A CONSTITUTIVE MODEL FOR NATURAL SOFT CLAY WITH STRAIN SOFTENING

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ABSTRACT

A constitutive model of natural soft clay with strain softening, which is an important feature of overconsolidated clay, has been developed based on the concept of elasto-plasticity with memory. A new stress history tensor, which denotes a memory of stress with respect to strain, has been introduced by use of a strain measure. The stress history tensor is effective for describing the deformation feature of frictional materials with strain softening. By using this stress history tensor, we extend the elasto-plasticity theory of overconsolidated clays previously proposed by Oka et al. The proposed model is capable of describing the strain-hardening and strain-softening behavior of natural soft clays in the overconsolidated region, and was applied to the triaxial drained and undrained behavior of a Champlain sea clay from eastern Canada. The comparison between volumetric and axial strain responses of experiment and simulation shows good agreement.

Key words : clay, constitutive equation of soil, over consolidation, plasticity, strain softening, stress-strain curve, triaxial compression (IGC : D 6/D 5)

INTRODUCTION

Although many constitutive relations of clays have been proposed in recent years (ISSMFE Subcommittee, 1985), there are still several problems to be solved. One of them relates to the strain-softening behavior during the deformation process of overconsolidated clay and soft rocks. In general, natural overconsolidated clays and rock-like materials show strain-hardening and strain-softening behavior during the plastic deformation process. The stress-strain curves obtained on natural soft clay after consolidation

at small stresses exhibit strain-softening after peak strength is reached while CID tests carried out after consolidation at higher stresses show strain hardening behavior (e. g. Tavenas and Leroueil, 1977). Softening behavior is an important phenomenon for the design problem in relation to the estimation of bearing capacity, stability problems and the capability of deformation.

In the framework of continuum mechanics, there are several approaches to describe the strain-hardening and strain-softening behavior. Many researchers have explained the strain-softening behavior by using the theory

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of plasticity (Nayak and Zienkiewicz, 1972 ; Prévost and Höeg, 1977 ; Bannerjee and Stipho, 1979 ; Dragon and Mróz, 1979 ; Kawahara et al., 1981 ; Matsumoto and Ko, 1982). Pietruszczak and Mróz (1981) dealt with the softening problem by introducing the smear element in finite element method to avoid the mesh size dependency of the analysis. Valanis (1982) showed that endochronic theory also can predict the strain-softening behavior of soil. Valanis (1985) presented the uniqueness of solution of initial value problem in softening materials, and Willam et al. (1987) addressed the uniqueness and stability of strain-softening computations. For the mesh size dependency problem, Bažant et al. (1988) developed a partially nonlocal continuum formulation to overcome

this difficulty.

Oka (1985) developed an elasto-viscoplasticity theory with internal variables and memory based on the generalized simple body (Wang, 1969) to take into account the acceleration creep behavior of materials. Thereafter, Oka and Adachi (1985) proposed a rate-independent elasto-plastic constitutive equation of soft rocks with memory in order to explain the strain-hardening and strain-softening behavior based on the concepts of generalized simple body and the intrinsic time measure used in the endochronic theory (Valanis, 1971).

The aim of the present paper is to construct a constitutive model of natural over-consolidated clay with strain-softening extending Oka and Adachi's approach (1985 b).

Fig. 1. Triaxial test results obtained on the Saint-Vallier clay, a-CIU tests ; b-CID tests (After Lefebvre, 1970)

The proposed model has been applied to simulate the drained and undrained triaxial compression test results obtained on a natural soft overconsolidated clay from Louiseville (Québec). Comparison of experimental and calculated results shows that the proposed model effectively simulates the deformation behavior of natural soft clay.

