AN EXTENSION OF THE PARTICULATE MODEL OF STABILITY ANALYSIS FOR MUDFLOWS

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ABSTRACT

Mudflows, debris flows and solifluction sheets are shallow instability forms with a distinct texture, this being a mixture of clay or rock lumps and a soft matrix of mud. The general equations of a new approach, previously introduced in a limited version, designed to calculate the stability of this type of mixture when located on a slope are presented. The new approach takes into consideration the particulate structure of the mixture as well as the shear strength provided by the lumps and the muddy matrix. Using the new approach, the least slope angles at which two reported mudflows got mobilized in the field were predicted well.

Key words : mudflows, particulate structure, slope stability, stability analysis (IGC : E 6)

INTRODUCTION

Shallow instability forms (categorized by Skempton in 1953 as such when the ratio of their thickness and length is less than 0.06) in soil slopes are often analyzed by the infinite slope stability method developed by Skempton and Delory (1957). This method considers the failing soil as a rigid, homogeneous and continuous mass, sliding on a plane assumed parallel to the ground surface (Fig.1(a)). The soil shear strength, s , and the shear stress, τ , on the slip surface are obtained from the following relationships in function of effective strength parameters

$$s = c' + (\gamma - m\gamma_w)d \cos \beta \tan \phi' \quad (1)$$

$$\tau = \gamma d \sin \beta \quad (2)$$

where c' , ϕ' and γ are the effective cohesion intercept, effective angle of internal friction and bulk unit weight of the soil. γ_w is the unit weight of water, m is the ratio between the height of the water table above the slip surface and the depth of the slip surface, d is the thickness of the sliding mass, and β is the slope inclination.

In function of total stresses, the shear strength s can be obtained from (Skempton and Hutchinson, 1969)

$$s = c_u \quad (3)$$

where c_u is the undrained shear strength of the soil.

The factor of safety, FS , for the infinite slope can be obtained from

$$FS = \frac{s}{\tau} \quad (4)$$

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Fig. 1. (a) Slope geometry and parameters used in the rigid-homogeneous-continuous approach of stability analysis (Skempton and Delory, 1957)
 (b) Slope geometry and parameters used in the particulate model of stability analysis (Vallejo, 1979, 1980 a)

Table 1. Materials involved in shallow failures

Shallow failure	Solid phase	Fluid phase	References
Subaerial mudflows	Large and stiff clay lumps	Liquid-like soil slurry (mud)	Hutchinson (1970); Hutchinson and Bhandari (1971); Hutchinson et al. (1974); Prior (1977); Keefer (1977).
Subaerial mudslides and debris flows	Rock fragments	Mud	Chandler (1970, 1972); Morton and Campbell (1974); Azimi and Desvarreux (1974); Pierson (1980); Francis (1980).
Submarine mudflows	Large and stiff clay lumps	Mud	Prior and Coleman (1978); Suhayda and Prior (1978); Prior and Suhayda (1979, 1979 a).
Solifluction sheets	Large and stiff clay lumps or rock fragments	Mud	Chandler (1970, 1972); McRoberts and Morgenstern (1974); Skempton and Weeks (1976); Vallejo and Edil (1981).
Flow slides (subaerial and submarine)	Sand or coal particles	Water or air	Koppejan et al. (1948); Jones et al. (1961); Andresen and Bjerrum (1968); Bishop (1973); Taylor and Morrel (1979).

Fig. 2. Mudflow structure at the location of Beltinge, England (Skempton and Hutchinson, 1969). The clay lumps volume concentration ratio, C , is equal to 0.8

However, instability forms such as subaerial and submarine mudflows, debris flows, solifluction sheets and flow slides are masses which can not be considered rigid, but rather highly deformable masses; nor are they homogeneous and continuous but are instead formed of a mixture of large particles of different shapes representing the solid phase and a fluid matrix (Table 1 and Fig. 2). Therefore, the rigid-homogeneous-continuous approach represented by Eqs. (1), (2) and (3) can not be rigorously applied to analyze the stability of soil masses that have a particulate and a deformable structure.

