ELASTO-PLASTIC BEHAVIOR OF K_0 -CONSOLIDATED CLAY IN TORSION SHEAR TESTS

WON PYO HONG¹⁾ and POUL V. LADE¹⁾

ABSTRACT

A series of torsion shear tests was performed along various stress-paths on hollow cylinder specimens of K_0 -consolidated clay to investigate the influence of rotation of principal stresses on the stress-strain and strength characteristics. The effects of stress-paths and reorientation of principal stresses were mainly observed in the prefailure stress-strain behavior. The experimentally obtained failure surface from torsion shear tests could practically be modeled by an isotropic failure criterion. Coupling effects between stresses and strains were observed when torsion shear and vertical normal stresses were applied. The work-space for torsion shear tests was illustrated, and the relation between stresses and strain increments was also studied in the work-space.

Key words : clay, consolidated undrained shear, failure, plasticity, shear strength, torsion, (IGC : D 6)

INTRODUCTION

The stress-strain behavior of natural clay deposits depends on the cross-anisotropy produced by K_0 -consolidation in the field. Tri-axial tests with independent control of the three principal stresses on cubical specimens have been performed to investigate the influence of principal stresses, especially the intermediate principal stress, on the behavior of clay (e. g., Lade and Musante, 1978).

Construction on or in clay deposits causes rotation of principal stress directions as well

as changes in the magnitudes of stresses. The reorientation and change in magnitude of principal stresses during loading may greatly affect the stress-strain behavior of such deposits. To enhance the knowledge and understanding of soil behavior, the effects of reorientation and change in magnitude of principal stresses should be investigated. However, in triaxial tests, including cubical triaxial tests, the directions of principal stresses cannot be rotated during loading.

The purpose of the present study is to investigate the influence of rotation of prin-

¹⁾ Associate Professor, Department of Civil Engineering, College of Engineering, Chung-Ang University, Seoul, 156-756, Korea.

¹⁾ Professor, Department of Civil Engineering, School of Engineering and Applied Science, University of California, Los Angeles, California 90024, USA.

Manuscript was received for review on June 21, 1988.

Written discussions on this paper should be submitted before January 1, 1990, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

principal stresses on the stress-strain, pore pressure and strength characteristics of K_0 -consolidated clay. A series of undrained and drained torsion shear tests were performed along various stresspaths on hollow cylinder specimens. The stress-paths were designed to cover the full range of vertical deviator stresses from compression to extension. The relation between stresses and strain increments was also studied in the work-space.

PREVIOUS STUDIES

The effects of rotation of principal stress directions have been investigated by use of the simple shear device and the torsion shear apparatus. The behavior of soil specimens in the simple shear device has been studied extensively by Roscoe (1953), Roscoe et al. (1967), and Bjerrum and Landva (1966). However, Wright et al. (1978) showed that the uniformity of the shear stresses on the central plane of the simple shear specimen is questionable. Generally, the simple shear device has the following disadvantages: Complementary shear stresses cannot be sustained on the vertical boundaries, the stress and strain states within the specimen lack uniformity, and the evolution of the horizontal, normal stress during shear is generally unknown (Saada and Townsend, 1981).

Broms and Casbarian (1965) performed three series of consolidated-undrained torsion shear tests on hollow cylinder specimens of a remolded kaolinite clay to study the influence of stress rotation and the intermediate principal stress on the strength characteristics. Torsion shear tests have since been performed to a limited extent (Lade, 1975, 1976, 1981; Saada and co-workers, 1967, 1973, 1975, 1984; Geiger and Lade, 1979; Hight et al., 1983; Symes et al., 1984; Tatsuoka et al., 1986; Hicher and Lade, 1987).

The main advantage of the torsion shear test is that it allows inclination of the major principal stress in any desired direction while complementary shear stresses are ideally applied in the specimen. The angle between the major principal stress and vertical, Ψ ,

is tied to the parameter b ($=(\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$), which indicates the relative magnitude of the intermediate principal stress σ_2 . The relation between b and Ψ is given by $b = \sin^2 \Psi$ (Geiger and Lade, 1979). Therefore, the influence of σ_2 on the strength characteristics can be also investigated in the torsion shear apparatus.

In a torsion shear test, the hollow cylinder specimen is exposed to a plane stress state when the inside and outside pressures are equal. In order to achieve this state of plane stress, the stresses and strains should be distributed uniformly in the specimen (Saada and Townsend, 1981). The uniformity can be maximized by employment of appropriate dimensions for the specimen. Experimental and theoretical studies (Lade, 1981; Wright et al., 1978; Hight et al., 1983) have been performed to produce appropriate specimen geometries in which the non-uniformity in stresses and strains within the specimen have been minimized.

