SYSTEM COMPLIANCE CORRECTION FROM PORE PRESSURE RESPONSE IN UNDRAINED CYCLIC TRIAXIAL TESTS

Koiji Tokimatsu

ABSTRACT

The effects of system compliance on pore pressure response during undrained cyclic triaxial tests are examined based on a review of previous studies. The presence of system compliance decreases the amplitude of pore pressure fluctuation caused by cyclic changes in the axial load, making the stress path slanted. On the basis of theoretical studies, equations are presented for determining the system compliance ratio of a specimen using its pore pressure coefficients $\hat{A}$ and $B$. $\hat{A}$-value can readily be determined from pore pressure fluctuation during an undrained cyclic triaxial test. The system compliance ratios given by the proposed equations are in good agreement with those measured by a special test. A simplified procedure is then presented for correcting for the system compliance effects on triaxial liquefaction test results. The proposed method requires neither special test conditions nor additional tests, and can therefore conveniently be used in many cases.

Key words: gravel, laboratory test, liquefaction, membrane penetration, sand, shear strength, system compliance, undrained test (IGC: D7/D6)

INTRODUCTION

The system compliance in undrained cyclic triaxial tests, because of its volume change with decreasing effective stress, has a significant influence on the generation of pore water pressure. Previous studies have shown that not only the volume changes due to pore pressure measuring system and tubing but also membrane penetration is the major cause of the compliance. Since the resulting errors in liquefaction resistance are on the unsafe side, an appropriate correction appears indispensable for a test with significant system compliance.

There are basically two methods to eliminate or correct for such compliance effects in an undrained cyclic triaxial test.

1) Elimination or minimization of system compliance during the test: A treated membrane (e.g., Kiekbusch and Schuppener, 1977) and fine particles smeared on the surface of the test specimen are effective for reducing membrane compliance, but ineffective for eliminating volume changes in the pore pressure measuring system and tubing. Besides, the volume changes of treated specimens due to membrane penetration are not always negligibly small and yet vary from specimen to specimen, thus requiring the re-assessment of system compliance for each treated specimen.

1) Associate Professor, Tokyo Institute of Technology, O-okayama, Meguro-ku Tokyo 152. Manuscript was received for review on April 27, 1989.

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Tokimatsu and Nakamura (1986) proposed a computer controlled test apparatus which can basically eliminate any unfavorable volume changes during a triaxial liquefaction test. Although effective, the method needs the knowledge of the volume change characteristics of the system compliance for each test specimen. This in turn requires a special test to determine such volume change characteristics.

2) Corrections of the test result of a specimen after the test has been made conventionally: Martin et al. (1978) first proposed this type of correction. Their method is based on analytical studies with few experimental data and tends to result in overcompensation of resistance, raising questions concerning its general applicability. Recently, Tokimatsu and Nakamura (1987) proposed a refined version of correction procedure of which validity is supported by a comparison with laboratory test results. Both of these methods require a specific test to evaluate system compliance characteristics of the test specimen as well.

All of the above procedures thus require special test conditions to evaluate and/or to minimize volume changes caused by system compliance. Difficulty in conducting these tests routinely severely restricts the general applicability of the aforementioned procedures. Besides, the difficulty in determining system compliance of a specimen without affecting its liquefaction resistance also poses a problem. In addition, the accuracy of the corrected liquefaction resistance of a specimen depends on the accuracy of the system compliance volume change characteristics used for correction.

It seems therefore that a more reliable and yet simple method is required to evaluate and if needed to correct for the system compliance effects on a triaxial liquefaction test result. Hopefully such evaluation could be done without any information other than liquefaction test results. This situation is particularly desirable when the system compliance effect has become known to be significant after the test. It is the purpose of this paper to present an extremely simple procedure for the system compliance correction.

EFFECTS OF SYSTEM COMPLIANCE ON PORE PRESSURE RESPONSE

In order to derive a relationship between system compliance and pore pressure response in undrained cyclic triaxial tests, the study made by Tokimatsu and Nakamura (1986) is re-examined. They performed liquefaction tests with a conventional cyclic triaxial test apparatus coupled to a computer controlled system which can compensate for unwanted volume changes due to system compliance. Thus either with or without activating the system, liquefaction tests with and without system compliance were possible under otherwise the same conditions.

