stress-controlled cyclic loading ($|\dot{q}|/p_0=10^{-3}$ per unit time) by using the properties in Table 2 except μ

proach a line as η increases.

The numerical results for cyclic triaxial tests at a constant $|\dot{\eta}|$ are shown in Fig. 12. The dilatancy loops (in $\eta-v$ curve) are exhibited only when the amplitude of the stress ratio is greater than μ . The re-

sponses approach (pseudo-)elastic behaviour with an increasing number of cycles.

Undrained Triaxial Tests ($\dot{v}=0$, $\dot{\gamma}=\dot{\epsilon}_a$, $\dot{\xi}>0$)

The third type of simulation deals with

Fig. 14. Simulation for undrained triaxial tests with two-way stress-controlled cyclic loading ($|\dot{q}|/p_0=10^{-3}$ per unit time) by using the properties in Table 2 except μ and $\bar{q}$

undrained triaxial tests, where also $\dot{\xi} > 0$ all the time. The results are shown in Figs. 13 and 14 for stress-controlled loading and in Figs. 15 and 16 for strain-controlled loading. The main simulations are performed for the above soil B. Note that the material param-

eters μ , $\bar{q}$ and initial q do not have the same values in all of the simulations indicated in the diagrams; because the influence of these parameters is of some interest. When $\mu = M$, the soil might be regarded as a loose sand.

Fig. 15. Simulation for an undrained triaxial test with strain-controlled cyclic loading ($|\dot{\epsilon}_a| = 10^{-3} \%$ per unit time) by using the properties in Table 2 but with $\mu = 1.5$

Fig. 16. Simulation for undrained triaxial tests with strain-controlled monotonic loading by using the properties in Table 2 except μ and initial q

Fig.13 presents the numerical results for monotonic and one-way cyclic loadings, and the influence of the relation between μ and M is observed. When $\mu < M$, the response approaches (pseudo-)elastic behaviour with the increase in the number of cycles. When $\mu = M$, with the increasing number of cycles, the response approaching elastic behaviour is followed by the growth of shear deformation, until ultimately $\gamma = \infty$.

Fig.14 presents the numerical results for two-way cyclic loadings. A wide difference in stress and strain paths is observed according to the different relation between μ and M . A marked anisotropy is exhibited for $\bar{q} \neq 0$.

Fig.15 presents the numerical results for

a two-way cyclic loading at a constant $|\dot{\epsilon}_a|$. It is seen that p , the amplitude of q and the slope of the loop of the q - γ curve decrease with the increase in the number of cycles.

Fig.16 presents the numerical results for different $|\dot{\epsilon}_a|$ -constant loadings along with the results calculated by using Eq. (26) in order to disregard the term of time. The effect of strain rate is observed. Anisotropic paths due to the initial stress anisotropy ($\sigma_r/\sigma_a = 0.5$) are also observed. When $\mu < M$, the phase transformation line appears and strain hardening occurs. When $\mu = M$, with the exception of the case disregarding the term of time, strain softening occurs.

Fig. 17. Comparison between predicted and measured (Andersen et al., 1980) behaviour of normally consolidated Drammen clay in a strain-controlled ($\dot{\epsilon}_a = 3\%/\text{hr}$) undrained triaxial compression test: (a) pore pressure response; (b) stress path; (c) stress-axial-strain curve

Undrained Triaxial Tests on Drammen Clay

Finally, the model predictions for undrained triaxial tests on normally consolidated Drammen clay are compared with the experimental results of Andersen et al. (1980).

The relations among the mean normal stress p , the normal stress difference q , the axial strain ϵ_a and the pore pressure u in slow strain-controlled triaxial compression ($\dot{\epsilon}_a = 3\%/hr$) are shown in Fig. 17. The values of the constitutive parameters α , ξ_0 and G_0 were not given explicitly in the paper by Andersen et al. (1980). These values were determined through a trial-and-error procedure so that the predicted behaviour may reproduce the measured behaviour with a reasonable accuracy. The calculation

whose results are shown in Fig. 17 is, rather, that which was used to determine the material parameters required in the following prediction for cyclic loading.

Fig. 19 shows the development of mean axial strain and mean pore pressure with an increasing number of cycles of one-way stress-controlled loading. Cyclic loads applied here are the sinusoidal pulses with a period of 10 seconds, as shown in Fig. 18(a). The mean axial strain represents the average axial strain in a cycle as shown in Fig. 18(b), and so does the mean pore pressure.