Geiger and Lade (1979) studied the behavior of cohesionless soil during large stress reversals and reorientation of principal stresses in torsion shear tests. On the basis of a limited number of torsion shear tests performed on medium-loose sand, Symes et al. (1984) showed that the initial anisotropy could be changed significantly by the loading path. An excellent review and study of the behavior of sand in torsion shear tests was presented by Tatsuoka et al. (1986).

Saada and his co-workers have performed several investigations (1967, 1973, 1975, 1984) to study the influence of anisotropy on the stress-strain behavior of clay in torsion shear tests. The influence of cyclic loading on the behavior of clay in torsion shear tests has also been studied in the past (Hicher and Lade, 1987).

TORSION SHEAR APPARATUS

Fig. 1 shows a three-dimensional drawing of a torsion shear apparatus designed and constructed to permit individual control of confining pressure, vertical deviator stress

Fig. 1. Torsion shear apparatus

and shear stress to a hollow cylinder specimen. The design and operational principles of this apparatus are given by Lade (1981).

The hollow cylinder specimen employed for testing of clay has a height of 25 cm, an average diameter of 20 cm, and a wall thickness of 2 cm. Based on previous studies (Lade, 1981) these dimensions were found to be sufficient to neglect nonuniformities in stresses and strains in the specimen. The specimen is confined between cap and base rings and between inside and outside rubber membranes with thickness of 0.03 to 0.04 cm. The inside membrane was formed directly on the specimen while the outside prefabricated membrane was placed using a membrane stretcher. In order to form the inside membrane, the specimen was first

sprayed with diluted rubber cement to seal it off. The inside surface was then sprayed or painted with several layers of fluid latex rubber. The fluid rubber was allowed to dry before another layer was applied. In order to transfer shear stresses from the cap and base rings to the specimen and to avoid slippage at their interfaces, full friction surfaces were provided on the rings. These consisted of a layer of sand grains glued to the rings by epoxy.

The entire setup is contained in a pressure cell, and the integrated hydraulic loading system is located below the table on which the apparatus sits. The same confining pressure is applied to the inside and outside surfaces of the specimen through the cell water that surrounds the specimen.

For the purpose of performing torsion shear tests on clay specimens, some modifications were made to the vertical loading cylinder and the torsion shear loading cylinders. An upward loading capability was added to the vertical loading cylinder. And the strokes of the torsion shear loading cylinders were increased so that larger torsional displacements can be imposed on the specimen. The maximum strains that can be achieved in this apparatus for a 25 cm tall specimen are $\epsilon_z = \pm 25\%$ for vertical strains and $\gamma_{z\theta} = \pm 40\%$ for torsion shear strains, respectively.

The vertical load and the torque can be applied independently to the specimen through the center shaft and the cap plate, which is attached to the cap ring, as shown in Fig.1. Both the vertical load and the torque can be either stress- or strain- controlled. The resulting deviator stress, together with the cell pressure, provides for a vertical, normal stress either larger or smaller than the confining pressure. Two sets of two interconnected pressure cylinders supply the horizontal load in either clockwise or counterclockwise direction to the torsion arm connected to the center shaft.

During the torsion shear tests, the vertical load, the torque, the cell pressure, the vertical deformation, the change in thickness of

the wall, the outside and inside diameter changes, and the shear deformation were measured in addition to the pore pressure in undrained tests or the volumetric strain in drained tests.

CLAY TESTED

All tests were performed on Edgar Plastic Kaolinite (EPK) clay whose specific gravity was 2.62. The particle size distribution indicated that the clay consisted of 40% silt and 60% clay size particles. Atterberg limit tests produced $LL=60$ and $PL=30$. The activity was 0.5.

SPECIMEN PREPARATION

Remolded normally consolidated specimens at the K_0 -stress condition were prepared from a slurry of EPK clay mixed at a water content of twice the liquid limit. The clay slurry was poured into a special consolidometer and consolidated at a vertical pressure of 2.00 kgf/cm^2 (196 kN/m^2). The special consolidometer was built to make hollow cylindrical specimens for torsion shear tests. The consolidometer has a height of approximately 60 cm, an outer diameter of 26 cm, and an inner diameter of 16.5 cm. Two drainage plates consisting of porous plastic were placed on top and bottom of the clay slurry. In addition, long drainage strips made of porous plastic were attached along the outside wall to form a spiral from top to bottom, so it could be compressed easily as the clay slurry consolidated. After consolidation, the oversized hollow cylindrical clay sample was removed from the consolidometer and trimmed to its final dimensions in a specially designed trimming device with a rotating table and common trimming tools.

The specimen was then installed in the torsion shear apparatus and again consolidated under K_0 -stress conditions by maintaining the volumetric strain equal to the vertical strain. Slotted filter paper drains were installed on the outside face of the specimen. The maximum drainage path was therefore

equal to the thickness of the specimen wall (2 cm). During K_0 -consolidation, a cell pressure of 4.00 kgf/cm^2 (392 kN/m^2) and a back pressure of 2.00 kgf/cm^2 (196 kN/m^2) were applied. The average value of K_0 for all tests was 0.55. The B -values ($=\Delta u/\Delta \sigma_{\text{cell}}$) measured after consolidation indicated that the specimens were fully saturated.