Takahashi (1978) and Vallejo (1979) using the findings of Bagnold (1954, 1956) have presented a method of stability analysis for the case of mudflows taking into consideration their particulate structure (Fig. 1 (b), Fig. 2). The method predicted well the low angle of mobilization of reported mudflows

Table 2. Undrained shear strength values for the muddy matrix

Shallow instability	Undrained shear strength, c_u , of muddy matrix kN/m ³	Reference
Subaerial mudflow on the London Clay slopes at Beltinge, England	4.8–7.2 (avg. 6)	Hutchinson (1970)
Subaerial mudflow, Davila Hill Complex, California	2–8.8	Keefer (1977)
Subaerial debris flow, North Canterbury, New Zealand	1.3–2.4	Pierson (1980)
Submarine mudflow, Mississippi Delta	4.8	Prior and Suhayda (1979)

(Vallejo, 1980, 1980 a; Vallejo and Edil, 1981). In the analysis presented by Vallejo (1979), the mixture of large particles and fluid (mud) was assumed to be located on an infinite slope with the fluid phase being continuous and coincident with the free surface of the slope (Fig. 1 (b)). The shear stress, τ , produced by gravity on the mixture of large particles and fluid and measured on the plane of failure can be obtained from (Vallejo, 1979)

$$\tau = [\gamma_f + (\gamma_s - \gamma_f)C]d \sin \beta \quad (5)$$

where γ_f is the unit weight of the fluid phase (mud), γ_s is the bulk unit weight of the large particles (solid phase), C is the ratio of the volume occupied by the large particles and the volume of the whole mass, and the rest of the terms as defined before.

The shear resistance, s , to the movement of the mixture was *assumed to be provided entirely by frictional forces between the large particles, neglecting therefore the shear resistance provided by the fluid phase (mud)*. The shear resistance can be obtained from (Vallejo, 1979, 1980 a)

$$s = (\gamma_s - \gamma_f)C d \cos \beta \tan \phi' \quad (6)$$

where ϕ' is the effective angle of internal friction between the large particles (Fig. 1 (b)).

However, according to Hutchinson (1970), Keefer (1977), Prior and Suhayda (1979), Pierson (1980) (Table 2), and Rodine and Johnson (1976), the *fluid phase* (mud) forming part of mudflows and debris flows has

Fig. 3. Slope geometry and parameters used in the extended version of the particulate model of stability analysis

some shear strength that will have some influence on their stability. Therefore, the shear strength of the fluid phase can not be neglected in the stability calculations involving mudflows, debris flows and solifluction sheets.

In the present paper the original approach presented by Vallejo (1979) on the stability analysis of slopes for shallow depths, taking into consideration their particulate structure, will be extended to cover a) the shear strength of the fluid phase, b) the fluctuations in level of the fluid phase (mud) inside the slope, and c) the effect of the environment, subaerial or submarine, in which the instabilities take place. Applications of the extended approach are also presented.

STABILITY ANALYSIS USING THE EXTENDED PARTICULATE APPROACH

For the stability analysis of a slope with a particulate structure, Fig. 3 is utilized. The slope is assumed to be infinite and with a failure surface parallel to the ground surface. Above the failure surface one part of the slope is saturated by the fluid (upper fluid) forming the environment on which it is located (air or water), and with the other part being saturated by a fluid (the lower fluid) other than air or water (mud for example). The stability of the slope will be

analyzed first when it is located on a subaerial environment and next when it is located on a submarine environment.

Subaerial Environment

Unit weights

When the slope with a particulate structure represented by Fig. 3 is saturated up to the free surface ($m=1$ in Fig. 3) by a fluid other than air or water (mud for example), the submerged unit weight of the mixture (grains plus mud) forming the slope when it is located in a *subaerial* environment can be obtained, according to Bagnold (1954) and Hubbert and Rubey (1959), from the following expression

$$\begin{aligned}\gamma'_{\text{mix}_1} &= [\gamma_f(1-C) + \gamma_s C] - [\gamma_a] \\ &= [(\gamma_f - \gamma_a) + (\gamma_s - \gamma_f) C] \quad (7)\end{aligned}$$

where γ'_{mix_1} is the submerged unit weight of the mixture in a *subaerial* environment, γ_f is the unit weight of mud, γ_a is the unit weight of air reported by Bagnold (1941) to be equal to 0.012 kN/m³, and the rest of the terms as defined before.

