TECHNICAL NOTE

PREDICTION OF BENDING MOMENT OF A BATTER PILE IN SUBSIDING GROUND

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ABSTRACT

Prediction of bending moments of a batter pile installed in subsiding clay ground is presented. The analysis is based on the assumption that loads acting on the upper part of a pile are similar to the case of a positive project conduit and the lower part of the pile behaves as an elastic beam on Winkler's model. The computed values are compared with the measured values on the prototype of piles tested in a clay ground. From the result obtained, it is concluded that the theoretical values are somewhat larger than the measured values. Diagrams with nondimensional parameters are prepared for designing of pile foundations.

Key words: batter pile, design, field test, foundation, pile, settlement (IGC: E4/H1)

THEORETICAL CONSIDERATIONS

Where clay ground subsides under load such as an embankment, there is possibility that a batter pile installed will yield due to excess bending stress. To estimate this bending moment of the pile in such a case, several methods have been proposed so far by Sato et al. (1970), Shibata et al. (1982), Takahashi (1985), and Sato et al. (1987). In this report, the estimation method based on a realistic concept at loading zone will be presented.

Looking at the surface of the subsiding ground where batter piles are installed, we can often see the heave of the ground surface right above the piles. This finding may be explained by the reason that the pile prevents the soil (represented by "loading zone" in Fig.1) right above it from subsiding. This phenomenon seems to be similar to the case of a positive project conduit. Therefore,

Fig. 1. Analytical model of a batter pile

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the vertical loads acting on the upper part of a batter pile near the ground surface can be assumed to be the earth pressure which can be calculated by Spangler's formula. On the other hand, the lower part of the pile is assumed to be an elastic beam on Winkler's model.

If the top of the pile is at the same level as the ground surface and the distance to the limit of loading zone from the top along the pile is referred to $X_1$ as shown in Fig.1, in the range of $X_1 \geq X \geq 0$,

$$EI \frac{d^4 Y_1}{dX^4} = B^2 Y_1 \sin^2 \theta \left( \exp \frac{2K \mu \cos \theta}{B} X - 1 \right)$$  \hspace{1cm} (1)

where $Y_1$ is deflection of the upper part of the pile, $EI$ flexural rigidity of the pile, $B$ width of the pile, $\gamma$ unit weight of the soil, $\theta$ angle of the pile to the vertical, $X$ distance from the top along the pile, $K$ coefficient of lateral earth pressure, and $\mu$ coefficient of friction between the soil right above the pile and the surrounding soil.

On the other hand, in the range of $1 \geq X \geq X_1$,

$$EI \frac{d^4 Y_1}{dX^4} = B_k (S \sin \theta - Y_2)$$  \hspace{1cm} (2)

where $Y_2$ is deflection of the lower part of the pile, $B_k$ horizontal coefficient of subgrade reaction, $S$ settlement at the depth of $X$, and $l$ length of the pile. Here, it is supposed that $S$ decreases in linear relation with the depth and there is no subsidence at the level of the pile toe,

$$S = S_o \left( 1 - \frac{X}{l} \right)$$  \hspace{1cm} (3)

where $S_o$ is settlement of the ground surface.

After substituting Eq. (3) into Eq. (2), Eqs. (1) and (2) are solved using the boundary conditions:

at $X=0$; $Y_1 = \frac{d^3 Y_1}{dX^3} = 0$ at $X=X_1$;

at $X=X_1$;

$$Y_1 = Y_b, \quad \frac{dY_1}{dX} = \frac{dY_2}{dX}, \quad \frac{d^2 Y_1}{dX^2} = \frac{d^2 Y_2}{dX^2}, \quad \frac{d^3 Y_1}{dX^3} = \frac{d^3 Y_2}{dX^3}, \quad \frac{d^4 Y_1}{dX^4} = \frac{d^4 Y_2}{dX^4}$$

$$Y_1 = l; \quad \frac{d^2 Y_2}{dX^2} = 0$$

where the top and the toe of the pile are assumed to be hinges. $X_1$ can be determined by the boundary conditions. However, as no closed solutions can be obtained (Sawaguchi, 1986), numerical calculation should be performed and, as a result, the normalized maximum bending moments ($M_{ul}/EI$) are expressed in diagrams with nondimensional parameters, as shown in Figs. 2. In these figures,

$$\beta l = l \sqrt{\frac{E_I}{4EI}}, \quad \nu l = \frac{2K \mu \cos \theta}{B},$$

$$\xi = \frac{B^2 Y_1 \sin^2 \theta}{2K \mu EI}, \quad \xi = \frac{S_o}{l} \sin \theta$$

The ranges of these parameters used in the figures are as follows: $\beta l = 3 \sim 8$, $\nu l = 20 \sim 50$, $\xi = 0 \sim 0.12$.