BEHAVIOR OF NATURAL SOFT CLAYS

Many studies on the stress-strain behavior of natural soft clays have been performed in the last two decades. They generally confirm the validity of the basic concepts of limit and critical states first suggested by Roscoe et al. (1958), but they also show that, for natural clays, it is necessary to extend these basic concepts to incorporate anisotropy, structure and time or strain rate effects (Tavenas and Leroueil, 1977 ; Leroueil et al., 1979 ; Larsson, 1981 ; Magnan et al., 1982 ; Graham et al., 1983 ; Tavenas and Leroueil, 1987).

When testing natural clays in their overconsolidated state, the behavior obviously reflects all these phenomena. The results obtained by Lefebvre (1970), Lefebvre and La Rochelle (1974), on the Saint-Vallier clay from the Province of Québec will be used to illustrate the observed behavior. More detailed analysis of this behavior is given by El Moustassim (1986).

Fig.1(a) shows CIU(consolidated isotropically undrained triaxial test) test results. All the stress-strain curves present well defined

Fig. 2. Stress paths and limit state curve-Saint Vallier clay (After Lefebvre, 1970)

Fig. 3. Consolidation tests on the Saint-Vallier clay (After Lefebvre, 1970)

peaks at small strain, followed by strain softening. The pore water pressure generated increases with the consolidation stress. As shown on Fig.2, on which the stress path are plotted, the peak strength define a curved peak-strength envelope. At large strains, all the stress paths converge towards the same stress area well below the peak strength envelope.

Fig.3 shows the results of two oedometer tests and of triaxial isotropic consolidation tests ; both curves have well defined yielding stresses at a vertical stress respectively of 183 and 112 kPa for the oedometric and isotropic consolidation tests. The ratio of these two yield stresses, equal to 1.63 is typical of natural soft clays (Leroueil and Tavenas, 1977 ; Leroueil et al., 1983). These yielding stresses together with the peak strengths obtained in CIU tests and experience with similar clays allow the drawing of the entire yield curve which is more or less centered on the $K_{0, n.c.}$ line (Fig.2).

Fig.1(b) shows CID (consolidated isotropically drained triaxial test) test results. The specimen consolidated under 3.5 kPa shows a well defined peak with an important strain-softening ; the peak and large strain strength are indicated on Fig.2. The specimen consolidated under 103 kPa presents first a quasi-elastic behavior until the stress path reaches the yield curve (point y on Fig.2), then it develops a strain hardening behavior,

Fig. 4. Interrelation between normalized limit state and shear moduli in the Saint-Alban clay (from Tavenas and Leroueil, 1979)

going towards the ϕ' line of the normally consolidated clay. The specimen consolidated under 69 kPa (Fig. 2) reaches the yield curve and the ϕ' line at the same time and consequently presents an elastic perfectly plastic behavior (Fig. 1(b)). The peaks obtained in these CID tests are slightly lower than the peaks obtained in CIU tests, which is due to strain rate effects, the rate used in these CID tests being five times slower than in CIU tests (Lo and Morin, 1972; Tavenas and Leroueil, 1977). As indicated by Lefebvre and La Rochelle (1974) and by Tremblay (1983) the strength envelope obtained in CID tests at large strain is curved (Fig. 2).

It is finally interesting to mention that the stress-strain relations before yielding are not linear and that the initial shear modulus G_0 is not a constant in the overconsolidated range. As indicated by Tavenas and Leroueil (1979), G_0 is strongly influenced by the shape of peak strength envelope (Fig. 4).

GENERAL DESCRIPTION OF THE CONSTITUTIVE MODEL

Rate-independent deformation process of natural soft clay is assumed to be described

by the stress, stress-history tensor and internal variables. Total strain increment tensor $d\varepsilon_{ij}$ is assumed to be composed of three components, the external elastic strain increment tensor $d\varepsilon_{ij}^{ex}$, plastic strain increment tensor $d\varepsilon_{ij}^p$ and the internal elastic strain increment tensor $d\varepsilon_{ij}^{ie}$. The concept of internal elastic strain is introduced since the failure of microstructure generally relates to the elastic properties of the material. On the other hand, the microstructural change related to plastic deformation is not influenced by external elastic strain.