When the mixture of grains and mud ($m=1$ in Fig. 3) is located in a *submarine* environment, the submerged unit weight of the mixture can be obtained according to Bagnold (1968) from the following expression

$$\begin{aligned}\gamma'_{\text{mix}_2} &= [\gamma_f(1-C) + \gamma_s C] - [\gamma_w] \\ &= [(\gamma_f - \gamma_w) + (\gamma_s - \gamma_f) C] \quad (8)\end{aligned}$$

where γ'_{mix_2} is the submerged unit weight of the mixture in a *submarine* environment, and γ_w is the unit weight of the ocean water which has been reported by Capurro (1970) to be equal to 10.1 kN/m³.

If the lower fluid (mud) with unit weight γ_f , which was assumed to saturate the slope in Fig. 3 up to its free surface ($m=1$) in order to develop Eqs. (7) and (8), is replaced partially in the slope by either air or water (upper fluids), γ_f in Eqs. (7) and (8) has to be replaced by γ_a or γ_w for that depth of the slope in which the replacement took place in order to calculate its submerged unit weight in either a *subaerial* or a *submarine* environment. Eqs. (7) and (8) are used next to calculate the shear stress, τ ,

and the shear strength, s , on the failure surface assumed to be parallel to the free surface of the slope depicted in Fig. 3.

The shear stress, τ

The shear stress, τ , produced by gravity on the grain-fluid mixture forming part of an infinite slope located on a *subaerial* environment and measured at the location of the failure surface can be obtained from (Fig. 3)

$$\begin{aligned}\tau &= [(\gamma_s - \gamma_a) C][d - md] \sin \beta \\ &\quad + [(\gamma_f - \gamma_a) + (\gamma_s - \gamma_f) C][md] \sin \beta \\ &= \{[(\gamma_s - \gamma_a) C (1 - m)] + [(\gamma_f - \gamma_a) \\ &\quad + (\gamma_s - \gamma_f) C] m\} d \sin \beta \quad (9)\end{aligned}$$

The shear strength, s

The shearing resistance, s , at the location of the failure surface for the slope shown in Fig. 3 is provided by a) frictional resistance at the contact area between the large soil fragments and the surface over which failure takes place, assumed to be of the same material as the large soil fragments, and b) by the shear resistance of the lower fluid (mud) (this assumes that the adhesion between the mud and the surface on which failure takes place is greater than the shear strength of the mud)

From cone penetrometer tests performed by Rodine and Johnson (1976) on mudflows and debris flows, it was found that the fluid phase (mud) forming part of their structure (Table 1) behaved like a purely cohesive material ($\phi=0$). Therefore the resistance provided against shear by the fluid phase (mud) will be a function of its undrained shear strength, c_u .

In order to estimate the fraction of the failure surface (Fig. 3) occupied by the contact area between the large soil fragments and the plane of failure, and the fraction of the failure surface occupied by the lower fluid (mud), the following analysis is made. If one takes the segment L of the failure surface of the slope shown in Fig. 3, the volume, V , of the mixture of grains plus fluid above the segment L per unit distance perpendicular to the plane of the paper is

equal to

$$V = Ld \quad (10)$$

The fraction of L occupied by the contact area between the large soil fragments and the plane of failure can be obtained from the expression defining the grain concentration ratio, C . This expression is

$$C = \frac{V_s}{V} \quad (11)$$

where V_s is the volume of the large soil fragments present in the volume, V , of grains plus fluid above the segment L as shown in Fig. 3.

If one replaces Eq. (10) into Eq. (11), the following expression is obtained

$$V_s = CLd \quad (12)$$

The porosity, n , of the grain-fluid mixture shown in Fig. 3 can be related to the grain concentration ratio, C , if the following expression is used

$$n = (1 - C) = \frac{V_v}{V} \quad (13)$$

where V_v is the volume of the voids present in the volume, V , of grains plus fluid above the segment L as shown in Fig. 3. If Eq. (10) is replaced into Eq. (13) the following relationship is obtained

$$V_v = nLd = (1 - C)Ld \quad (14)$$

In the present analysis, the volume porosity of the grain-fluid mixture, n , is assumed to be equal to the boundary porosity measured at the interface between the grain-fluid mixture and the surface on which the mixture moves (Fig. 3). Taking this into consideration, if one compares Eq. (14) with Eq. (10), the product $(1 - C)L$ in Eq. (14) represents the fraction of the segment L on the failure plane occupied by the lower fluid (mud).

