YIELDING AND FLOW OF SAND UNDER PRINCIPAL STRESS AXES ROTATION

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ABSTRACT

In order to investigate the influence of inherent anisotropy on the yielding and plastic flow of loose and dense sand, a series of experiments involving a wide range of principal stress axes directions were carried out on hollow cylindrical specimens of Toyoura sand using a torsional shear test apparatus. Stress paths involving cycles of stress reversal and stress axis rotation were performed in order to identify the states of stress at which plastic flow begins to take place. The test results revealed well-defined families of yield loci having identical shapes in the stress space independent of the density of sand. Experimental evidence presented here shows that the direction of the plastic strains for different stress increments is not uniquely established indicating the non-existence of a unique plastic potential. Consequently, within the framework of conventional theory of plasticity, the yielding and flow of sand can only be explained with the concept of multiple yield surfaces.

Key words: anisotropy, deformation, drained shear, plastic flow, sand, stress-strain curve, yield (IGC : D 6)

INTRODUCTION

It is well known that the loading induced by earthquakes, traffic and waves, involves cyclic changes in the direction of the principal stresses axes. Concerns about the likelihood of liquefaction and settlements have created the needs for accurate predictions of sand behavior with sophisticated numerical models. However since most of the proposed constitutive models are expressed in terms of stress and strain invariants, the pore pressures and deformations related to stress axes rotation can not be predicted. This effect should not be ignored. Laboratory investigations (Arthur et al., 1980; Ishihara and Towhata, 1983 a) have shown the effects of the rotation of principal stress directions during cyclic loading; these studies have revealed that strains steadily accumulate and that under undrained conditions there is a continuous development of pore water pressures.

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Experimental investigations involving testing with fixed directions of principal stresses (Arthur et al., 1981; Miura et al., 1986; Symes et al., 1984) have shown that inherent anisotropy plays an important role and showed the influence of the orientation of the principal stress axes on the deformability and the shear strength of sands. The absence of coaxiality between stresses and strain increment directions has also been observed experimentally (Miura et al., 1986). However in tests with fixed directions of stresses the deviations are relatively small and for modeling purposes coaxiality might be considered as a reasonable assumption.

APPARATUS AND TEST PROCEDURES

The torsional shear test device used in this study is an improved version of the one previously employed by Ishihara and Towhata (1983a) as shown in Fig. 1. It allows the individual control of the axial force $F$, torque $T$, inner cell pressure $p_i$ and outer cell pressure $p_o$. Two load cells allowed to monitor the forces creating the axial and torsional loads. After introducing the necessary corrections such as friction forces, back pressure and the effects of the cell pressure, the effective loads $F'$ and $T'$ acting on the top of the specimens were obtained. Two high capacity differential pressure transducers (HC-DPT) connected to the cell and to the specimen's drainage lines were used to measure the effective inner and outer pressures $p_i'$ and $p_o'$ acting on the soil specimen.

The hollow cylindrical specimens used in this study had the nominal dimensions of an inner radius of $r_i=30 \text{ mm}$, an outer diameter of $r_o=50 \text{ mm}$ and a height of $h=193 \text{ mm}$. The dimensions $r_i$ and $r_o$ are the same as previously employed, whereas the height of the specimen was almost doubled from 104 mm in order to minimize the effects of end restraint. It appeared that this modification of the geometry had a considerable influence on the strength, although the deformation pattern was unchanged in agreement with prior observations.

The average deformations were computed based on the changes in height $\Delta h$, in torsional angle $\Delta \theta$, and in inner and outer radii $\Delta r_i$ and $\Delta r_o$ of the specimen. The volume change of the specimen was measured with two low capacity differential pressure transducers (LC-DPT) connected to burettes. The measurements were corrected for membrane penetration and for tube compress-
ibility; the average values of $\Delta r_t$ and $\Delta r_o$ were calculated at each step of loading using the corrected values of the volume change and the actual dimension of the specimen.