TESTING PROGRAM

Torsion Shear Tests

Fifteen undrained and two drained tests were performed on hollow cylinder specimens consolidated under K_0 -stress conditions. Each test was conducted with constant confining pressure, σ_r , according to predetermined stress-paths, as shown in Fig. 2. The diagrams in this figure indicate the relations between the vertical stress difference, $(\sigma_z - \sigma_\theta)$, and the torsion shear stress, $\tau_{z\theta}$. In order to follow these stress-paths to failure and beyond, either the vertical stress difference or the torsion shear stress was applied under strain-control, and the other stress was stress-controlled. The vertical stress difference could be either increased or decreased. Strain rates of approximately 0.3 %/h for undrained tests and 0.06%/h for drained tests were employed for both vertical strain and shear strain in the strain-controlled directions. Based on a previous study of EPK clay (Kirkgard, 1981), these strain rates were found to be sufficiently slow for equalization of pore pressures in undrained tests and pore pressure dissipation in drained test.

The shear stress due to the torque produces reorientation of the principal stress directions as well as a stress state with three unequal principal stresses. If shear stresses are not applied to the specimen, the confining pressure σ_r becomes the minor principal stress σ_3 in a compression test, or the major principal stress σ_1 in an extension test. With torsional shear stresses the confining pressure becomes the intermediate principal stress σ_2 .

The angle between σ_1 and vertical, Ψ , can be calculated from the following expression

Fig. 2. (a) Stresses in hollow cylinder specimen, (b) and (c) stress-paths in torsion shear tests

$$\tan 2\psi = \frac{2 \cdot \tau_{z\theta}}{\sigma_z - \sigma_\theta} \quad (1)$$

The value of ψ can be varied between 0° in triaxial compression to 90° in triaxial extension. The value of b ($=\sin^2 \psi$) can be chosen between zero and unity, which covers the full range of the intermediate principal stress.

Stress-Paths

All shear tests were started after K_0 -consolidation in the torsion shear apparatus. The stress-paths projected on the plane of $\sigma'_r = 1.00$ kgf/cm² (98 kN/m²) are shown in Figs. 2. Test No. 11 was performed as a triaxial compression test by increasing the vertical load under strain-control. Test Nos. 3, 4, 5, 6 and 13 were performed by increasing

the torque under strain-control with only small changes of the vertical load. Test Nos. 2, 7, 9, 10 and 15 were performed by adjusting the vertical stress difference to the predetermined values under stress-control and then increasing the torque under strain-control. A leak developed in Test No. 7, and it could not be continued, as indicated in Fig. 2 (b). Test Nos. 8, 16 and 17 were performed by increasing the torsion shear stress under stress-control and then decreasing the vertical load under strain-control. Test Nos. 1, 12 and 14 were performed by decreasing the vertical load under strain-control to produce triaxial extension tests.

All except two tests were performed under undrained conditions. Test Nos. 14 and 17 were drained tests.

Triaxial Compression Tests

For comparison with the torsion shear tests, a series of triaxial compression tests with lubricated end plates were performed on cylindrical specimens with diameter=7.1 cm and height-to-diameter ratio of 1.0. The specimens were consolidated under isotropic and K_0 -stress conditions and sheared at strain rates of 0.52%/h in undrained tests

and 0.26%/h in drained tests.

In addition, triaxial compression tests were performed in a companion study on vertical and horizontal specimens of K_0 -consolidated EPK clay. These produced stress-strain relations and undrained strengths that indicated negligible influence of orientation. The laboratory prepared clay was therefore taken to behave in an isotropic manner, although

Fig. 3. (a) and (b) stress-strain, (c) excess pore pressure, and (d) effective stress ratio variations obtained in two typical torsion shear tests on Edgar Plastic Kaolinite

it had initially been K_0 -consolidated. More detailed studies of the clay behavior might have indicated some anisotropic features, but these were not likely to be as pronounced as found in natural clays.

Natural clays have been produced by deposition of individual clay particles or clusters of particles, each allowed to settle and come to equilibrium with preferred orientation (most likely horizontal), and consolidated under K_0 -conditions over long time periods, thus allowing the majority of creep to occur under current in-situ stresses. In comparison, the slurry of EPK clay mixed at two times the liquid limit may not have provided the individual particles with sufficient space to allow extensive alignment in a preferred direction. Further, sufficient time was allowed for the slurry to consolidate, but the amount of time allowed for creep after the end of primary consolidation was small. The EPK clay prepared for the present study therefore exhibited little anisotropy as a result of the initial K_0 -consolidation. The experimental results of the torsion shear tests are consequently evaluated in view of isotropic plastic behavior.