OUTLINE OF FIELD TEST

A field test on the prototype of piles was conducted. The length of the piles is 33.65 m, the width 50.8 cm, the flexural rigidity $1.835 \times 10^{11}$ kgf·cm² (1.7983 × 10⁸ kN·m²) and the angle of batter 15°. The two batter piles (Piles C, and D) were installed in a couple with a hinged condition at the top. The subsoil is composed of a clay layer from the surface to the depth of 30 m and it is encountered by sandy soil or stiff clay below. Unconfined compressive strength $q_u$ of the clay soil is expressed as follows:

$$q_u = 3.18 \times r - 7.22 \quad (kN/m^2)$$  \hspace{1cm} (4)

where $r$ (m) is the depth from the ground surface. The pile toes are penetrated into the sandy soil about 5 m in depth. The load on the ground was an embankment whose height was 2.5 m. The size of the embankment was 40 m × 25 m. The bending stress of the piles was measured by wire strain gauges and the settlement of the ground surface by leveling plane surveys. The tests were continued for one year and the settlement of the ground surface finally amounted to 26.3 cm. The details of the piles and the soils conditions may be referred to Ref. 4).
Fig. 2. (a)～(1) Diagrams for maximum bending moment of piles

or 5).

ANALYSIS OF TEST RESULTS

To examine the test results by the proposed method, it is required that the soil constants for use should be set forth. $k_n$ value was estimated by using an empirical correlation $k_n=1.2q_u$ (Sawaguchi, 1986), and $q_u$ was supposed to be the value at the middle level of the clay layer. $K$ was determined by Jaky's formula $K=1-\sin \phi$, where $\phi$ was estimated by the expression

$$
\frac{c_u}{\rho} = \frac{(1-\sin \phi + A_f \sin \phi) \sin \phi}{1 + (2A_f - 1) \sin \phi}
$$

in which $c_u = q_u/2$, $\rho$ effective overburden pressure, $\phi$ angle of shear resistance of the clay, and $A_f$ pore pressure coefficient at failure. In this case, $A_f=0.75$ was supposed, and $\mu$ was taken to be $\tan \phi$, supposing that the vertical plane between the soil right above the pile and the surrounding
Table 1. Parameters of the pile and soil conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$k_u$</td>
<td>0.6 kgf/cm$^3$ (5.88 MN/m$^3$)</td>
</tr>
<tr>
<td>$r$</td>
<td>1.50 tf/m$^3$ (14.7 kN/m$^3$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>25$^\circ$</td>
</tr>
<tr>
<td>$K$</td>
<td>0.43 (1.7963 x 10$^6$ kN.m$^{-1}$)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.7</td>
</tr>
<tr>
<td>$I$</td>
<td>33 m 65 cm</td>
</tr>
<tr>
<td>$\theta$</td>
<td>15$^\circ$</td>
</tr>
<tr>
<td>$S_h$</td>
<td>26.3 cm</td>
</tr>
<tr>
<td>$E_l$</td>
<td>1.835 x 10$^4$ kgf.cm$^2$</td>
</tr>
</tbody>
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Fig. 3. Comparison between the calculated and the measured values on bending moments of the batter pile tested (1 tf-m = 9.8 kN-m)

- Pile C measured
- Pile D calculated

Table 2. Comparison between the calculated values by three methods and the measured on maximum bending moments (1 tf-m = 9.8 kN-m, 1 tf/m$^3$ = 9.8 kN/m$^3$)

<table>
<thead>
<tr>
<th>Author</th>
<th>$M_{\text{max}}$</th>
<th>Remarks</th>
</tr>
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<tbody>
<tr>
<td>Sato, S. et al. (1987)</td>
<td>34(tf-m) 30</td>
<td>$N_e=7$</td>
</tr>
</tbody>
</table>
| Takahashi, K. (1985) | 34 33 28~42 33 | $X(4)=18.65~26.93$  
|                | 34 33 28~42 33 | $r=1.48~14.8$ tf/m$^3$  
|                | 34 33 28~42 33 | $k_u=200~500$ tf/m$^3$  
|                | 34 33 28~42 33 | $k_l=400~800$ tf/m$^3$  |

Note: The values obtained from the writer's diagrams

The measured maximum bending moments and the calculated by Sato, S. et al., Takahashi, K., and the author who have analyzed the same test results by their respective method. In remarks, the inputs with some range are indicated. From this table, it will be evident that any calculated values are close to the measured ones, but it should be noted that the author's method is based on a realistic concept at loading zone.

CONCLUSIONS

The values of the bending moment of batter piles installed in the subsiding clay ground were estimated by the theoretical method which was based on the assumption that the upper part of the pile is loaded by overburden pressure similar to the case of a positive project conduit and that the lower part of it is an elastic beam on Winkler's model. These values were compared with the measured values on the prototype of test piles. As a result, we came to the conclusions that the theoretical value of the maximum bending moment was somewhat larger than the measured values, but that a wide difference did not lie between them.

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REFERENCES