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{ex} + d\varepsilon_{ij}^{in} \quad (1)$$

$$d\varepsilon_{ij}^{in} = d\varepsilon_{ij}^{ie} + d\varepsilon_{ij}^p \quad (2)$$

in which $d\varepsilon_{ij}^{in}$ is the internal strain increment tensor that affects the plastic deformation or microstructural change.

In order to define the stress history tensor, the strain measure which is similar to the intrinsic time measure advocated by Valanis (1971, 1982), is used instead of real time. Herein, the strain measure z is defined by

$$dz = P_{ijkl} d\varepsilon_{ij}^{in} d\varepsilon_{kl}^{in} \quad (3)$$

in which P_{ijkl} is a fourth order positive definite tensor.

We introduce a stress history tensor, S^* , which is a functional of stress history. If the stress history tensor S^* is a continuous functional of the stress history S^z , the following form can be taken.

$$S^* = \int_{-\infty}^z K(z-z', [W]) S^z(z') dz' \quad (4)$$

$$S^z = (S(z-z')) ; 0 \leq z' < \infty \quad (5)$$

in which S is the stress tensor and $[W]$ denotes other state variables. In Eq. (4), the scalar kernel function $K(\cdot)$ can be generally expressed by a distribution function $D(z)$ as

$$K(z) = \int_0^\infty \frac{D(\tau')}{\tau'} \exp(-z/\tau') d\tau' \quad (6)$$

All constitutive functionals are assumed to be functional of the present stress, the stress history tensor and the internal state variables. The plastic strain tensor is introduced as a second order internal variable tensor. The evolutionary equation is introduced for

the plastic deformation on the basis of the non-associated flow rule. Indeed experimental data on natural clays are indicating the deviations from the normality (Wong and Mitchell, 1975 ; Fauveau-Brucy, 1977 ; Graham et al., 1983 ; Felix et al., 1985). For overconsolidated clay, Pender (1977) showed the effectiveness of non-associated flow rule, and thereafter, Stipho (1980), Ohmaki (1980) and Akai, Adachi and Oka (1981) constructed constitutive models based on the non-associated flow rule. It must be recognized that the non-associated flow rule demands the increase in number of material parameters, however, it gives a great advantage to explain the stress-strain behavior of soil (see Lade, 1987). The non-associated flow rule is given by

$$d\varepsilon_{ij}^p = h \frac{\partial f_p}{\partial S_{ij}} df_v \quad (7)$$

in which f_v is the yield function, f_p the plastic potential function, h the strain-hardening and strain-softening function, S_{ij} the stress tensor and $d\varepsilon_{ij}^p$ the plastic strain increment tensor.

The subsequent yield function f_v is in general an intricate function of the state variables. In this paper, the subsequent yield function is a function of the stress history tensor, stress tensor and the strain-hardening and strain-softening parameters.

$$f_v(S_{ij}^*, S_{ij}, \kappa_s) = 0 \quad (8)$$

In Eq. (8), strain-hardening and strain-softening scalar parameter κ_s is related to the plastic strain through the following equation.

$$\kappa_s = \kappa_s(\varepsilon_{ij}^p) \quad (9)$$

Taking into account the external and internal elastic components of strain increment tensor, we get an elasto-plastic constitutive model with stress-memory with respect to strain measure.

CONSTITUTIVE MODEL FOR NATURAL OVERCONSOLIDATED CLAY

Although many constitutive models have been proposed for normally consolidated clay,

Fig. 5. Overconsolidated boundary surface and plastic potential

theoretical studies on constitutive model for overconsolidated clay are relatively limited. Natural clays have been overconsolidated to some degree due to many reasons, such as stress release, cementation, thixotropic hardening, variation in water level or drying. The strain-hardening and strain-softening behavior of soft natural clay is essentially associated with overconsolidation. To describe the strain-softening behavior of clay, some models have been developed in the framework of continuum mechanics. However, these models have to be validated with comparison with experimental data and improved to couple shearing and dilatancy, and incorporate three-dimensional behavior. Adachi and Oka (1984) proposed a constitutive equation for sand and overconsolidated clay without strain-softening based on the elasto-plastic theory.