TEST RESULTS

The results from Test Nos. 10 and 16 are shown in Fig. 3 to demonstrate the type of behavior obtained in the torsion shear tests. The vertical stress difference, $(\sigma_z - \sigma_\theta)$, the torsion shear stress, $\tau_{z\theta}$, the pore pressure, Δu , and the effective principal stress ratio, σ'_1/σ'_3 , are plotted versus the torsion shear strain, $\gamma_{z\theta}$, in this figure. The stress-paths for these two tests are shown in Fig. 2.

The torsion shear stress in Test No. 10 was gradually increased causing increasing shear strain until the specimen failed, as shown in Fig. 3 (b). Strain softening developed after peak failure. In Test No. 16 the vertical stress difference was decreased while the shear strain increased until the specimen failed, as shown in Fig. 3 (a). The points corresponding to failure according to the maximum effective stress ratio are indicated

on each curve. Although the specimens failed at significantly different values of torsion shear stress and vertical stress difference, there is only little difference in shear strain at failure.

Fig. 3 (c) shows that a negative pore pressure developed in Test No. 10 due to the reduction in vertical deviator stress, followed by a gradual increase caused by the applied torsion shear stress. The pore pressure in Test No. 16 increased due to the applied torque during the first part of the stress-path. The pore pressure then decreased during the initial reduction of the vertical deviator stress causing a negative pore pressure at the time of zero deviator stress. However, the pore pressure increased again as the vertical stress difference became negative.

Fig. 3 (d) shows that the effective principal stress ratio, σ'_1/σ'_3 , in Test No. 10 first decreased slightly with decreasing vertical stress and then increased with increasing torque until the specimen failed. σ'_1/σ'_3 in Test No. 16 increased slightly with application of torque, then decreased with decreasing vertical load. However, σ'_1/σ'_3 again began to increase when the vertical stress difference became negative.

A set of stress-strain and pore pressure relations was obtained from each test, as exemplified by the diagrams in Fig. 3. It is the interpretation of these relations that are significant in understanding soil behavior. The experimental results are therefore presented and interpreted below in terms of several aspects of soil behavior.

STRENGTH CHARACTERISTICS

In order to study the effective strengths obtained in the torsion shear tests, the strength results have been compared internally and with those from the triaxial compression tests. Since the laboratory prepared clay exhibited little anisotropy as a result of the initial K_0 -consolidation, the strength results were also compared with those predicted by an isotropic three-dimensional failure

criterion for soils. For torsion shear tests with the same inside and outside pressure, the effects of b ($=\sin^2 \Psi$) and of the orientation of σ_1 (as measured by Ψ) cannot be separately examined. However, since the clay was essentially isotropic in behavior, the effect of b is described by the isotropic, three-dimensional failure criterion (Lade and Musante, 1978).

Failure Criterion

A three-dimensional failure criterion expressed in terms of stress invariants has previously been developed for soils (Lade, 1977). This isotropic failure criterion is expressed as :

$$(I_1^3/I_3 - 27) \cdot (I_1/p_a)^m = \eta_1 \quad (2)$$

in which I_1 and I_3 are the first and the third invariants of the stress tensor, which for triaxial compression and torsion shear tests can be expressed as :

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_z + \sigma_r + \sigma_\theta \quad (3)$$

$$I_3 = \sigma_1 \cdot \sigma_2 \cdot \sigma_3 = \sigma_z \cdot \sigma_r \cdot \sigma_\theta - \sigma_r \cdot \tau_{z\theta} \cdot \tau_{\theta z} \quad (4)$$

and p_a is atmospheric pressure expressed in the same units as the stresses. The parameters η_1 and m are constants.

Fig. 4 shows the relation between $(I_1^3/I_3 - 27)$ and (p_a/I_1) at failure in a log-log diagram. On this diagram η_1 is the intercept with $(p_a/I_1)=1$ and m is the slope of the straight line. The data from both torsion shear and triaxial compression tests on EPK clay are plotted on Fig. 4. The values of

$\eta_1=27.1$ and $m=0.42$ are determined from a regression analysis for the best fitting straight line on this diagram.

Comparison of Failure Criterion and Test Data

The strength results from the torsion shear tests are presented on a diagram of torsion shear stress, $\tau_{z\theta}$, versus vertical stress difference, $(\sigma_z - \sigma_\theta)$, as shown in Fig. 5. The solid curve on this diagram represents the failure surface defined by Eq. (2). The failure surface is egg-shaped with a slightly more pointed end near the extension stress state. Whereas only one half of the surface is shown in Fig. 5, the other half is sym-

Fig. 4. Determination of material parameters η_1 and m involved in failure criterion for Edgar Plastic Kaolinite

Fig. 5. Stress points at failure and failure surface for torsion shear tests on Edgar Plastic Kaolinite

metrical with that shown around the horizontal axis. Note that the failure surface is not an ellipse. An ellipse is obtained for the Mohr-Coulomb failure criterion.