Therefore, the shear resistance, s , per unit area of the failure surface ($L = 1$ in Eqs. (10), (12), and (14)) of the slope shown in Fig. 3 can be obtained from

$$\begin{aligned} s &= \{[(\gamma_s - \gamma_a)(d - md) + (\gamma_s - \gamma_f)md] \\ &\quad \times [\cos \beta \tan \phi'] [C]\} + \{[c_u][1 - C]\} \\ &= \{[(\gamma_s - \gamma_a)(1 - m) + (\gamma_s - \gamma_f)m] \end{aligned}$$

$$\times [Cd \cos \beta \tan \phi']\} + \{[1 - C]c_u\} \quad (15)$$

where c_u is the undrained shear strength of the lower fluid (mud) and the rest of the terms as defined before.

As a checking procedure, Eqs. (5) and (6) can be obtained from the more general Eqs. (9) and (15) if the assumptions established by Vallejo (1979) are introduced into these equations. The assumptions are: a) the unit weight of air, γ_a , as well as the undrained shear strength, c_u , of the lower fluid (mud) are both negligible (Fig. 3), b) the fluid phase (other than air or water) with unit weight, γ_f , fills the pore space of the granular slope up to its free surface ($m = 1$ in Fig. 3).

The factor of safety, FS , against failure of the slope shown in Fig. 3 and defined by Eq. (4), can be obtained by dividing the values of the shear strength, s , and the shear stress, τ , obtained from Eqs. (15) and (9) respectively.

Submarine Environment

If the slope represented by Fig. 3 is located in a *submarine* environment, Eqs. (9) and (15) can still be used to analyze its stability if one replaces in these equations the unit weight of air, γ_a , by the unit weight of the ocean water, γ_w .

APPLICATION OF THE EXTENDED PARTICULATE APPROACH OF STABILITY ANALYSIS

A stability analysis of slopes in granular materials will now be made using the extended particulate approach. The slope instabilities analyzed are mudflows. These instabilities take place in slopes that can be considered infinite, involve shallow depths and also granular materials with their pore space filled by a fluid which has some shear strength according to Hutchinson (1970), Keefer (1977), Prior and Suhayda (1979), Pierson (1980) (Table 2).

Table 3. Shear strength as a function of grain concentration ratio, C

Mixture*	Type of test	Grain concentration ratio, C			Reference
		Mixture shear strength $\approx$ Frictional shear resistance between grains*	Mixture shear strength = Frictional plus matrix shear strength*	Mixture shear strength $\approx$ Matrix shear strength*	
Sand and clay	Triaxial	>0.70	$0.70-0.50$	<0.50	Paduana (1966).
Sand and clay	Direct Shear	>0.80	$0.80-0.62$	<0.62	Kurata and Fujishita (1960).
Sand and clay	Cone penetrometer		? -0.45	<0.45	Rodine and Johnson (1976).
Sand and clay	Vane		? -0.58	<0.58	Vallejo (1981).
Sand and ice	Triaxial		? -0.42	<0.42	Goughnour and Andersland (1968).
Gravel and clay	Triaxial	>0.90	$0.90-0.80$	<0.80	Rico and Orozco (1975).
Gravel and clay	Vane		? -0.49	<0.49	Vallejo (1981).
Average Values		>0.80	$0.80-0.55$	<0.55	

* Sand or Gravel=Grains
Clay (kaolinite) or Ice=Matrix

Fig. 4. Structural arrangements and parameters used for the case of mudflows

Stability Analysis of Mudflows

Mudflows, both subaerial and submarine, are shallow instability forms composed of two different phases (Table 1 and Fig. 2). One is a continuous fluid phase represented by a liquid-like soil slurry (muddy matrix). The other is a solid phase represented by large and stiff lumps of clay.

Kurata and Fujishita (1960), Paduana (1966), Goughnour and Andersland (1968), Rico and Orozco (1975), Rodine and Johnson (1976), Vallejo (1981) found from laboratory investigations on the shear strength of two-phase systems such as sand-clay, gravel-clay and sand-ice mixtures that there are percentages of the granular material in the mixtures, defined by the grain concentration

ratio, C (volume of the grains over volume of mixture), that will have either a total, partial or a nil influence on the shear strength of the mixture as shown in Table 3.