From Fig. 19, qualitatively good correspondence is obtained between the model predictions and the actual measurements. There is room for quantitative improvement, and the actual values of α and ξ_0 may be smaller

Fig. 18. Illustration of stress application and axial strain response :
(a) stress pulse ; (b) definition of mean axial strain ϵ_{am}

Fig. 19. Comparison between predicted and measured (Andersen et al., 1980) behaviour of normally consolidated Drammen clay subjected to one-way stress-controlled cyclic loading (sinusoidal pulses with a frequency of 0.1 Hz, Fig. 18(a)) in undrained triaxial tests : development of (a) mean axial strain ; (b) mean pore pressure

than those used here.

CONCLUSIONS

A viscoplastic constitutive model has been presented for soils subjected to monotonic and cyclic loading under general stress conditions. The relation $\mu=M$ holds for clays and loose sands, and $\mu<M$ holds for dense sands. Three kinds of anisotropy in stress and strain paths can be described: anisotropy due to anisotropic initial stress, anisotropy due to anisotropic initial structure and anisotropy due to loading reversal. The new stress-ratio parameter η^* is the key to such description. Anisotropy in the critical stress ratio is realized by the initial structural anisotropy ($\bar{s}_{ij}\bar{s}_{ij}\neq 0$).

Performance characteristics of the model have been investigated by simulating a number of typical triaxial tests on imaginary soils. Furthermore, comparisons with measurements on normally consolidated Drammen clay have been made. The results indicate qualitatively the applicability of the model to a wide range of loading conditions. Examination of the predictive capability of the present model shall be a subject for future study.

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APPENDIX 1

Derivation of Eq. (3)

When $p \geq p_c$, from Eq. (1),

$$v = \frac{e_0 - e}{1 + e_0} = \frac{\kappa}{1 + e_0} \ln\left(\frac{p}{p_0}\right) + f - f_c \quad (\text{A.1})$$

in which f and f_c are given by Eq. (3).

When $t \geq t_N$ ($p = \text{constant}$), from Eq. (2),

$$dv/dt = \alpha/t$$

Supposing that a series of points (e , p) at the end of the primary consolidation is the normal consolidation line, then

$$\frac{1}{\dot{v}_N} \frac{dv}{dt} = \frac{t_N}{t} \quad (\text{A.2})$$

Eqs. (2) and (A.2) yield

$$v = C_1 + \alpha \cdot \ln\left(\frac{t}{t_N}\right) = C_1 - \alpha \cdot \ln\left(\frac{1}{\dot{v}_N} \frac{dv}{dt}\right) \quad (\text{A.3})$$

where C_1 is an integral constant. On the above supposition, C_1 is found from Eqs. (A.1) and (A.3) to be given by

$$v_{(t=t_N)} = C_1 = \frac{\kappa}{1 + e_0} \ln\left(\frac{p}{p_0}\right) + f - f_c$$

Hence

$$v = \frac{\kappa}{1 + e_0} \ln\left(\frac{p}{p_0}\right) + f - f_c - \alpha \cdot \ln\left(\frac{1}{\dot{v}_N} \frac{dv}{dt}\right) \quad (\text{A.4})$$

When p is constant with time, the solution of Eq. (A.4) with respect to t can be given by

$$v = \alpha \cdot \ln \left[C_2 + \frac{\dot{v}_N}{\alpha} t \cdot \exp \left\{ \frac{\frac{\kappa}{1 + e_0} \ln\left(\frac{p}{p_0}\right) + f - f_c}{\alpha} \right\} \right] \quad (\text{A.5})$$

where C_2 is an integral constant which is given by assuming

$$v_{(t=0^+)} = \alpha \cdot \ln C_2 = \frac{\kappa}{1 + e_0} \ln\left(\frac{p}{p_0}\right)$$

Therefore

$$\left. \begin{aligned} v &= v^E + v^V, \quad v^E = \frac{\kappa}{1+e_0} \ln\left(\frac{p}{p_0}\right) \\ v^V &= \alpha \cdot \ln\left[1 + \frac{\dot{v}_N}{\alpha} t \cdot \exp\left(\frac{f-f_c}{\alpha}\right)\right] \end{aligned} \right\} \quad (\text{A.6})$$

However, when $p \neq \text{constant}$, Eq. (A.5) cannot be derived, and likewise neither can Eq. (A.6). The nature of the Sekiguchi-Ohta model, in which the initial state is defined as the reference state just prior to the change of p and the initial time is redefined every moment, enables Eq. (A.5) to hold true. When the sole initial state ($t=0$, $e=e_0$, $p=p_0$) is defined, the problem will be settled by modification of Eq. (A.6) to Eq. (3).

For reference to the above derivation, see Sekiguchi (1984).