In this section, this model will be combined with the constitutive model proposed in the previous section to construct a constitutive model for the behavior of soft natural clay that exhibits strain-hardening and strain-softening behavior.

An overconsolidated boundary surface that defines the boundary between overconsolidated ($f_b < 0$) and the normally consolidated ($f_b \geq 0$) regions as shown in Fig. 5 has been introduced. This boundary surface is similar to what has been called limit state surface in the first part of the paper. In this paper,

we have assumed that the limit state surface is equal to the boundary surface.

The boundary surface is written by

$$f_b = \bar{\eta}_{(0)} + M_m^* \ln[(\sigma_m' + b)/(\sigma_{mb}' + b)] = 0, \quad (10)$$

in which b is the material parameter corresponding to the intersection of the upper part of the boundary surface with the σ_m' axis (Fig. 5) and $\bar{\eta}_{(0)}$ is the relative stress ratio parameter, which is a variant of stress ratio parameter η first introduced by Sekiguchi and Ohta (1977), expressed by

$$\bar{\eta}_{(0)} = [(\eta_{ij} - \eta_{ij(0)}) (\eta_{ij} - \eta_{ij(0)})]^{1/2} \quad (11)$$

$$\eta_{ij} = s_{ij} / \sigma_m' \quad (12)$$

$$\eta = (\eta_{ij} \eta_{ij})^{1/2} \quad (13)$$

In the above relations, η_{ij} is the stress ratio tensor, s_{ij} is the deviatoric stress tensor and σ_m' is the mean effective stress, and $\eta_{ij(0)}$ is the value of η_{ij} at the end of K_0 consolidation in the normally consolidated range.

In Eq. (10), σ_{mb}' can be determined by the vertical preconsolidation stress σ_p' and $K_{0,n.c.}$ value, and is also related to σ_{mc}' in Fig. 5 through the following relation:

$$\sigma_{mb}' = \sigma_{mc}' \exp(-\eta_{(0)} / M_m^*) \quad (14)$$

where $\eta_{(0)}$ is a value of η at point P in Fig. 5.

According to Eq. (4), the stress history tensor σ_{ij}^* is expressed by use of an exponential type kernel function as follows:

$$\sigma_{ij}^* = \frac{1}{\tau} \int_0^z \exp\left(\frac{-(z-z')}{\tau}\right) \sigma_{ij}(z') dz' \quad (15)$$

in which z is a strain measure defined by

$$dz = (de_{ij} de_{ij})^{1/2} \quad (16)$$

where de_{ij} is a deviatoric internal strain increment tensor.

In Eq. (15), τ is a material parameter, which expresses the retardation of transformed stress σ_{ij}^* to real stress σ_{ij} with respect to strain measure z and controls the degree of strain-softening. The difference between σ_{ij}^* and σ_{ij} depends on the value of τ . This type of stress history tensor satisfies the principle of fading memory (Christensen, 1982). This stress history tensor controls the frictional component of strength. In general, in the stiff geological

materials, the degree of damaging is small in the early part of the straining and the frictional resistance is also small. On the other hand, the degree of damaging becomes large and the frictional resistance develops with the increase of strain. Correspondingly, the value of σ_{ij}^* is small in the early part of deformation and becomes large in the large strain state. Now, we are considering the shear deformation. Therefore, we used the strain measure by deviatoric strain. However, the definition of the strain measure can be generalized or specified taking account of the physical process of deformation. In this paper, the external elastic strain rate is neglected for simplicity, because we do not deal with behavior under the cyclic loadings.

