TECHNICAL NOTE

A NEW METHOD FOR STABILITY OF SLOPES

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ABSTRACT

Using the method of characteristics, for soils with nonhomogeneity in angle of internal friction mobilized, contours of angle of internal friction mobilized and stability numbers are obtained. Stability numbers obtained give higher values of factor of safety when compared to Taylor's values.

Key words : landslide, safety factor, shear strength, slope stability, soil mechanics, stability analysis (IGC : E 6)

INTRODUCTION

The problem of stability of slopes has been studied by several investigators. Fellenius (1936) gave an analysis based on circular failure surface. Taylor (1937) and Bishop (1955) used friction circle method and method of slices, respectively. In all these methods some assumptions were made regarding the failure surfaces or interslice forces and the conditions of equilibrium are not fully satisfied. An attempt is made herein to arrive at the failure surfaces satisfying fully the conditions of equilibrium.

$$\text{For } F_c = F_\phi = F_s \text{ to occur ; } \frac{c}{c_m} = \frac{\tan \phi}{\tan \phi_m} \quad (1)$$

after Taylor (1937)

$$\text{Hence, } c \cot \phi = c_m \cot \phi_m \quad (2)$$

where F_c , F_ϕ , F_s = factors of safety with

respect to cohesion, angle of internal friction and shear strength, respectively.

Basic Assumptions :

1. The slope problem is a two dimensional one.
2. The value of the angle of internal friction is fully mobilized along the failure surface and is a constant.
3. The Mohr-Coulomb failure criterion is valid at these points.

ANALYSIS

Fig.1 shows the schematic diagram of the problem considered. A slope having an angle β_0 with the vertical with height H_1 is in equilibrium. The surface OB is horizontal as shown. The point O is taken as the origin.

The equations of equilibrium are

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Manuscript was received for review on September 7, 1988.

Written discussions on this note should be submitted before October 1, 1990, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4 F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

using the method of finite differences, Eqs. (8) through (11) are solved to construct the characteristic grid and the solution of the differential equations at the grid points.

The method of analysis is that of iteration and by trial and error. Some value of ϕ_m is assumed along the $(\theta + \mu)$ surface BC. With the assumed value of ϕ_m the values at grid points are found out satisfying Eqs. (8) through (11). Then at the last point C, if the values do not satisfy the boundary conditions, again another value of ϕ_m is assumed and the procedure repeated. For calculation purposes a small surcharge is applied along or and care is taken to see that the effect of surcharge is negligible.

At the point O, $dr=0$ and $dz=0$. Using the above, Eq. (11) reduces to

$$d\sigma + 2\sigma \tan \phi_m d\theta = 0 \quad (12)$$

Integrating

$$\sigma_{OA} = \sigma_{Or} \exp\{2 \tan \phi_m (\theta_{Or} - \theta_{OA})\} \quad (13)$$

Along the slope, the shear stress and the normal stress are equal to zero. Hence angle between OA and direction of major principal stress is zero and

$$\sigma_{OA} = \frac{H}{1 - \sin \phi_m}; \quad \sigma_{Or} = \frac{q + H}{1 + \sin \phi_m} \quad (14)$$

Substituting σ_{OA} and σ_{Or} in Eq. (13)

$$q = \frac{1 + \sin \phi_m}{1 - \sin \phi_m} H \exp(2 \beta_0 \tan \phi_m) - H \quad (15)$$

For the analysis done, the value of ϕ_m at singular point is chosen as 0.01 radians and q for this value and for a given slope angle β_0 , is found out using Eq. (15).

The $(\theta + \mu)$ characteristic shrinks to a point at the singular point O (see Fig. 1). For any given value of θ , the value of σ at point O can be found from Eq. (13). Hence, the solution for the $(\theta + \mu)$ characteristic at point O is known. The solution along a typical $(\theta + \mu)$ characteristic beyond the singular point is found using the following procedure.

The solution along the $(\theta + \mu)$ characteristic passing through points 1, 6 and 7, let us say, has been earlier obtained at all grid points along it. To obtain the solution along the next $(\theta + \mu)$ characteristic passing through points 2, 3, 4 and 5, a value of ϕ_m

is assumed along it. For this assumed value of ϕ_m , the values of σ , θ , r and z at point 2 (Fig. 1) are arrived for given r and z at point 2. The values of the variables at points 1 and 2 are represented by σ , θ , r and z with subscripts 1 and 2. The variables at the point of intersection of the $(\theta - \mu)$ and $(\theta + \mu)$ characteristics passing through points 1 and 2, respectively, are denoted by σ_3 , θ_3 , r_3 and z_3 and are found from:

$$r_3 = \frac{z_2 - z_1 + r_1 \tan(\theta_1 - \mu_1) - r_2 \tan(\theta_2 + \mu_2)}{\tan(\theta_1 - \mu_1) - \tan(\theta_2 + \mu_2)};$$