The stress states at failure in the torsion shear tests did not correspond to the same value of the effective radial stress σ'_r . However, it is desirable to compare the data points on the same effective radial stress plane such that a common basis for the comparison exists. The experimental points shown in Fig. 5 have therefore been projected to a common σ'_r -plane corresponding to $\sigma'_r = 1.00$ kgf/cm² (98 kN/m²). The adjusted stress states were obtained by projection of each individual stress point along a curved failure envelope. This is done mathematically using a technique involving Eq. (2) for the failure criterion.

The stresses at failure from all torsion shear tests are plotted on Fig. 5. The test results with compressive axial loads are slightly inside the failure envelope, while the test results close to $b=1.0$ were slightly outside the failure envelope. The largest difference occurred in the undrained tests close to $b=1.0$ in which the values of the effective minor principal σ'_3 were very small. Two drained tests were therefore performed to establish the failure envelope near the extension condition. The results of these two tests appear to steady the failure envelope near extension, as shown in Fig. 5. It may be seen that the failure envelope represents the failure points obtained from the torsion shear tests with reasonable accuracy. Thus, the stress conditions of K_0 -consolidated clay at failure were not much affected by stress-paths and rotation of principal stress directions. The experimentally obtained failure surface from torsion shear tests can therefore be modeled with good accuracy by the isotropic failure criterion defined by Eq. (2).

As indicated on Figs. 3 and 5, the effects of stress-paths and reorientation of principal stresses were mainly observed in the pre-failure stress-strain behavior of K_0 -consoli-

Fig. 6. Effective friction angles and failure surface shown in ϕ - b diagram for torsion shear tests on Edgar Plastic Kaolinite

dated clay, whereas sufficient changes in the clay fabric had occurred at large strains to produce failure conditions that adhered to an isotropic failure criterion.

Effective Friction Angle

The variation of the measured effective friction angle with the value of b is shown in Fig. 6. The friction angle ϕ' was computed from

$$\sin \phi' = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} = \frac{2\sqrt{\tau_{z\theta}^2 + (\sigma_z - \sigma_\theta)^2/4}}{(\sigma'_z + \sigma'_\theta)} \quad (5)$$

where $\sigma'_\theta = \sigma'_{\text{cell}}$. The friction angle is affected by the b -value which in turn represents the intermediate principal stress. The friction angle was smallest for $b=0.0$ and it increased initially with increasing magnitude of the intermediate principal stress σ'_2 until b reached 0.6. The friction angle was largest at $b=1.0$ in undrained tests. However, as discussed in connection with Fig. 5, the results of the undrained tests were less reliable than those from the drained tests. The friction angles from the drained tests, corresponding to b -values near 1.0,

were smaller than those from the undrained tests. Thus, a small decrease in friction angle from $b=0.6$ to 1.0 is indicated. Cubical triaxial tests performed on clay and sand have shown the same tendency (Lade and Duncan, 1973; Lade and Musante, 1978; Ochiai and Lade, 1983; Tsai and Lade, 1985).

The solid line on the diagram is the failure surface described by the criterion in Eq. (2). This failure criterion appears to account fairly well for the experimental data. Therefore, any influence of the orientation of σ_1 (as measured by ψ) appears to be negligible as expected for an isotropic material.

RELATIONS BETWEEN STRAINS

Work-Space

Fig. 7 illustrates a suitable space in which to view the results of torsion shear tests. The vertical axis represents the effective confining pressure, σ'_r , and the horizontal axis represents the vertical stress difference, $(\sigma_z - \sigma_\theta)$, and the torsion shear stress, $\tau_{z\theta}$. In this diagram the failure surface defined by Eq. (2) forms a cone with origin at $\sigma'_r=0$. The cross-section of the failure surface on a plane with $\sigma'_r=\text{const.}$ resembles an egg. The egg-shaped failure surface grows from

Fig. 7. Three-dimensional stress space suitable for study of data from torsion shear tests

the origin of the $(\sigma_z - \sigma_\theta)$ and $\tau_{z\theta}$ -axes, as indicated in Figs. 2 and 5. Failure does not occur for stresses inside this three-dimensional, conical surface.