Japanese standard sand known as Toyoura sand was used as the test material. It is made of particles of subrounded to subangular shape, and has a specific gravity of 2.65 and a mean particle size of 0.17 mm. Its minimum and maximum void ratios were measured to be 0.597 and 0.977, respectively. Specimens were prepared by air pluviation and the specimen density was controlled by varying the height of fall. The void ratios obtained ranged from 0.691 to 0.716 with an average relative density of 70% for the dense specimens and from 0.827 to 0.857 with an average relative density of 32% for the loose specimens.

The state of stress within the specimen is shown in Fig.2. Tests were performed under the condition of equal radial and tangential stresses $\sigma_r'$ and $\sigma_\theta'$. Because of experimental limitations some small discrepancies in the order of $\sigma_r'=\sigma_\theta'\pm 0.2$ kPa occurred. The specimens were all isotropically consolidated and then sheared under a constant mean effective pressure of $p'=98.1$ kPa where:

$$ p' = \sigma_r' + \sigma_\theta' + \sigma_z' $$(1)

STRESS AND STRAIN WITHIN THE SPECIMEN

Since the distribution of the stress across the wall of a hollow cylindrical specimen is unknown, some uncertainty exists in the evaluation of the stress $\sigma_r$, $\sigma_\theta$, $\sigma_z$ and $\sigma_\phi$ (Fig.2). A comprehensive review and analysis of existing formulas was performed by Pradhan et al. (1988) who showed that the values of the average stress calculated from different formulas can vary considerably. However, when the ratio of inner to outer cell pressure is close to unity, as is the case in this study, all existing solutions converge and the discrepancy between the different solutions vanishes. In this study the average radial, tangential and axial stresses were calculated, respectively, as follows:

$$ \sigma_r' = \frac{p_o' \cdot r_o + p_i' \cdot r_i}{r_o + r_i} + \Delta \sigma_r' $$(2)

$$ \sigma_\theta' = \frac{p_o' \cdot r_o - p_i' \cdot r_i}{r_o - r_i} + \Delta \sigma_\theta' $$(3)

$$ \sigma_z' = \frac{F'}{A} + \frac{1}{2} \cdot \gamma' \cdot h + \Delta \sigma_z' $$(4)

In the above formulas, $\Delta \sigma_r$, $\Delta \sigma_\theta$ and $\Delta \sigma_z$ designate the membrane force corrections based on the theory of elastic thin shells (Tatsuoka et al., 1986). $A$ is the average cross-sectional area of the specimen, and $\gamma'$ is the submerged unit weight of the specimen.

For any stress distribution the circumferential stress satisfies the condition of equilibrium:

$$ T' = \int_0^{2\pi} \int_{r_i}^{r_o} [\sigma_{zz}(r') \cdot r'] \cdot r dr \cdot d\theta $$

Based on this condition the average stress corresponding to the linear elastic and to the perfectly plastic distribution is obtained as follows:

$$ (\bar{\sigma}_{zz})^l = \frac{4T' \cdot (r_o^3 - r_i^3)}{3\pi \cdot (r_o^4 - r_i^4) \cdot (r_o^4 - r_i^4)} $$ (5)

$$ (\bar{\sigma}_{zz})^p = \frac{3 \cdot T'}{2\pi \cdot (r_o^4 - r_i^4)} $$ (6)

Each of the above formulas has an optimum range of applicability. At low shear stress levels the linear elastic distribution gives the best results and Eq. (5) should be used. However at levels close to failure a perfectly plastic distribution seems more accurate and the use of Eq. (6) becomes more appropriate. The maximum deviation induced by considering either of these distributions is a function of the geometry. With the adopted specimen dimension it was evaluated to be less than 2% and therefore small. In this study Eq. (7) as follows, was used to obtain the average circumferential stress. It is the average of the linear elastic solution given by Eq. (5) and the perfectly plastic solution expressed by Eq. (6) and includes a correction for membrane forces.
The average strains, $\varepsilon_z$, $\varepsilon_r$, and $\varepsilon_\theta$, were obtained assuming the specimen to remain a straight cylinder throughout the loading in the experiment. Therefore:

$$\varepsilon_z = \frac{\Delta h}{h}$$

$$\varepsilon_r = -\frac{\Delta r_0 + \Delta r_1}{r_0 + r_1}$$

$$\varepsilon_\theta = -\frac{\Delta r_0 - \Delta r_1}{r_0 - r_1}$$

The average circumferential strain is obtained assuming a linear distribution across the wall of the hollow cylindrical specimen. Hence:

$$\varepsilon_{z0} = \frac{1}{\pi \cdot (r_0^3 - r_1^3)} \int_{r_1}^{r_0} \int_0^{2\pi} \varepsilon_{z0}(r) \cdot r \cdot d\theta dr$$

$$\varepsilon_{r0} = \frac{1}{\pi \cdot (r_0^3 - r_1^3)} \int_{r_1}^{r_0} r \cdot \frac{\Delta \theta}{2h} \cdot 2\pi r \cdot dr$$

$$\varepsilon_{\theta0} = \frac{\Delta \theta \cdot (r_0^3 - r_1^3)}{3h \cdot (r_0^3 - r_1^3)}$$

It is interesting to note that though the stresses, $\sigma_r$, $\sigma_\theta$, and $\sigma_{z0}$, acting on planes perpendicular to the radial and tangential directions, respectively, are different, the strains $\varepsilon_r$ and $\varepsilon_\theta$ are always almost identical. The usual deviation of $(\varepsilon_r - \varepsilon_\theta)$ from the computed values was smaller than 0.1% and the maximum deviation $(\varepsilon_r - \varepsilon_\theta)_{\text{max}} = 0.3\%$ was observed on a test when $\varepsilon_r - \varepsilon_\theta = 7\%$ and $\varepsilon_\theta = 3\%$. It is therefore useful to introduce the following average strain:

$$\varepsilon_m = \frac{\varepsilon_r + \varepsilon_\theta}{2}$$

The use of this average strain is also justified from energy consideration; the increment of strain energy $dW$ is given by:

$$dW = \sigma'_r \cdot d\varepsilon_r + \sigma'_\theta \cdot d\varepsilon_\theta + 2 \sigma_{z0}' \cdot d\varepsilon_{z0}$$

Considering $\sigma'_r = \sigma'_\theta$, then:

$$dW = \frac{1}{3} \cdot (\sigma'_r + \sigma'_\theta + \sigma'_z) \cdot (d\varepsilon_r + d\varepsilon_\theta + d\varepsilon_z) + 2 \cdot \sigma_{z0}' \cdot d\varepsilon_{z0}$$

$$= \frac{2}{3} \cdot (\varepsilon_r - \varepsilon_m) \cdot (d\varepsilon_r + d\varepsilon_\theta + d\varepsilon_z) + 2 \cdot \sigma_{z0}' \cdot d\varepsilon_{z0}$$

Fig. 3. Mohr circle of stress

In the above expression the first right-hand term gives the volumetric contribution to energy, while the second and third terms define the contribution from shear distortion. This equation reflects that in torsional shear two very distinct shearing modes are possible; one is the triaxial mode created by the stress difference $\sigma_z - \sigma_\theta$ and the other one is the torsional mode due to the stress $\sigma_{z0}$. Note that the maximum deviator stress can be obtained as illustrated in Fig. 3 from the Mohr circle of stress as:

$$\tau_{\text{max}} = \frac{\sigma_z - \sigma_\theta}{2} = \sqrt{\left(\frac{\varepsilon_z - \varepsilon_\theta}{2}\right)^2 + \varepsilon_{z0}^2}$$