Fig. 1 compares liquefaction resistance curves with and without system compliance for Kinugawa sand with a mean grain size, $D_{50}$, of 1 mm and a relative density, $D_r$, of 65 %. The test specimen is approximately 150 mm high and 75 mm in diameter. Its physical properties are summarized in Table 1. The figure shows that the liquefaction resistance with compliance becomes much higher than would be expected with zero compliance.

Fig. 2 compares typical time histories of pore pressure and strain for the two specimens, which are tested at the same cyclic stress ratio of 0.167. Despite the same density and stress

![Fig. 1. Effects of system compliance on liquefaction resistance for sand with $D_{50}=1.0$ mm (after Tokimatsu and Nakamura, 1986)]
Table 1. Physical properties of soils tested

<table>
<thead>
<tr>
<th>Soil</th>
<th>Specific gravity of Solids $G_s$</th>
<th>Mean grain size $D_{50}$ (mm)</th>
<th>Effective grain size $D_{10}$ (mm)</th>
<th>Coefficient of uniformity $U_c$</th>
<th>Maximum dry density $\rho_{max}$ (g/cm$^3$)</th>
<th>Minimum dry density $\rho_{min}$ (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyoura sand</td>
<td>2.64</td>
<td>0.17</td>
<td>0.12</td>
<td>1.5</td>
<td>1.61</td>
<td>1.34</td>
</tr>
<tr>
<td>Kinugawa sand</td>
<td>2.65</td>
<td>0.5</td>
<td>0.36</td>
<td>1.5</td>
<td>1.60</td>
<td>1.32</td>
</tr>
<tr>
<td>Kinugawa sand</td>
<td>2.83</td>
<td>1.0</td>
<td>0.72</td>
<td>1.5</td>
<td>1.61</td>
<td>1.37</td>
</tr>
</tbody>
</table>

$(1 \text{g/cm}^3=1 \text{Mg/m}^3)$

![Graph of Kinugawa Sand][1]

Fig. 2. Time histories of pore pressure and strain during liquefaction tests for samples with and without system compliance (after Tokimatsu and Nakamura, 1986) $(1 \text{kgf/cm}^2=98 \text{kPa})$

ratio, the rate of pore pressure increase of the specimen with compliance is significantly lower than the other. As a result, the number of cycles to cause liquefaction for the specimen with compliance is about one order of magnitude greater than that without it.

The stress paths before the initial liquefaction for the two specimens are compared in Fig. 3 in which the mean effective stress, $\sigma$, and the deviator stress, $q$, are defined by:

\[ \sigma = (\sigma_a + 2\sigma_r)/3 - u \quad (1) \]

\[ q = \sigma_a - \sigma_r \quad (2) \]

in which $\sigma_a$=axial stress, $\sigma_r$=lateral stress, and $u$=pore water pressure.

The stress path for the specimen with system compliance shown in Fig. 3 (a) gradually shifts leftward, with a marked slant in one side. In contrast, the stress path without compliance shown in Fig. 3 (b) shifts faster to the left and does not seem to slant in a definite direction. A similar trend can be seen in the study by Pradhan et al. (1989).

![Graph of Stress Paths][2]

Fig. 3. Stress paths during liquefaction tests for specimens with and without system compliance $(1 \text{kgf/cm}^2=98 \text{kPa})$
Not only the rate of shift but also the degree of slant of the stress path is considered to reflect the system compliance of the test specimen, for which reason may be explained as follows.

The pore pressure change in Eq. (1) can be written by

$$\Delta u = B \sigma_v + \bar{A} (\Delta \sigma_a - \Delta \sigma) \tag{3}$$

in which $\bar{A}$ and $B$ are the pore pressure coefficients defined by Skempton (1954). In triaxial shearing with constant lateral stress ($\Delta \sigma_r = 0$), the changes in pore pressure and mean effective confining pressure can be reduced to:

$$\Delta u = \bar{A} \sigma_a \tag{4}$$

$$\Delta p = \sigma_a / 3 - \Delta u = (1/3 - \bar{A}) \sigma_a \tag{5}$$

The pore pressure change in the above equations consists of two components: one is the accumulation of pore pressure due to plastic soil deformation under cyclic shearing, and the other is the elastic pore pressure change due to a change in the axial stress. The former corresponds to the rate of shift to the left, and the latter to the degree of slant of the stress path.