Referring to previous works on overconsolidated clay for constitutive relations (Pender, 1977; Ohmaki, 1980; Oka and Washizu, 1981; Adachi and Oka, 1984), it is assumed that the material behaves elastically in the overconsolidated region only when the stress ratio is kept constant. Considering these previous works, the plastic yield function can be written as:

$$f_y = \hat{\eta}^* - \kappa_s = 0. \quad (17)$$

where $\hat{\eta}^*$ is a stress ratio history invariant given by

$$\hat{\eta}^* = (\hat{\eta}_{ij}^* \hat{\eta}_{ij}^*)^{1/2} \quad (18)$$

$$\hat{\eta}_{ij}^* = s_{ij}^* / \sigma_m'^* \quad (19)$$

in which s_{ij}^* is a deviatoric component of σ_{ij}^* and $\sigma_m'^*$ is spherical component of σ_{ij}^* . For loading condition, it is implicitly assumed that the plastic deformation occurs only when $df_y > 0$. In this case, dz is always positive. From a physical point of view, this assumption corresponds to the fact that the straining is a cause of the micro-structural change or plastic deformation of the materials. Strain-hardening and -softening parameter κ_s is assumed to increase with the increase of the plastic deviatoric strain invariant as

$$\kappa_s = \frac{M_f^* G' \gamma^{p*}}{M_f^* + G' \gamma^{p*}} \quad (20)$$

The hyperbolic type of stress-strain relation has been widely accepted for explaining the

deformation behavior of soils by many researchers (Kondner, 1963; Nishi, 1978; Adachi and Oka, 1984 etc.). In Eq. (20), M_f^* is a value of stress ratio parameter at the large strain strength state and γ^{p*} is the plastic strain invariant;

$$\gamma^{p*} = (e_{ij}^p e_{ij}^p)^{1/2} \quad (21)$$

where e_{ij}^p is a plastic deviatoric strain tensor.

Based on the previous study on overconsolidated clays, the following plastic potential function is used.

$$f_p = \eta + \tilde{M}^* \ln[(\sigma_m' + b)/(\sigma_{mc}' + b)] = 0 \quad (22)$$

where $\tilde{M}^*$ can be obtained by

$$\tilde{M}^* = -\frac{\eta}{\ln[(\sigma_m' + b)/(\sigma_{mc}' + b)]} \quad (23)$$

In the above relations, σ_{mc}' is obtained from the preconsolidation pressure through Eqs. (10) and (14). We add to Eq. (23) the assumption that $\tilde{M}^* = M_m^*$ is always valid after the stress state has reached the normally consolidated region $f_b \geq 0$.

Taking into account the elastic component of strain rate, we get the total strain rate. The elastic strain increment is given by

$$d\epsilon_{ij}^e = \frac{1}{2G} ds_{ij} + \frac{\kappa}{3(1+e)\sigma_m'} d\sigma_m' \delta_{ij} \quad (24)$$

where G is an elastic shear modulus, e is a void ratio and κ is a swelling index.

It is worth noting that the stress history tensor σ_{ij}^* becomes the stress tensor σ_{ij} when $\tau \rightarrow 0$. In this case, the proposed theory automatically becomes the usual elastic-plastic constitutive model by Adachi and Oka (1984) with no strain softening. As for the method of the determination of the value of τ , we have taken a curve-fitting method. However, it is possible to use a suitable optimization technique to determine the value of τ .

SIMULATION OF CHAMPLAIN CLAY BEHAVIOR

In this section, the proposed theory is applied to the triaxial tests carried out on the Louiseville clay. Louiseville is located

on the North shore of the Saint-Laurent river, about 170 km from Quebec city and 88 km from Montreal. The clay deposit has been formed 12000–8500 years before present and is rather massive and homogeneous (Tavenas et al., 1983).