It is therefore natural to define the state of shear in a hollow cylindrical specimen in terms of the stress parameters $\sigma_z - \sigma_\theta$ ann $\sigma_{z0}$. The definition of the strain-increment parameters $\varepsilon_z - \varepsilon_\theta / 3$ and $\gamma_{z0}$ can be obtained from the condition that when the corresponding stress and strain-increment is multiplied together it correctly gives the incremental work per unit volume (Eq. 13). Hence:

$$dW = \rho' \cdot d\varepsilon_z + (\sigma_z - \sigma_\theta) \cdot \left(\sigma_z - \sigma_\theta - \frac{1}{3} \varepsilon_m\right) + \sigma_{z0}' \cdot d\gamma_{z0}$$

where:

$$\varepsilon_m = \varepsilon_z + \varepsilon_r + \varepsilon_\theta$$

$$\frac{2}{3} (\varepsilon_z - \varepsilon_m) = \varepsilon_z - \frac{1}{3} \varepsilon_m$$

$$\gamma_{z0} = 2 \cdot \varepsilon_{z0}$$
DETERMINATION OF THE YIELD LOCI

The method followed in this investigation assumes that at every stage of loading the yielding condition can be approximated with a yield function \( f \) expressed in terms of stresses \( \sigma_{ij} \) and of hardening variables \( a_k \). Yielding occurs when a stress increment \( d\sigma_{ij} \) exceeds the boundary of the yield surface \( f=0 \), i.e., when:

\[
f(\sigma_{ij}, a_k) = 0 \quad \text{and} \quad \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} \geq 0 \quad (19)
\]

When the above conditions are fulfilled, the yield surface defined as \( f(\sigma_{ij}, a_k) = 0 \) is erased and replaced by a new surface \( f(\sigma_{ij} + d\sigma_{ij}, a_k + da_k) = 0 \); where \( da_k \) contains the increment of the hardening variables. For a given set of hardening variables \( a_k \) the function \( f=0 \) defines a convex hypersurface in the six-dimensional stress space known as the yield surface. Sections of this surface can be obtained experimentally, and several studies have been performed to this end using the conventional and cubical triaxial test devices (Poorooshasb et al., 1967; Tatsuoka and Ishihara, 1974; Tatsuoka and Molem-kamp, 1983; Yamada and Ishihara, 1982).

In this investigation stress paths such as the one illustrated in Fig. 4 were employed to study the yielding behavior of sand when stress axes rotate. Note that in this figure the torsional shear stress \( \sigma_\theta \) and the stress difference \( \sigma_z - \sigma_\theta \) are normalized with respect to the mean effective pressure \( p' = 98.1 \text{kPa} \). These stress paths consist first of a cycle of loading and unloading with a fixed direction of principal stress \( \sigma_p \), followed by the rotation of the stress axis by an angle \( \Delta \alpha_p = 15^\circ \), and finally of reloading along the direction \( \alpha_p + \Delta \alpha_p \). The tests were conducted for two different densities of sand having a void ratio between 0.701 and 0.714 with \( D_v = 70\% \) for the Y tests, and a void ratio between 0.827 and 0.877 with \( D_v = 32\% \) for the L tests as illustrated in Fig. 5. In what follows, the tests are identified by a combination of the type of loading and its direction on the plane in Fig. 5, e.g., Y 90 is a test performed on dense sand in which the direction of the stress state varies periodically between \( 2\alpha_p = 90^\circ \) and \( 2\alpha_p + \Delta \alpha_p = 120^\circ \). The tests were performed such that each of them allowed to determine two or three yield loci.

Examining the stress–strain behavior it can be noticed that yielding occurs throughout well defined portions of the stress path. For the loading scheme such as the one shown in Fig. 4 the sand behavior was observed to be plastic between points 1 and 2, then essentially elastic between 3 and 5, and again plastic between points 5 and 6. The stress at which plastic strains reappear, i.e. point 5, was defined as the yield point. Since the yield condition (Eq. 19) has been defined as the end point of the previous stage of yielding, points 2 and 5 are considered to
belong to the same yield surface and define what is known as a yield locus (Fig. 4).