The increment in work done by the stresses in torsion shear tests can be expressed as follows:

$$dW = \sigma_\theta \cdot \dot{\epsilon}_\theta + \sigma_z \cdot \dot{\epsilon}_z + \sigma_r \cdot \dot{\epsilon}_r + \tau_{zr} \cdot \dot{\gamma}_{zr} + \tau_{r\theta} \cdot \dot{\gamma}_{r\theta} + \tau_{z\theta} \cdot \dot{\gamma}_{z\theta} \quad (6)$$

where $\dot{\epsilon}_\theta$, $\dot{\epsilon}_z$, and $\dot{\epsilon}_r$ are linear strain increments, and $\dot{\gamma}_{zr}$, $\dot{\gamma}_{r\theta}$, and $\dot{\gamma}_{z\theta}$ are engineering shear strain increments. The boundary conditions in a torsion shear test are such that:

$$\begin{aligned} \tau_{zr} &= \tau_{r\theta} = 0 \\ \sigma_\theta &= \sigma_r = \sigma_{\text{cell}} \\ \dot{\gamma}_{zr} &= \dot{\gamma}_{r\theta} = 0 \end{aligned}$$

Thus,

$$dW = \sigma_\theta \cdot \dot{\epsilon}_\theta + \sigma_z \cdot \dot{\epsilon}_z + \sigma_r \cdot \dot{\epsilon}_r + \tau_{z\theta} \cdot \dot{\gamma}_{z\theta}$$

This expression can be further specialized for the conditions in the torsion shear test:

$$\begin{aligned} dW &= (\sigma_z - \sigma_\theta) \cdot \dot{\epsilon}_z + \sigma_\theta \cdot \dot{\epsilon}_z + \sigma_\theta \cdot \dot{\epsilon}_\theta + \sigma_r \cdot \dot{\epsilon}_r + \tau_{z\theta} \cdot \dot{\gamma}_{z\theta} \\ dW &= (\sigma_z - \sigma_\theta) \cdot \dot{\epsilon}_z + \sigma_r \cdot (\dot{\epsilon}_\theta + \dot{\epsilon}_z + \dot{\epsilon}_r) + \tau_{z\theta} \cdot \dot{\gamma}_{z\theta} \end{aligned}$$

Since the volumetric strain increment $\dot{\epsilon}_v = \dot{\epsilon}_\theta + \dot{\epsilon}_z + \dot{\epsilon}_r$,

$$dW = (\sigma_z - \sigma_\theta) \cdot \dot{\epsilon}_z + \sigma_r \cdot \dot{\epsilon}_v + \tau_{z\theta} \cdot \dot{\gamma}_{z\theta} \quad (7)$$

Therefore, to study the type of behavior observed in the space shown in Fig. 7, the strain increments, $\dot{\epsilon}_z$, $\dot{\epsilon}_v$, and $\dot{\gamma}_{z\theta}$ are superimposed on the stress axes corresponding to $(\sigma_z - \sigma_\theta)$, σ_r , and $\tau_{z\theta}$, respectively. The strain increments form a vector, $\dot{\epsilon}$, as exemplified by that initiating at the failure surface on Fig. 7.

The results of torsion shear tests on hollow metal tubes presented by Taylor and Quinney (1931) were analyzed in view of the following equation for the increment in work:

$$dW = \sigma_z \cdot \dot{\epsilon}_z + \tau_{z\theta} \cdot \dot{\gamma}_{z\theta} \quad (8)$$

Since the metal tubes were unconfined, $\sigma_\theta = \sigma_r = 0$, and for solid metals $\dot{\epsilon}_v = 0$. Eq. (8) is therefore a special case of the more general expression in Eq. (7) which allows for volumetric strains in the material.

Fig. 8. Relations between axial strains and engineering shear strains obtained in torsion shear tests on Edgar Plastic Kaolinite

Relations Between Axial and Shear Strains

The axial strains, ϵ_z , are plotted versus the engineering shear strains, $\gamma_{z\theta}$, in Fig. 8. The points corresponding to failure are indicated as black points on each curve in this diagram. The axial strains are compressive for tests with b -values smaller than 0.5 and expansive for larger b -values.

Note that considerable axial strains occurred in the tests in which the axial stress was held essentially constant. This coupling effect between shear stress and axial strain was observed in most cases when shear stresses were applied. Comparison of stress-paths in Fig. 2 (b) with strains in Fig. 8 (a) indicates this effect. A similar coupling effect between shear strains and axial stresses was observed in tests in which shear stress was applied and held constant and the axial stress was varied. This may be seen by comparison of stress-paths in Fig. 2 (c) with strains in Fig. 8 (b). These coupling effects between stresses and strains are typical of materials that behave plastically. They cannot be predicted by theory of elasticity.

Directions of Strain Increment Vectors

It is of interest to study the directions of plastic strain increment vectors relative to the failure surface in the work-space. If the failure surface is taken as the ultimate yield surface, it may be deduced whether associated flow is obtained for the clay. The increment

Fig. 9. Directions of strain increment vectors at failure in torsion shear tests on Edgar Plastic Kaolinite

axes of vertical strain, $\dot{\epsilon}_z$, and torsion shear strain, $\dot{\gamma}_{z\theta}$, are therefore superimposed on the vertical stress difference, $(\sigma_z - \sigma_\theta)$, and torsion shear stress, $\tau_{z\theta}$, respectively, in the work space shown in Fig. 7. Only the directions of the strain increment vectors at failure will be considered. All strain increments are plastic at failure, because the changes in stress at failure are negligible, thus resulting in negligible elastic strain increments. The directions of the strain increment vectors are obtained from the slopes of the relations between ϵ_z and $\gamma_{z\theta}$ shown in Fig. 8.