Considering the average of the values reported in Table 3, it can be concluded that for values of the grain concentration ratio, C , greater than 0.8, the shear strength of the mixtures will be governed mainly by frictional shear resistance between the grains. When the grain concentration ratio, C , has values between 0.8 and 0.55 the shear strength of the mixtures will be dictated in part by frictional resistance between the grains and in part by the shear strength of the clay or ice matrices. If the value of the grain concentration ratio, C , is below 0.55

the shear strength of the mixtures will be dictated *solely* by the shear strength of the matrix (clay or ice).

If the above results are applied to analyze the shear strength and therefore the stability of mudflows when located on a slope it can be concluded that the shear strength preventing the mobilization of the mixtures will be provided totally or partially by the solid or the fluid phases.

Structural arrangements

Depending on the concentration of the hard clay lumps in a mudflow structure, three possible arrangements of the solid and fluid phases can take place in a mudflow assumed to be saturated up to its free surface by one fluid phase (mud) ($m=1$ in Fig. 3). They are shown in Fig. 4.

The first arrangement is the grain or solid phase supported structure (Fig. 4 (a)). In this structural arrangement the solid phase predominates and the mudflow behaves like a rigid body sliding down the slope. This type of arrangement takes place according to Prior and Coleman (1978), Suhayda and Prior (1978), Prior and Suhayda (1979 a), and Vallejo (1979) during the initial stages of development of a mudflow. The resistance to the movement of the mixture on the failure surface will be provided mainly by frictional forces between the clay lumps and in less degree by the undrained shear strength of the fluid phase (mud) for values of C greater than 0.8 following the average values of Table 3.

The second structural arrangement is the grain-matrix supported structure (Fig. 4 (b)). In this type of structure neither the solid nor the fluid phase predominates. This type of arrangement takes place after a mudflow has traveled for some distance down a slope, with the original clay lumps having decreased in size because part of their soil mixed with the water forming part of the fluid phase. The resistance to the movement of the grain-matrix supported structure is provided in part by frictional forces between the clay lumps and in part by the undrained

shear strength of the fluid phase for values of C between 0.8 and 0.55 following the average values of Table 3.

The third structural arrangement involves a matrix (mud) supported structure (Fig. 4 (c)). In this arrangement the fluid phase predominates and the stability of the structure is provided *entirely* by the undrained shear strength of the fluid phase for values of C less than 0.55 following the average results of Table 3. According to Prior and Coleman (1978), Suhayda and Prior (1978), Prior and Suhayda (1979 a), and Vallejo (1979, 1980 a) this type of structure occurs in mudflows after they have traveled for long distances down a slope and a large part of the soil from the large clay lumps has mixed with the muddy water (fluid phase). In this structure, the solid phase is dispersed (i.e. grains are not in contact with each other or with the plane of failure, therefore no frictional forces exist between the inclusions or between the inclusions and the plane of failure) in the fluid phase and represents only a small portion ($C < 0.55$) of the total mixture.

Therefore as a mudflow moves down a slope, the three structural arrangements shown in Fig. 4 can take place at different times and locations during its development. The difference between the three arrangements is based on the concentration of the large clay fragments as well as the concentration of fines in the fluid phase. As soil from the large clay lumps is added to the fluid phase, its overall water content decreases resulting in an increase of its undrained shear strength, c_u . The increase in fines concentration in the fluid phase will also cause an increase in its unit weight that in turn will reduce the immersed unit weight of the large clay lumps. This increase in buoyancy of the fluid phase will help to keep the large particles of clay in dispersion and afloat.