With the present loading scheme two shearing phenomena occur simultaneously. One is due to torsional shear and involves $\sigma_{zz}$, and the other due to the stress difference $\sigma_z - \sigma_y$. Hence, the following discussions are made treating each of them separately. For any given experiment, two yield points are obtained based on the following stress-strain curves:

1. $\frac{\sigma_y}{p'}$ versus $\gamma_{2B}$
2. $\frac{\sigma_z - \sigma_y}{p'}$ versus $\varepsilon_z - \frac{1}{3} \varepsilon_y$

Fig. 6 shows the deformation pattern of sand in the torsional mode which was observed for the stress path in Fig. 7. The point at which irreversible strains reappear is clear and there is no uncertainty in defining the position of the yield point and of the yield locus. This can also be observed for the tests in Figs. 8 and 9. Such a sharp change between the elastic and plastic response can not always be observed. Furthermore, in reality there is always a smooth transition between the two types of behavior,
Fig. 11. Stress-strain behavior in Y 120 test

Fig. 12. Stress path in Y 120 test

Fig. 13. Stress-strain behavior in L 150 test

as it is shown schematically in Fig.10. In certain tests the strain in the transition zone is very small and the points 1, 2 and 3 in Fig.10 appear to be identical as it is the case for the stress-strain curves illustrated in Figs.6 and 8. Such a sharp yield point was not observed in all the tests, and in many cases the plastic strain in the transition zone was too large to locate accurately a yield point as shown in Figs.11 through 14.

Fig. 14. Stress path in L 02 test

Fig. 15. Experimental yield loci of dense sand

Fig. 16. Experimental yield loci of loose sand

Under such circumstances the derivation of the yield point is subject to some errors and requires some assumptions regarding the
stress–strain behavior. For the determination of the yield loci the behavior was assumed to be bilinear and the yield point was obtained by locating an intersection between idealized elastic and plastic responses such as the point 2 in Fig. 10. A linear slope following the last stress reversal was adopted for the elastic branch, whereas the plastic response was taken non-linear and obtained by extending the observed post-yield stress–strain curve as illustrated in Figs. 10, 11 and 13.

The yield loci obtained in the above manner for the loose and dense specimens are shown in Figs. 15 and 16. It can be noticed that different stress–strain relationships give slightly different loci. These deviations are to be related to both the very complex phenomena involved in the yielding of sand, and to approximate nature of the aforementioned technique for yield locus determination. Although some differences do exist the yield loci in Figs. 15 and 16 have a very unique shape that defines smooth elliptic shaped yield surfaces in the stress space for both dense and loose Toyoura sand.

It is important to note that yielding was seldom observed during the rotation process, i.e. between points B and C in Figs. 6 through 14. Although this observation is limited to the case where \( \Delta \alpha_s = 15^\circ \), it implies that stress axes rotation does not necessarily produce plastic strains. Yielding is therefore associated with a stress magnitude corresponding to a given direction of principal stress axes. In the study by Ishihara and Towhata (1983 b), the behavior of specimens subjected to half circular rotation of the principal stress axes was analyzed. In these tests the principal stresses were rotated such that the angle \( \alpha_s \) varied cyclically between \(-45^\circ\) and \(45^\circ\). They observed that at the beginning of stress reversal the direction of strain increments and of stress increments coincides, hence implying elastic behavior. Their tests revealed that the domain where the increments in stress and strain coincide in direction corresponded to a variation of the angle \( \alpha_s \) in the order of \(20^\circ\). In the present investigation an angle \( \Delta \alpha_s \) higher than \(20^\circ\) would have produced plastic strains during the rotation process, thus creating a sizable transition zone (Fig. 10) which would have brought uncertainty to the determination of yield points. However, this problem did not occur since the angle \( \Delta \alpha_s \) was limited to \(15^\circ\), and therefore well within the elastic domain suggested by the experiments of Ishihara and Towhata (1983 b).