The directions of strain increment vectors on the plane corresponding to $\sigma'_r = 1.00 \text{ kgf/cm}^2$ (98 kN/m^2) are shown in Fig. 9 together with the projected failure stress points and the failure surface defined by Eq. (2). The projected directions corresponding to associated flow are also indicated on this figure. These directions were derived mathematically as the normals to the failure surface. Note that these projected directions are not quite normal to the failure surface in Fig. 9. It may appear that the experimental directions for practical purposes coincide in direction with these normals for b -values smaller than 0.5, but coincidence is not obtained for most of the tests with larger b -values.

The experimental strain increment vectors are entirely contained in the plane in Fig. 9. This is because they correspond to undrained tests in which $\dot{\epsilon}_v = 0$. However, the directions expected for associated flow are not contained in this plane but projected onto the diagram. This is because yield surfaces for soils near failure most likely are inclined in a manner similar to the failure surface, but they cross the failure surface at a very shallow angle (Tatsuoka and Ishihara, 1974; Lade and Kim, 1989). The normal to the yield surface at failure is therefore almost perpendicular to the failure surface. Fig. 7 shows a normal to the failure surface. It is clearly not contained in the $(\sigma_z - \sigma_\theta) - \tau_{z\theta}$ plane. Thus, the experiments indicate that nonassociated flow most likely is necessary to model the observed behavior. A similar con-

clusion was reached by Lade and Musante (1977) on the basis of cubical triaxial tests performed on normally consolidated Grundite clay.

CONCLUSIONS

A series of torsion shear tests was performed on remolded K_0 -consolidated Edgar Plastic Kaolinite under drained and undrained conditions to investigate the influence of reorientation of principal stress directions on the behavior of the clay. The influence of rotation of principal stresses on the stress-strain, and strength characteristics was observed as follows:

(1) The effects of stress-paths and reorientation of principal stresses were mainly observed in the prefailure stress-strain behavior.

(2) The failure surface obtained from torsion shear tests could practically be modeled by an isotropic failure criterion proposed by Lade (1977).

(3) The most accurate results of torsion shear tests close to $b=1.0$ were obtained under drained conditions.

(4) Coupling effects between shear stress and axial strain and between axial stress and shear strain were observed in most cases when shear and axial stresses were applied.

(5) The experimental strain increment vectors in the work-space were not perpendicular to the yield surface at failure. Non-associated flow may therefore be required for models developed to capture the behavior of normally consolidated clay.

ACKNOWLEDGMENT

A portion of the study presented here was supported by the National Science Foundation, U.S.A., under Grant No. ENG-78-20239. The study was continued while the first writer was on leave at the University of California, Los Angeles, California. Support for the first writer was provided by the Korea Science and Engineering Foundation, Korea. Grateful appreciation is expressed to

both Foundations for their support of this study.