$$z_3 = z_1 + (r_3 - r_1) \tan(\theta_1 + \mu_1) \quad (16)$$

$$\sigma_1 - \sigma_2 \tan^2 \phi_3 \theta_3^2 + X_5 \theta_3 + X_6 = 0 \quad (17)$$

$$\text{and} \quad \sigma_3 = \frac{X_1 - \sigma_2 \theta_2 \tan \phi_3}{X_2 + \theta_3 \tan \phi_3} \quad (18)$$

in which

$$A_1 = \{(z_3 - z_1) - (r_3 - r_1) \tan \phi_3\} \gamma;$$

$$B_1 = \frac{1}{2} \left\{ \frac{\partial \phi_3}{\partial r_3} (z_3 - z_1) - \frac{\partial \phi_3}{\partial z_3} (r_3 - r_1) \right\}$$

$$X_3 = \sigma_1 - \sigma_1 \theta_1 \tan \phi_3 + A_1 + \sigma_1 B_1$$

$$X_4 = 1 + \theta_1 \tan \phi_3 - B; \quad A_2 = [(z_3 - z_2) + (r_3 - r_2) \tan \phi_3] \gamma$$

$$B_2 = -\frac{1}{2} \left[(z_3 - z_2) \frac{\partial \phi_3}{\partial r_3} - (r_3 - r_2) \frac{\partial \phi_3}{\partial z_3} \right];$$

$$X_1 = \sigma_2 + \sigma_2 \theta_2 \tan \theta_2 + \sigma_2 B_2 + A_2;$$

$$X_2 = 1 - \theta_2 \tan \phi_3 - B_2$$

$$X_5 = (\sigma_1 X_2 + X_3 + \sigma_2 X_4 + X_1) \tan \phi_3;$$

$$X_6 = X_2 X_3 - X_1 X_4$$

As the solution at grid points along the $(\theta + \mu)$ characteristic passing through points 1, 6 and 7 is known, using the above equations, the values of unknowns at all the other grid points, except at grid point 5, of the $(\theta + \mu)$ characteristic passing through points 2, 3, 4 and 5 are obtained. To arrive at the solution at grid point 5 which is along the slope OA (Fig. 1) Eqs. (10) and (11) are used along with

$$r_5 = -z_5 \tan \beta_0 \quad \text{and} \quad \theta_5 = \frac{\pi}{2} + \beta_0$$

The value of σ_5 obtained from the above equations should tally with $\sigma_5 = H/(1 - \sin \phi_m)$ which is the value along the free boundary OA (Fig. 1). If not, another value of ϕ_m is assumed along the $(\theta + \mu)$ characteristic

passing through points 2, 3, 4 and 5 and the iteration is repeated till the required accuracy is obtained. The same procedure is used to get the solution along the other $(\theta + \mu)$ characteristics. The partial derivatives are found using the method given by Sabzevari and Ghahrmani (1972). For better accuracy Larkin (1968) procedure is used in arriving at the values of unknowns at the intersections of the characteristics.

RESULTS

For the calculation purposes all the equations are nondimensionalized using characteristic stress ' H ' and characteristic length ' l ' such that $l = H/\gamma$.

$$h_l' = \frac{h_l}{l}; \quad \therefore h_l = h_l' l$$

$$N_s = \frac{c_m}{\gamma l h_l'}, \text{ but } \gamma l = H = c_m \cot \phi_m$$

$$\therefore N_s = \frac{c_m}{c \cot \phi_m h_l'} = \frac{\tan \phi_m}{h_l'}$$

The results are presented as the curves of N_s versus the ϕ_m for different slope angles. Fig. 2 shows the ϕ_m contour pattern for slope angle $\beta_0 = 5^\circ$. It can be seen that the $(\theta + \mu)$ lines go deeper as the value of ϕ_m increases.

The results are presented in Fig. 3 as curves

Fig. 2. Failure pattern for $\beta_0 = 5^\circ$

for stability numbers versus ϕ_m mobilized for different slope angles. The horizontal axis denotes the value of ϕ_m .

As β_0 increases, i.e. as the slope becomes flatter the stability number decreases at the same value of ϕ_m . It indicates that at the

Fig. 3. Stability chart for various values of β_0

Table 1. Percentage difference between Taylor's (1937) values and values of present analysis

ϕ_m , deg (1)	Slope angle β_0			
	0° (2)	10° (3)	20° (4)	30° (5)
5	1.36	1.44	9.34	10.77
10	4.59	5.91	9.38	17.86
15	6.00	6.66	10.71	17.40
20	6.59	7.53	12.5	21.06
25	7.23	7.69	13.46	25.00

same value of ϕ_m , more height of flatter slope is possible.

Table 1 shows the percentage variations between the values from the present analysis and Taylor's (1937) values. The percentage difference increases with increase in β_0 and ϕ_m .

CONCLUSION

A method for stability of slopes is developed satisfying the equations of equilibrium and arriving at contours of different values of

angles of internal friction developed. The numerical results for stability numbers give higher values of factor of safety compared to Taylor's values.

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