Fig. 4 shows schematic diagrams showing the effect of the latter component on the stress paths starting from a condition where $\sigma_a = \sigma_r$. For an ideal case in which an isotropic and elastic soil specimen without compliance is saturated with an incompressible pore fluid ($\bar{A} = 1/3$ and $B = 1$), Eq. (5) yields $\Delta p = 0$, which means that the mean effective stress does not change and thus the stress path is perpendicular to the abscissa as shown in Fig. 4 (a).

For an isotropic and elastic specimen with system compliance, the pore pressure change is less than that specified by Eq. (4) as a result of an unwanted volume change in the system. Consequently, the mean effective stress with compliance increases during compression and decreases during extension, resulting in the slant of the stress path as shown in Fig. 4 (b). This suggests a possibility that the system compliance of a test specimen can be determined based on its pore pressure response in an undrained cyclic triaxial test.

**EVALUATION OF SYSTEM COMPLIANCE**

**System Compliance in Non-Destructive Test**

In undrained triaxial tests, the total volume change of the closed system consisting of a saturated soil specimen with surrounding membrane, and a pore pressure measuring system and tubing, must be zero (e.g., Lade and Hernandez, 1977). This leads to the following equation assuming incompressibility of soil particles:

$$\varepsilon_v + \frac{\Delta \sigma_r - \Delta u}{K_G} - \frac{n \Delta u}{K_w} - \frac{\Delta u}{K_p} = 0 \tag{6}$$

in which $n =$ porosity, $\varepsilon_v =$ volumetric strain of the soil, $K_G =$ pressure required for unit volume increase of the closed system due to flexibility of membrane and tubing within the triaxial cell which are subjected to lateral stress, $K_P =$ pressure required for unit volume increase of the closed system due to flexibility of the remaining part of the pore pressure line including pore pressure transducer that is not subjected to lateral stress, and $K_w =$ bulk modulus of water.

Assuming an isotropic and elastic soil mass saturated with water in undrained triaxial shear with constant lateral stress ($\Delta \sigma_r = 0$), the axial and lateral volumetric strains of the soil, $\varepsilon_a$ and $\varepsilon_r$, are given by:

$$\varepsilon_a = -\frac{1}{E} [(\Delta \sigma_a - \Delta u) + 2 \nu \Delta u] \tag{7}$$

$$\varepsilon_r = -\frac{1}{E} [-\Delta u - \nu (\Delta \sigma_a - 2 \Delta u)] \tag{8}$$

in which $E =$ Young's modulus, and $\nu =$ Poisson's...
ratio. Assuming $K_s$ is the bulk modulus of the soil and noting $K_s = E/3(1-2\nu)$, the volume change of the soil is given by:

$$\varepsilon_v = \varepsilon_a + 2 \varepsilon_r = \frac{\Delta \sigma_a - 3 \Delta u}{3K_s}$$

(9)

From Eqs. (4), (6), and (9) and $\Delta \sigma_r = 0$, one obtains

$$\bar{A} = \frac{\Delta u}{\Delta \sigma_a} = \frac{1/3}{1+K_s/K_C+nK_s/K_W+nK_s/K_P}$$

(10)

Similarly, for undrained compression in which all-round pressures are increased by $\Delta \sigma_r$, the volumetric strain of the soil is defined by:

$$\varepsilon_v = \frac{\Delta \sigma_r - \Delta u}{K_s}$$

(11)

Substitution of Eq. (11) into Eq. (6) and noting $\Delta \sigma_a = \Delta \sigma_r$ yield

$$B = \frac{\Delta u}{\Delta \sigma_r} = \frac{1+K_s/K_C}{1+K_s/K_C+nK_s/K_W+nK_s/K_P}$$

(12)

$\bar{A}$ and $B$ values given above are identical to those proposed by Lade and Hernandez (1977) except minor difference in the definition of modulus. Combining Eqs. (10) and (12) results in

$$\bar{A} = \frac{B}{3(1+C_R)}$$

(13)

in which

$$C_R = K_s/K_C$$

(14)

$C_R$ is equal to the system compliance ratio proposed by Martin et al. (1978), assuming that the compliance of the pore pressure line which is not subjected to lateral stress is negligibly small (i.e., $K_s \ll K_P$). This assumption appears acceptable for up-to-date triaxial test apparatus with rigid pore pressure transducer.