REFERENCES

- 1) Bjerrum, L. and Landva, A. (1966) : "Direct simple-shear tests on a Norwegian quick clay," *Geotechnique*, Vol. 16, No. 1, pp. 1-20.
- 2) Broms, B. B. and Casbarian, A. O. (1965) : "Effects of rotation of the principal stress axes and of the intermediate principal stress on shear strength," *Proceedings, 6th International Conference on Soil Mechanics and Foundation Engineering*, Montreal, Canada, Vol. I, pp. 179-183.
- 3) Geiger, E. and Lade, P. V. (1979) : "Experimental study of the behavior of cohesionless soil during large stress reversals and reorientation of principal stresses," Report No. UCLA-ENG-7917, University of California, Los Angeles.
- 4) Hicher, P.-Y. and Lade, P. V. (1987) : "Rotation of principal directions in K_0 -consolidated clay," *Journal of Geotechnical Engineering*, ASCE, Vol. 113, No. 7, pp. 774-788.
- 5) Hight, D. W., Gens, A. and Symes, M. J. (1983) : "The development of a new hollow cylinder apparatus for investigating the effects of principal stress rotation in soils," *Geotechnique*, Vol. 33, No. 4, pp. 355-383.
- 6) Kirkgard, M. M. (1981) : "Consolidation characteristics determined during the constant rate of strain consolidation test," M. S. Thesis, University of California, Los Angeles.
- 7) Lade, P. V. (1975) : "Torsion shear tests on cohesionless soil," *Proceedings, 5th Panamerican Conference on Soil Mechanics and Foundation Engineering*, Buenos Aires, Argentina, Vol. I, pp. 117-127.
- 8) Lade, P. V. (1976) : "Interpretation of torsion shear tests on sand," *Proceedings, 2nd International Conference on Numerical Methods in Geomechanics*, Blacksburg, Virginia, Vol. I, pp. 381-389.
- 9) Lade, P. V. (1977) : "Elasto-plastic stress-strain theory for cohesionless soil with curved yield surfaces," *International Journal of Solids and Structures*, Pergamon Press, Inc., New York, N. Y., Vol. 13, pp. 1019-1035.
- 10) Lade, P. V. (1981) : "Torsion shear apparatus for soil testing," *Laboratory Shear Strength of Soil*, ASTM STP 740, R. N. Yong and F. C. Townsend, Eds., American Society for Testing and Materials, pp. 145-163.
- 11) Lade, P. V. and Duncan, J. M. (1973) : "Cubical triaxial tests on cohesionless soil," *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 99, No. SM 10, pp. 793-812.
- 12) Lade, P. V. and Kim, M. K. (1988) : "Single hardening constitutive model for frictional materials, II. Yield criterion and plastic work contours," *Computers and Geotechnics*, Vol. 6, pp. 13-29.
- 13) Lade, P. V. and Musante, H. M. (1977) : "Failure conditions in sand and remolded clay," *Proceedings, 9th International Conference on Soil Mechanics and Foundation Engineering*, Tokyo, Japan, Vol. I, pp. 181-186.
- 14) Lade, P. V. and Musante, H. M. (1978) : "Three-dimensional behavior of remolded clay," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 104, No. GT 2, pp. 193-209.
- 15) Macky, T. A. and Saada, A. S. (1984) : "Dynamics of anisotropic clays under large strains," *Journal of Geotechnical Engineering*, ASCE, Vol. 110, No. 4, pp. 487-504.
- 16) Ochiai, H. and Lade, P. V. (1983) : "Three-dimensional behavior of sand with anisotropic fabric," *Journal of Geotechnical Engineering*, ASCE, Vol. 109, No. 10, pp. 1313-1328.
- 17) Roscoe, K. H. (1953) : "An apparatus for the application of simple shear to soil samples," *Proceedings, 3rd International Conference on Soil Mechanics and Foundation Engineering*, Zurich, Switzerland, Vol. I, pp. 186-191.
- 18) Roscoe, K. H., Bassett, R. H. and Cole, E. R. L. (1967) : "Principal axes observed during simple shear of a sand," *Proceedings, the Geotechnical Conference*, Oslo, Norway, Vol. I, pp. 231-237.
- 19) Saada, A. S. and Baah, A. K. (1967) : "Deformation and failure of a cross anisotropic clay under combined stresses," *Proceedings, 3rd Panamerican Conference on Soil Mechanics and Foundation Engineering*, Venezuela, Vol. I, pp. 67-88.
- 20) Saada, A. S. and Bianchini, G. F. (1975) : "Strength of one dimensionally consolidated clays," *Journal of the Geotechnical Engineering Division*, ASCE, Vol. 101, No. GT 11, pp. 1151-1164.
- 21) Saada, A. S. and Ou, C.-D. (1973) : "Stress-strain relations and failure of anisotropic clays," *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 99, No. SM 12, pp. 1091-1111.
- 22) Saada, A. S. and Townsend, F. C. (1981) : "State-of-the-art : Laboratory strength testing

- of soils," Laboratory Shear Strength of Soil, ASTM STP 740, R.N. Yong and T.C. Townsend, Eds., American Society for Testing and Materials, pp. 7-77.
- 23) Saada, A. S. and Zamani, K. K. (1973) : "The mechanical behavior of cross-anisotropic clays," Proceedings, 7th International Conference on Soil Mechanics and Foundation Engineering, Moscow, USSR, Vol. 1, pp. 351-359.
- 24) Symes, M. J. P. R., Gens, A. and Hight, D. W. (1984) : "Undrained anisotropy and principal stress rotation in saturated sand," *Geotechnique*, Vol. 34, No. 1, pp. 11-27.
- 25) Tatsuoka, F. and Ishihara, K. (1974) : "Yielding of sand in triaxial compression," *Soils and Foundations*, Vol. 14, No. 2, pp. 63-76.
- 26) Tatsuoka, F., Sonoda, S., Hara, K., Fukushima, S. and Pradhan, T. B. S. (1986) : "Failure and deformation of sand in torsional shear," *Soils and Foundations*, Vol. 26, No. 4, pp. 79-97.
- 27) Taylor, G. I. and Quinney, H. (1931) : "The plastic distortion of metals," *Phil. Trans. Roy. Soc.*, Vol. A 230, pp. 323-362.
- 28) Tsai, J. I. and Lade, P. V. (1985) : "Three-dimensional behavior of remolded overconsolidated clay," Report No. UCLA-ENG-85-09, University of California, Los Angeles.
- 29) Wright, D. K., Gilbert, P. A. and Saada, A. S. (1978) : "Shear devices for determining dynamic soil properties," Proceedings, ASCE Specialty Conference on Earthquake Engineering and Soil Dynamics, Pasadena, California, Vol. 2, pp. 1056-1075.