APPENDIX 2

Explanation of Eq. (6)

Consider the right-hand side of the third equation in Eq. (5). Since f^+ is constant from t_r to t , it follows that

$$\begin{aligned} & \frac{\dot{v}_N}{\alpha} \int_{t_r}^t \exp\left(\frac{f-f^+}{\alpha}\right) dt \\ &= \frac{\dot{v}_N}{\alpha} \left(\frac{p_0}{p_c}\right)^\beta \cdot \exp\left(-\frac{1}{\alpha} \int_0^{t_r} \dot{v}^V dt\right) \cdot \int_{t_r}^t \exp\left(\frac{f}{\alpha}\right) dt \end{aligned}$$

where

$$\beta = \frac{\lambda - \kappa}{(1+e_0)\alpha}$$

In the above, the term $\dot{v}_N(p_0/p_c)^\beta$ is a constant. Similarly, the term $\dot{v}_N(p_s/p_0)^\beta$ is also a constant. Hence, a set of constants p_c , p_s and $\dot{v}_N$ in Eq. (5) can be replaced by the other set of constants p_0^+ , p_0^- and $\dot{v}_0$ if $\dot{v}_0(p_0/p_0^+)^\beta = \dot{v}_N(p_0/p_c)^\beta$ and $\dot{v}_0(p_0^-/p_0)^\beta = \dot{v}_N(p_s/p_0)^\beta$. Note that these relations are identical to Eq. (6).

When evaluated experimentally, three parameters p_0^+ , p_0^- and $\dot{v}_0$ will be variable according to the rate of loading, but two lumped parameters $\dot{v}_0(p_0/p_0^+)^\beta$ and $\dot{v}_0(p_0^-/p_0)^\beta$ will be constant independently of the loading rate. This means that these lumped parameters should be chosen as the material parameters.

APPENDIX 3

Equations Derived from Eqs. (13)

From the definition of Eqs. (13),

$$\begin{aligned} \frac{\partial \eta^*}{\partial p} &= -\frac{q^* - \bar{q}}{p^2} = -\frac{\eta^*}{p} \quad \text{and} \\ \frac{\partial \eta^*}{\partial s_{ij}} &= \frac{\partial \eta^*}{\partial q^*} \frac{\partial q^*}{\partial q_{kl}} \frac{\partial q_{kl}}{\partial s_{ij}} = \frac{1}{p} \frac{q_{kl}}{q^*} \sqrt{\frac{3}{2}} \delta_{kl} \delta_{ij} \\ &= \frac{1}{p} \sqrt{\frac{3}{2}} \frac{q_{ij}}{q^*} \end{aligned}$$

which induce Eq. (15).

The following presents η^* in a special case:

Let the stress and the stress rate be in an axi-symmetric state, and suppose that their shear components do not exist. Then

$$\left. \begin{aligned} s_{ij} &= \dot{s}_{ij} = 0 \quad (\text{for } i \neq j) \\ q &= s_a - s_r = s_a/3/2, \quad \dot{q} = \dot{s}_a/3/2 \end{aligned} \right\} \quad (\text{A.7})_1$$

Supposing that the deviatoric stress due to the initial structure is the same,

$$\left. \begin{aligned} \bar{s}_{ij} &= 0 \quad (\text{for } i \neq j) \\ \bar{q} &= \bar{s}_a - \bar{s}_r = \bar{s}_a/3/2 \end{aligned} \right\} \quad (\text{A.7})_2$$

In the above, s_a and s_r are respectively the axial and radial components of s_{ij} , and likewise $\bar{s}_a$ and $\bar{s}_r$ are components of $\bar{s}_{ij}$. Moreover the definition

$$\left. \begin{aligned} \eta' &= q'/p, \quad \dot{\eta}' = (\dot{q}' - \eta' \dot{p})/p \\ q' &= (s_a - \bar{s}_a) - (s_r - \bar{s}_r) = q - \bar{q}, \quad \dot{q}' = \dot{q} \end{aligned} \right\} \quad (\text{A.8})$$

is given. In this case, throughout the stress paths as shown in Figs. 11-19, η^* defined by Eqs. (13) can be rewritten in the simple form:

$$\left. \begin{aligned} \eta^* &= \eta' \quad \text{when } \dot{\eta}' > 0 \\ \eta^* &= -\eta' \quad \text{when } \dot{\eta}' < 0 \end{aligned} \right\} \quad (\text{A.9})$$

It therefore seems that Eqs. (A.7) are satisfied in a triaxial specimen sampled vertically from a natural deposit which is horizontally homogeneous. Eqs. (A.7) will be nearly satisfied in a triaxial specimen sampled horizontally from such a deposit. Let $\bar{q}$ in the former be $\bar{q}_V$, and let $\bar{q}$ in the latter be $\bar{q}_H$. Then it will be true that $\bar{q}_V = -\bar{q}_H$.