SOIL PROPERTIES

The main properties of the clay studied, taken at a depth of 10 m, are given in Table 1; it is a highly plastic clay with a water content of 69% and a liquidity index of 1.07. The preconsolidation pressure σ_p' is equal to 180 kPa and the sensitivity is equal to 22. Fig. 6 shows the limit state curve of the clay as defined from three CIDC, two CIUC and one anisotropic consolidation ($\sigma_3'/\sigma_1' = 0.5$) triaxial tests. The swelling index κ was determined from this last test; the volumetric plastic strain parameter could also be defined from this test in the normally consolidated region, this aspect has not been considered. Because the present study is dealing only with the overconsolidated range.

As indicated previously, six triaxial tests have been performed on the Louiseville clay and they will be numerically simulated by use of the model proposed in the previous section. The material parameters used in the calculation are listed in Table 2; they can be described as follows: $-\sigma_{m(0)}'$ is the consolidation mean effective stress; except for the CAUC test, the consolidation was isotropic; $-G$, the elastic shear modulus varies with the applied stress (Tavenas and Leroueil, 1979); it has been chosen as shown in Fig. 4;

Table 1. Properties for Louiseville clay

Water content w_0 (%)	69
Void ratio e_0	1.90
Liquid limit w_L (%)	66
Plastic limit w_p (%)	26
Plastic index I_p (%)	40
Specific density G_s	2.77
Clay fraction (%)	73~85
Silt fraction (%)	15~27
Sensitivity S_t	22
Preconsolidation pressure σ_p' (kPa)	180
Friction angle (degree) in the normally consolidated range	30
Permeability k (m/s)	9×10^{-10}

Fig. 6. Limit state surface and stress paths for CIDC tests-Louisville clay

Table 2. Material parameters used in the calculation

Test No.	$\sigma_{m(0)'}^*$ kPa	G_0 kPa	τ	M_f^*	M_m^*	b kPa	G'
CIDC-1	10	17000	0.042	1.5	1.23	50	500
CIDC-2	35	19000	0.014	1.25	1.23	50	500
CIDC-3	60	20000	0.010	0.98	1.23	50	500
CIUC-1	23	18000	0.018	1.5	1.23	50	500
CIUC-2	44	19500	0.013	1.5	1.23	50	500
CAUC-1	36.6	19000	0.014	1.5	1.23	50	500

For CAUC-1, initial deviator stress $\sigma_{11}' - \sigma_{33}'$ is 28 kPa.

$\sigma_{m(0)'}^*$: an initial mean effective stress

G_0 : Elastic shear modulus (initial value of $dq/d\varepsilon_1$)

$\sigma_{m0}' = 198$ kPa

$\sigma_{m0}' = 120$ kPa

$-\tau$ indicates the degree of strain-softening of the material; it is large at low stresses and becomes equal to 0 at the intersection of the limit state curve with ϕ' line of the soil normally consolidated. Fig. 7 shows a typical relationship between τ and $\sigma_{m(0)}'$ and the data giving the best fit for the tests simulated.

$-M_f^*$ is the large strain strength parameter; its variation reflects the curvature of the strength envelope (Lefebvre and La Rochelle, 1974; Tremblay, 1983).

$-M_m^*$ is the strength parameter corresponding to the friction angle of the normally

Fig. 7. Parameter τ versus initial mean effective stress use of Louisville clay

consolidated soil ($\phi' = 30$ degree for the Louisville clay).

DRAINED TRIAXIAL COMPRESSION TEST

Figs. 8-10 show isotropically consolidated drained triaxial compression test results (CIDC test). For drained tests, the strain rate for shearing was 0.1%/h. Fig. 8 provides a typical stress-strain relation for CIDC

Fig. 8. Stress-strain relation for CIDC-1 test

Fig. 9. Stress-strain relation for CIDC-2 test

Fig. 10. Stress-strain relation for CIDC-3 test

test. Before peak, the material is elastic and after, the strength of the material decreases and approaches the large strain strength state. In this figure, the result of model simulation is also presented. Dash-dot-line denotes the relationship between the deviator stress history q^* and axial strain relation. With the increase of axial strain, deviator stress q becomes equal to q^* so that $q - q^*$ decreases during the deformation. q^* can be interpreted as the frictional component of shear strength and

$q - q^*$ corresponds to the component of the shear strength due to bonding, cohesion or reasons other than the frictional resistance. The strain-softening behavior can be interpreted as the process in which the material is destructured and the shear resistance due to cohesion decreases.