THEORETICAL CONSIDERATIONS

The validity of the yield loci in Figs. 15 and 16 will be discussed in the light of existing elastoplastic models for the development of constitutive laws. Conventional constitutive models such as those by Burland and Roscoe (1968), Lade and Duncan (1973), Lade (1977), Vermeer (1978), and Matsuoka (1976) assume the existence of one or two yield surfaces enclosing a huge elastic domain. These models are well adapted to soil behavior and give very reasonable prediction for various types of loading conditions. When cyclic loading and unloading is involved they predict a sharp change in the stress–strain behavior when switching from elastic to plastic response. For the stress path such as the one in Fig. 4, the predicted behavior is bilinear and identical to the idealized response shown in Fig. 10. Yield points derived from the intersection of a linear elastic and post-yield slopes (Fig. 10) give obviously the most appropriate position of the yield loci for this type of models.

Models using the isotropic-kinematic hardening formulation by Mroz (1967), such as the models by Prevost and Hoeg (1975), Aubry et al. (1982) and Mroz et al. (1979), make a distinction between primary and secondary yielding. Primary yielding occurs when a yield surface is mobilized for the first time at its initial state position. Secondary yielding is characterized by the reactivation of yield surfaces that have been previously mobilized. This concept is illustrated in Fig. 17 for a stress path similar to the one in Fig. 4. Between points O and A in Fig.
17(b) the activation of the yield surfaces \(f_0\) and \(f_1\) corresponds to primary yielding; during a stress reversal the behavior is elastic until the boundary of the first surface \(f_0\) is reached, such as the point B in Fig.17 (c); then surfaces previously activated are once again mobilized, such as \(f_0\) and \(f_1\) between points B and C in Fig.17 (d), hence, involving secondary yielding. During a cycle of loading, unloading, rotation of stress axes and reloading such as the one in Fig.4, both types of yielding are involved. In the reloading phase the yield point obtained with the aforementioned technique (e.g. point 2 in Fig.10), lies in between the points where plastic strains reappear (point 1 in Fig.10) and where the behavior is essentially plastic and primary yielding reoccurs (point 3 in Fig.10). Since the unloading portions of the stress path in this study are essentially within the elastic domain and the changes in loading direction are small, the experimental points equivalent to points B and C in Fig.17(d) are very close to each other. This explains why the transition zone (Fig.10) in Figs.6 through 14 is so small. Hence the experimental yield points are a good approximation of the location of primary yielding and therefore the yield loci shown in Figs.15 and 16 are also applicable for this type of models.

In the models using the bounding surface theory, such as those by Dafalias and Herrmann (1987) and Poorooshab (1986), the bounding surface (which is roughly the equivalent to primary yielding in the isotropic-kinematic hardening models) is allowed to either shrink or dilate during cyclic loading. A direct comparison between the experimental yield loci obtained here and the shape of the bounding surface can not be made. Nevertheless, it is considered reasonable to believe that the magnitude of the plastic strains during plastic unloading are not sufficiently great to have a significant change in the size and position of the bounding surface. Hence the yield loci presented here give a reasonable approximation of the shape of the bounding surface. However, a conclusion such as in the case of isotropic-kinematic hardening models cannot be made and the precision of this approximation needs to be discussed in case-by-case manner for each of the bounding surface models.

It is interesting to compare the yield loci obtained in this study with some of the most widely known elastoplastic models. Most models define the yield function \(f=0\) in terms of the three stress invariants:

\[
I_1 = \sigma_1 + \sigma_2 + \sigma_3 \\
I_2 = \sigma_1 \cdot \sigma_2 + \sigma_2 \cdot \sigma_3 + \sigma_3 \cdot \sigma_1 \\
I_3 = \sigma_1 \cdot \sigma_2 \cdot \sigma_3
\]

(20a) (20b) (20c)

where \(\sigma_1, \sigma_2\) and \(\sigma_3\) are the three principal stresses.