Critical slope angles

At limit equilibrium conditions, the least slope angle on which failure of the three

grain-fluid mixtures shown in Fig. 4 will take place can be obtained by combining Eqs. (9) and (15) with the following relationship

$$\tau = s \quad (16)$$

For the case of a *subaerial* mudflow with a grain ($C > 0.8$) or a grain-matrix (C between 0.55 and 0.8) supported structures, the least slope angle on which failure will take place can be obtained from (Figs. 4 (a) and 4 (b))

$$\tan \beta = \tan \phi' \left\{ \frac{(\gamma_s - \gamma_f)C}{(\gamma_f - \gamma_a) + (\gamma_s - \gamma_f)C} \right\} + \left\{ \frac{z(1-C)c_u}{d^2[(\gamma_f - \gamma_a) + (\gamma_s - \gamma_f)C]} \right\} \quad (17)$$

where z is the depth of the failure plane (Fig. 4) and ϕ' is the static angle of internal friction between the clay lumps, and the rest of the terms as defined before.

Similarly, for the case of a *submarine* mudflow with a grain or a grain-matrix supported structure (Fig. 4), the least slope angle on which failure will take place can be obtained after combining Eqs. (9), (15) and (16) and replacing γ_a by γ_w in these equations. The tangent of this angle is equal to

$$\tan \beta = \tan \phi' \left\{ \frac{(\gamma_s - \gamma_f)C}{(\gamma_f - \gamma_w) + (\gamma_s - \gamma_f)C} \right\} + \left\{ \frac{z(1-C)c_u}{d^2[(\gamma_f - \gamma_w) + (\gamma_s - \gamma_f)C]} \right\} \quad (18)$$

For a plastic clay it is well known that the remoulded shear strength increases with the rate of shear (Bishop, 1973). But since mudflows attain very low velocities during flowing (between 2 mm/day to 4.5 m/day according to Keefer, 1977), it can be assumed, therefore, that the undrained shear strength of the muddy matrix, c_u , at equilibrium conditions and during flowing can be considered the same.

For the case of a *subaerial* mudflow with a matrix supported structure ($C < 0.55$) (Fig. 4 (c)) the least slope angle of mudflow mobilization can be obtained after combining Eqs. (9), (15) and (16) and remembering that the resistance to movement of the mudflow is provided *entirely* by the undrained shear strength of the fluid phase, c_u . The least

slope angle can be obtained from

$$\sin \beta = \frac{c_u}{[(\gamma_f - \gamma_a) + (\gamma_s - \gamma_f)c]d} \quad (19)$$

Similarly, for a *submarine* mudflow with a matrix supported structure, the least slope angle can be obtained from

$$\sin \beta = \frac{c_u}{[(\gamma_f - \gamma_w) + (\gamma_s - \gamma_f)C]d} \quad (20)$$

Stability analysis of mudflows using the rigid-homogeneous-continuous approach

Hutchinson and Bhandari (1971) have calculated the critical value of the slope surface, β , on which a *subaerial* mudflow will take place by combining Eqs. (1) and (2) developed by Skempton and DeLory (1957) and the assumptions that: a) the *subaerial* mudflow behaves like a rigid-homogeneous-continuous translational mass of soil (Fig. 1 (a)), b) residual strength conditions take place on the sliding surface (c' equal to zero in Eq. (1)), c) the fluid phase is ground water that is coincident and flows parallel to the surface of the sliding mass, and d) the failure takes place in an infinite slope. β can be obtained from

$$\tan \beta = \tan \phi' \left(\frac{\gamma - \gamma_w}{\gamma} \right) \quad (21)$$

The ratio of the unit weights in the parenthesis of Eq. (21) can be taken as equal to 0.5.

In terms of total stress analysis and using the rigid-homogeneous-continuous approach, Skempton and Hutchinson (1969) after combining Eqs. (2) and (3) developed an equation from which the critical slope angle of failure involving shallow depths (i.e. mudflows) and *subaerial* conditions can be obtained. This equation is

$$\sin \beta = \frac{c_u}{\gamma d} \quad (22)$$

Comparison between the particulate and the rigid-homogeneous-continuous approaches

The difference between the particulate approach of stability analysis represented by Eq. (17) for the case of a *subaerial* mudflow with a *grain* or a *grain-matrix* supported

structure (Fig. 4) and the rigid-homogeneous-continuous approach represented by Eq. (21) is based on the grain concentration ratio, C . This parameter, C , is the basis of the particulate approach and is an indirect measure of the amount of clay lumps present in a mudflow structure (Fig. 2 and Fig. 4).