Substitution of Eq. (13) into Eq. (5) leads to

$$\frac{\Delta \sigma_a}{\Delta \sigma} = \frac{1}{3} \left(1 - \frac{B}{1+C_R}\right)$$

(15)

The slant of stress path in $p-q$ plane, $\Delta \sigma'/\Delta q$, is then given by

$$C_R = \frac{B}{3\bar{A} - 1}$$

(17)

Thus once knowing $\Delta \sigma'/\Delta q$ of the stress path in a non-destructive undrained triaxial test, one can readily determine the system compliance provided that the assumptions used to derive the equation are satisfied. Fig. 5 shows the relationship between $C_R$ and $\Delta \sigma'/\Delta q$ for $B=0.95$ and $1$.

Rearranging terms, Eq. (13) becomes

$$C_R = B/3\bar{A} - 1$$

(18)

This means that the system compliance ratio

$$C_R = \frac{B}{3\bar{A} - 1}$$

Fig. 5. Relationship between system compliance ratio and $\Delta p/\Delta q$

$$\frac{\Delta \sigma_a}{\Delta \sigma} = \frac{1}{3} \left(1 - \frac{B}{1+C_R}\right)$$

(16)

Rearranging the terms results in

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Rearranging terms, Eq. (13) becomes

$$C_R = B/3\bar{A} - 1$$

(18)

This means that the system compliance ratio

$$C_R = \frac{B}{3\bar{A} - 1}$$

(17)
can be expressed in terms of the pore pressure coefficients $\bar{A}$ and $B$ proposed by Skempton (1954). A similar equation but in more complicated form was presented by Tanaka and Suzuki (1988). $\bar{A}$-value equal to one third at $B=1$ yields $C_E=0$, indicating the absence of system compliance, whereas $\bar{A}$-values less than one third suggest the possible existence of system compliance as shown in Fig. 6.

**System Compliance in Liquefaction Test**

The test conditions involved in a conventional liquefaction test using a triaxial test apparatus are similar to those of the non-destructive test described above in a sense that the axial load is applied to the specimen at a constant lateral pressure. The only difference between the two test conditions is the strain level induced in the test. In this respect, Eqs. (17) and (18) cannot unconditionally be applied to liquefaction tests.

It is conceivable, however, that the soil behaves relatively elastic and isotropic in the middle stage of a liquefaction test for the following reasons: (1) both the induced strain and the increment of pore pressure per cycle at this stage are relatively small such as that shown in Fig. 2, (2) the stress path is far from the failure lines as shown in Fig. 3, and (3) a liquefaction test is a type of unloading test where the soil behavior is more isotropic and elastic than in virgin loading (Vaid and Negussey, 1984). Thus when the above requirements are satisfied, the system compliance ratio can be evaluated from the pore pressure response in an undrained cyclic triaxial test using Eq. (17) or (18).

In using Eq. (18), $\bar{A}$-value may be defined by the ratio between the fluctuation of pore pressure in a cycle, $\Delta u_d$, and the double amplitude of cyclic shear stress, $2\sigma_d$. These values can readily be read off from the time histories of shear stress and pore pressure such as those shown in Fig. 2.

### Table 2. Characteristics of test specimens

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Soil</th>
<th>$D_{90}$ (mm)</th>
<th>$D_r$ (%)</th>
<th>$C_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Toyoura sand</td>
<td>0.17</td>
<td>65</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>Kinugawa sand</td>
<td>0.5</td>
<td>65</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>Kinugawa sand</td>
<td>1.0</td>
<td>65</td>
<td>1.2</td>
</tr>
</tbody>
</table>

is studied using the previous study by Tokimatsu and Nakamura (1986) in which the system compliance of three specimens with different grain sizes were determined by measurements of volume change characteristics of soil in unloading (Vaid and Negussey, 1984). The soils tested included Toyoura Sand and size fractions of Kinugawa sand. The physical properties of the soils and the compliance ratios of the specimens are listed in Tables 1 and 2. All materials are poorly graded soils with the uniformity coefficient of about 1.5, and their mean grain sizes range from 0.17 to 1.0 mm.