The yield surfaces by Lade and Duncan (1973) and by Matsuoka (1976) are written, respectively, as:

\[
\frac{I_1}{I_3} = \kappa \\
\frac{I_1}{I_3} = \text{const.}
\]

(21a) (21b)

Fig.18 shows that Lade's and Matsuoka's yield conditions define surfaces which are different from the Mohr-Coulomb criterion:
PLASTIC FLOW OF SAND

In order to gather some information concerning the flow of granular materials, a series of tests were conducted on dense specimens of Toyoura sand with void ratios between 0.691 and 0.716 with $D_s=70\%$. Following the isotropic consolidation, the specimens were sheared along common stress paths OMP as shown in Fig. 22. Each specimen was then subjected to a cycle of loading and unloading of a small stress increment $d\sigma$ (Fig. 22). The magnitude, orientation and position of the stress probes performed in this study are illustrated in Fig. 23.

The plastic strains are defined as those which remain unrecovered after the cycle of application and removal of $d\sigma$. The directions of plastic strain increments are shown in Figs. 24 and 25 in terms of the strains $\varepsilon_p - \varepsilon_p/3$ and $\gamma_{pp}$.

The experimental results in Figs. 24 and 25
Fig. 22. Loading scheme in stress probe tests

Fig. 23. Stress probes in F tests

Fig. 24. Plastic strain increments observed in F 25 tests

Consequently, the use of a single plastic potential function in an elastoplastic formulation can not be satisfactory.

When $\sigma_f = \sigma_f'$ and $p' = 98.1 \text{kPa}$ the second derivative of plastic work $d^2W^p$ is obtained from Eq. (15) as follows:

$$d^2W^p = (d\sigma_z - d\sigma_y) \left(d\varepsilon_z^p - \frac{1}{3} d\varepsilon_v^p\right) + d\sigma_{yz}^p d\gamma_{yz}^p$$

(22)

It is interesting to note that although the strain increment vectors are not parallel in direction, $d^2W^p$ is always observed to be positive and therefore satisfying Drucker's stability condition (Drucker, 1951):

$$d^2W^p \geq 0$$

(23)

MULTIPLE MECHANISMS

The experimental results presented here show evidence of non-uniqueness of a single plastic potential (Figs. 23 to 25). Therefore, for accurate predictions of yielding and flow of sands with the theory of elastoplasticity, it is absolutely necessary to incorporate the concept of multiple mechanisms (Koiter, 1964).

For tests involving even small changes in the direction of the stress paths changes in the activation of mechanisms can be expected. This theory would therefore require the yield loci obtained from various stress-strain curves. This would of course explain the differences observed in Figs. 15 and 16 between the torsional and the triaxial mode.
Fig. 26. Direction of plastic flow observed in F tests

of yield loci. However, because of approximate nature of the yield loci determination method it is impossible to conclude whether this difference comes or not from the simultaneous activation of different mechanisms.

If only two different mechanisms are involved they would define a vertex in the stress plane in Fig. 26. Such type of vertex is widespread in the elastoplastic modelling of soil behavior and double hardening models such as those by Vermeer (1978), and Lade (1977) uses this concept in order to predict more accurately the volumetric and deviatoric behavior of soils. The range of observed plastic flow directions for two common stress points is illustrated in Fig. 26 together with the possible locations of the plastic potential. Note that for each of the common stress points there is a close relation between the position of yield locus, obtained from Fig. 15, and one of the limiting directions of the plastic potential.

CONCLUSIONS

In order to elucidate the influence of stress axes rotation on the yielding and plastic flow of sand, a series of experiments on loose and dense specimens of Toyoura sand were performed. The conclusions drawn are as follows:

(1) Yielding occurs when stresses reach a particular stress level for a given direction of principal stresses.

(2) The yield loci obtained from different stress-strain curves are not exactly identical.

(3) All the yield loci seem to define very smooth curves of elliptical shape that are independent of density.

(4) The direction of the plastic strains varies considerably, showing non-coaxiality and non-existence of a unique plastic potential.

(5) Within the framework of elastoplasticity all the above points can only be explained using multiple mechanisms.

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