If a mass of soil does not develop a particulate structure during failure at shallow depths, but instead moves down a slope as a rigid-homogeneous-continuous translational block, the parameter C will attain a value equal to one. If one neglects the unit weight of the air, and if a value of C is equal to one is replaced into Eq. (17), as expected Eq. (21) will be obtained, remembering that γ_s is now the bulk unit weight of the rigid translational mass (γ in Eq. (22)) and the fluid phase is considered to be water that fills the micropores of the rigid translational mass.

For the case of a failing soil mass like a *subaerial* mudflow, *entirely* supported by the fluid phase ($C < 0.55$) (Fig. 4 (c)), Eq. (19) developed to obtain the least slope angle of mudflow mobilization using the particulate approach, and Eq. (22), developed to obtain the same angle using the rigid-homogeneous-continuous approach, are the same since the summation inside the brackets forming the denominator of Eq. (19) represents the submerged bulk unit weight (γ'_{mix} of Eq. (7)) of the failing mass which is the same as γ in Eq. (22).

Therefore, Eqs. (21) and (22) representing the rigid-homogeneous-continuous approach of stability analysis can be obtained from the more general approach of stability analysis, the particulate approach, represented by Eqs. (9), (15) and their combinations, Eqs. (17) and (19).

Stability analysis of a reported subaerial mudflow

The particulate approach of stability analysis will be used to calculate the least slope angle of mobilization of a subaerial mudflow in the London Clay slopes at the location of Beltinge (Hutchinson, 1970 ; Hutchinson and

Bhandari, 1971). Also the least slope angle of mobilization will be also calculated using the rigid-homogeneous approach. A comparison of the results obtained for this angle using the two approaches and the least slope angle of mudflow mobilization measured in the field is also undertaken.

The grain concentration ratio, C . The mudflow at the location of Beltinge was recorded by Hutchinson (1970) to consist of a mixture of hard clay lumps and a matrix of very soft clay (mud). The c_u of the muddy matrix, according to Hutchinson (1970), ranged in value between 4.8 kN/m² and 7.2 kN/m², with an average of 6 kN/m². The c_u of the *mixture* of hard clay lumps and mud ranged in value between 10 kN/m² at the surface and 30 kN/m² at the base of the mudflow. Hutchinson (1970) obtained a cylindrical mudflow sample 10.16 cm in diameter, and took a photograph of the middle section. From this photograph, published also by Skempton and Hutchinson (1969), a drawing made and is shown in Fig. 2. From this drawing, using the principles of quantitative Stereology (Underwood, 1970), it can be easily demonstrated that the volume concentration ratio, C , of the hard clay lumps in the sample is equal to and can be obtained from the area concentration ratio of the hard lumps depicted in Fig. 2.

The area concentration ratio can be estimated using a transparent grid that is placed over Fig. 2 (Holliday, 1966). One then counts the area of the grid which is filled by the large soil fragments. The value of the solids area is divided by the total area of the photograph to estimate the grain concentration ratio, C . Using this technique a value of C equal to 0.8 was obtained. Therefore, according to the average values of C shown in Table 3, the mudflow at Beltinge is in the border between a *grain* supported structure and a *grain-matrix* supported structure (Fig. 4). Thus Eq. (17) will be used to calculate the least slope angle of mobilization for the mudflow. Eq. (21), representing the rigid-homogeneous-continuous approach of stability analysis in terms of

Table 4. Critical slope angles (limit equilibrium) using the particulate and rigid-homogeneous-continuous approaches

Site	τ_s kN/m ³	τ_f kN/m ³	τ'_{mix} * ² kN/m ³	ϕ' (deg.)	C	z m.	d m.	c_u kN/m ²	β : * ⁴ Particulate (deg.)	β : * ⁵ Rigid-Homog.- Continuous. (deg.)	β : Actual (deg.)
Subaerial mudflow in the London Clay Slopes* ¹	19.6	17.12	19.1	13.5	0.8	2.91 (avg.)	2.9 (avg.)	6 (avg.)	2.7	6.7	3.5 (avg.)
Submarine mudflow on the Mississippi Delta* ³	15.7	14.1	14.9	23	0.48	33	33	4.8	1.7	1.7	0.5—1.7

*¹ The values of τ_s , τ_f , ϕ' , z , d , c_u , β : Actual, were obtained from Hutchinson (1970), Hutchinson and Bhandari (1971) and Vallejo (1979).