It is notable that q^* is developed in the early stages of deformation before peak strength. From a physical point of view, it is reasonable that the microstructure change

Fig. 11. Volumetric strain versus axial strain relation for CIDC tests

or failure begins even in the low deviator stress level. Concerned with the components of strength as a function of strain, Kovari (1982) proposed a schematical figure for rocks, which is similar to Fig. 8. He set up the lower limit of strain where the micro-failure of material occurs. In the present model, the lower limit of strain for the development of frictional strength is not assumed. In Figs. 8-10, the experimental and predicted results of stress-strain curves in CIDC tests are plotted. These figures show that the proposed model can reasonably explain the behavior of O.C. clay during CIDC test. Going into details of stress-strain curves, during the strain-softening process just after the peak, the decreasing rate of deviator stress in experimental results is relatively lower than the calculated ones. In Fig. 11, the calculated volumetric strain-axial strain relations are compared with the experimental results. Compression is observed after peak strength is reached in CID-2 and CID-3, while the mean effective stress decreases.

Although the numerical result for case CIDC-1 is very close to the experimental result, compressive volumetric strains for cases CIDC-2 and CIDC-3 are slightly smaller than measured values. This difference between simulated and measured volumetric

Fig. 12. Stress-strain relation for CIUC-2 test

strains comes essentially from the shape of the plastic potential function, indicating that more theoretical work is necessary to improve the plastic potential function. However, the overall trend agrees well with the experimental results.

ISOTROPICALLY CONSOLIDATED UNDRAINED TRIAXIAL COMPRESSION TEST

In the CIUC-tests, clay specimens were sheared under undrained conditions. The strain rate was 0.5%/hour. Fig. 12 shows a typical comparison between the experimental stress-strain relation and the simulated one. As can be seen, just after the peak strength is reached, the reduction of the shear strength in simulated result is larger than that of experimental one. Fig. 13 shows the stress paths for CIUC and CAUC tests. The simulated stress paths are close to the experimental results.

ANISOTROPICALLY CONSOLIDATED UNDRAINED TRIAXIAL COMPRESSION TEST

In CAUC test, the clay specimen was

anisotropically consolidated under the condition $K=\sigma_3'/\sigma_1'$ equal to 0.75 and sheared under undrained conditions. The strain rate was 0.5%/h. Both simulated stress-strain curve (Fig. 14) and stress path (Fig. 13) show a good agreement with feature of CAUC test. As for CAUC test the decrease of the deviatoric stress after the peak strength is more rapid for the simulated test than for the experiment.

Fig. 13 also shows the trend for the three stress paths in undrained tests to go at large strain towards the same end point. This corresponds to one of the important experi-

Fig. 13. Stress paths for consolidated undrained triaxial tests

Fig. 14. Stress-strain relation for CAUC test

mental evidence.

CONCLUSIONS

In the present paper, a constitutive model for natural overconsolidated clay with strain-softening is developed based on the elasto/plastic constitutive theory for overconsolidated clay (Adachi and Oka, 1984) and the stress history tensor. The proposed constitutive model is applied to the triaxial behavior of a natural overconsolidated clay obtained from eastern Canada (Champlain sea clay). The comparison between the simulated results and the experimental results indicates that the proposed theory is capable of describing the strain-hardening and strain-softening plastic behavior which is important feature of natural soft clay. When applying the proposed model in FEM calculation, the generalization using the nonlocal continuum formulation (Bažant et al., 1988) might be useful to overcome the mesh size dependency.

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