As mentioned previously concerning Fig. 3, the stress path with system compliance slants in one direction. The value of $\Delta p/\Delta q$ for the specimen#3 is about 0.19, which corresponds to $C_E=1.3$ from Eq. (17) or Fig. 5. The value is in fairly good agreement with the measured value of 1.2 as indicated in Table 2.

The $\bar{A}$-values for the specimens with system compliance are plotted in Fig. 7 against their mean grain size. There is a well-defined trend in which $\bar{A}$-value decreases, i.e., the system compliance ratio increases, as the mean grain size increases. The trend is consistent.

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with the previous finding that the membrane penetration effects become more pronounced with increasing mean grain size.

The compliance ratios computed from Eq. (18) for the specimens with compliance are plotted in circles in Fig. 8 against those measured by Tokimatsu and Nakamura (1987). The computed values are generally in good agreement with the measured values, although the former are slightly higher than the latter. The computed system compliance ratios of the specimens tested with the compensation system proposed by Tokimatsu and Nakamura (1986) are also shown in Fig. 8 using square symbols, which are close to zero. This indicates that their compensation system did work well and that the isotropic and elastic soil properties assumed to derive Eqs. (17) and (18) are acceptable.

The proposed equations can therefore offer an extremely simple means to assess the system compliance ratio of a test specimen. Particularly attractive is the fact that the evaluation does not require any special tests. In fact, the liquefaction test itself gives the system compliance ratio. Although the proposed method may lead to a less accurate evaluation than other procedures, its simplicity should prove advantageous in many cases.

**CORRECTION OF LIQUEFACTION RESISTANCE CURVE**

Fig. 9 schematically compares liquefaction resistance curves and time histories of pore pressure for two specimens with and without system compliance. Tokimatsu and Nakamura (1987) have shown that the presence of system compliance increases the number of cycles to cause liquefaction from \( N_0 \) to \( N_c \) for the same stress ratio (Fig. 9 (a)), and that the cycle ratio defined by \( C_N = N_c / N_0 \) is independent of applied stress ratio (Fig. 9 (b)) and has a unique correlation with system compliance ratio as shown in Fig. 10. As a result, shifting the resistance curve horizontally leftward by a distance \( \log C_N \) results in the curve with zero compliance (Fig. 9 (a)).

Based on the above discussion, the error caused by system compliance in the liquefaction
to the cycle ratio of 10 for system compliance correction. The corrected resistance curve is shown in the figure for comparison. The corrected curve is in good agreement with that without compliance, showing the validity of the proposed method.

Different geometry and different volume change characteristics of a test specimen would result in different system compliance (e.g., Toki et al., 1988), and thereby yielding different errors in liquefaction characteristics. It is therefore required to assess and if needed to correct for the system compliance in each liquefaction test result. Thus it is a significant merit of the proposed procedure that the compliance correction can be done without any knowledge other than the liquefaction test result.

CONCLUSIONS

The effects of system compliance on pore pressure response during undrained cyclic triaxial tests are examined based on a review of previous studies. It is shown that the presence of system compliance decreases pore pressure fluctuation caused by cyclic changes in the axial stress, making the stress path slanted, and that the system compliance ratio of a specimen can be expressed in terms of the pore pressure coefficients $\tilde{A}$ and $B$. $\tilde{A}$-value can be determined by a ratio between the amplitude of pore pressure fluctuation and the amplitude of the cyclic axial stress during the test. The system compliance ratios evaluated from the proposed equations are in good agreement with those measured by the other method. A simplified procedure is then presented for correcting for the system compliance effects on triaxial liquefaction test results. The proposed method requires neither special test conditions nor additional tests, and can therefore conveniently be used in many cases.

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REFERENCES