*² Calculated using Eq. (7) for the subaerial mudflow using $\tau_a=0.012$ kN/m³, and using Eq. (8) for the submarine mudflow using $\tau_w=10.1$ kN/m³.

*³ The values of τ_s , τ_f , ϕ' , z , d , c_u , β : Actual, were obtained from Prior and Coleman (1978), Suhayda and Prior (1978) and Prior and Suhayda (1979, 1979 a). c_u measured in the 17m of soft clay sediment.

*⁴ Calculated using Eq. (17) for the subaerial mudflow, and Eq. (20) for the submarine mudflow.

*⁵ Calculated using Eq. (21) for the subaerial mudflow, and Eq. (22) for the submarine mudflow using the respective τ_{mix} of this Table instead of τ in Eqs. (21) and (22).

effective strength parameters will be used to obtain the critical slope angle of mobilization, since the undrained conditions ($\phi=0$) represented by Eq. (22) do not take place during failure for the mudflow structure shown in Fig. 2.

The calculated and measured least slope angles of mudflow mobilization are shown in Table 4, together with the rest of the parameters used to obtain the calculated values. A comparison of the calculated and measured least slope angles shows that the particulate approach predicts well the critical angle of mudflow mobilization in the field. However, this is not the case when using the rigid-homogeneous-continuous approach.

Stability analysis of a reported submarine mudflow

The particulate approach of stability analysis will now be used to calculate the least slope angle of mobilization of a submarine mudflow on the Mississippi River Delta in the Gulf of Mexico (Prior and Coleman, 1978 ; Suhayda and Prior, 1978 ; Prior and Suhayda, 1979, 1979 a). A comparison of this angle with the one obtained using the rigid-homogeneous-continuous approach and the one measured in the field is also undertaken.

The grain concentration ratio, C. From seismic data, Prior and Coleman (1978), and Prior and Suhayda (1979, 1979 a), found that

the depth of the failure plane in reported mudflows in the Mississippi Delta has an average value of 33 meters of which the top 16 meters is formed of a stiff clay which constitutes the so called "crust zone" underlaid by 17 meters of a zone of substantially soft clay sediment (mud). After failure takes place, the material forming the "crust zone" breaks into irregular shaped blocks which become *dispersed* and *supported* by the remoulded soft sediment which becomes a liquid-like soil slurry (*muddy matrix*) (Prior and Suhayda, 1979 a). Therefore, the volume concentration ratio, C , of the lumps of crust in the mudflow can be estimated from the ratio of the depth of the crust (16 meters) and the depth of the total failing mass (33 meters). This ratio is equal to 0.48.

Since the value of C for the Mississippi Delta mudflow is less than 0.55, according to the average results of Table 2, this mudflow can be classified as a *matrix supported* structure (Fig. 4 (c)).

Therefore, Eq. (20) and Eq. (22) will be used to calculate the least slope angle of mudflow mobilization. However, as was explained before, Eq. (20) representing the particulate approach and Eq. (22) obtained using the rigid-homogeneous-continuous approach will give the same results for the least slope angle of mudflow mobilization, if

one takes into consideration that the unit weight γ in Eq. (22) represents for the case of a submarine mudflow the submerged bulk unit weight of the mudflow structure given by Eq. (8), which is the denominator of Eq. (20).

The calculated and measured least slope angles of mudflow mobilization are shown in Table 4, together with the rest of the parameters used to obtain the calculated values. A comparison of the calculated and measured least slope angles shows that the particulate and the rigid-homogeneous-continuous approach predicted well the angle of mudflow mobilization in the field.

CONCLUSIONS

The general equations of a new approach to calculate the stability of a slope made of a mixture of fragments of either clay or rock and a very soft soil mass with some shear strength and involving shallow depths (i.e. mudflows, debris flows, solifluction sheets) have been presented. The new method takes into consideration the particulate structure of the mass. The new method predicts well the least slope angle of mobilization of a reported subaerial mudflow. However this was not the case when employing the rigid-homogeneous-continuous approach of slope stability analysis which has been used to date and considers mudflows to behave like a rigid-homogeneous-continuous translational mass which has been reported not to be the case in the field.

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