AN ELASTO-VISCOPLASTIC MODEL WITH COMBINED HARDENING OF ANISOTROPICALLY CONSOLIDATED COHESIVE SOILS

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ABSTRACT

A model introduced in the present paper is capable of adequately describing the time-dependent behaviour of anisotropically consolidated cohesive soils. The salient features of the proposed model are summarized as follows: i) Generalized forms of the Cambridge models are given to both yield function and plastic potential. ii) A combination of isotropic and kinematic hardening is used. iii) A non-associative flow rule is proposed. iv) The isotropic hardening is connected to the time history as well as to the viscoplastic strain history. v) The translation of yield surface is specified by extending Ziegler's rule of kinematic hardening. vi) The constitutive model has versatility and flexibility to describe expansion, translation, and rotation of yield surface in the stress space.

Several strain controlled undrained triaxial tests of anisotropically consolidated cohesive soils are simulated and the good agreement is observed between simulation and experimental data.

Key words: anisotropy, time-dependency, dilatancy, cohesive soil, consolidated undrained shear, constitutive equation, stress-strain curve, pore pressure-strain curve, effective stress path (IGC: D6/D7)

INTRODUCTION

Aspects of time-dependent anisotropic behaviour of soils are observed in many fields. Experimental investigations on a cohesive soil under anisotropically consolidated stresses have shown that the stress-strain relationships are influenced significantly by the initial degree of anisotropic consolidation. The time history exerts the great influence on the mechanical behaviour of materials, such as the creep phenomenon and the increase of the yield stress consorted with the increase of the loading rate.

Various constitutive models of cohesive soils have been developed to incorporate the influence of anisotropic initial condition on deformation and strength. Sekiguchi and Ohta (1977) proposed an anisotropical constitutive model by modifying the Cam clay theory for anisotropically consolidated clays. Pietruszczak and Mroz (1983) simulated the anisotropic behaviour of K_0 -consolidated clays by using multisurface model based on combined isotropic-kinematic hardening. Using an anisotropic, tensor to represent the rotation of yield surface, Kavvas (1983) proposed a non-associative stress-

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Manuscript was received for review on April 24, 1989.

Written discussions on this paper should be submitted before January 1, 1991, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

strain relationship of clays. The constitutive models to account for the initial anisotropy and subsequent evolution of induced anisotropy of clay are presented by Banerjee et al. (1984) and Anandarajah and Dafalias (1986) on the basis of the boundary surface plasticity. For anisotropically consolidated soils, the initial yield function and plastic potential in the stress space undergo a rotation as a result of previous anisotropical stress history. It may be considered that the subsequent yield surface expands, translates, and rotates in the stress space when general loadings are applied to a soil sample. Anisotropic hardening models are proposed by Hirai (1987a~e) in order to describe the induced anisotropy of isotropically consolidated sands. The combined hardening model (1988 a, b) can be easily applied to the simulation of anisotropically consolidated clays by taking into account a rotation of the yield surface and the plastic potential. This is because the combined hardening models proposed, which are based on the combination of isotropic and kinematic hardening, possess versatility and flexibility in describing expansion, translation, and rotation of yield surface in the stress space.

Attempting to unify the theory of linear viscoelasticity and the invicid incremental theory of plasticity, Naghdi and Murch (1963) showed that the plastic strain rate is dependent not only on the path history but also the time history of stress. For the time-dependent behaviour of normally consolidated clay, Adachi and Okano (1974) extended Roscoe's critical theory to explain the rate sensitive property of clays by using Perzyna's (1963 a, b, 1966) theory of elasto-viscoplasticity. Adachi and Oka (1982) generalized the constitutive models proposed by Adachi and Okano to describe not only time dependent behaviour of creep and strain rate effect, but also the phenomenon of secondary consolidation. Sekiguchi (1977) showed a constitutive theory of normally consolidated clay based on the newly developed elastoviscoplastic theory.

Tavenas and Leroueil (1979) and Graham, Grooks and Lau (1988) investigated empirically the yield locus of K_0 -consolidated clays, show-

ing that the typical shape of yield surface of anisotropically consolidated clays is more or less ellipsoidal and is centered on K_0 -line in the stress space. To study anisotropy of the undrained shear strength for cohesive soils of intermediate grading between sand and clay, Nakase and Kamei (1983, 1988), Nakase, Kamei and Tanaka (1984) and Nakase, Kamei and Kusakabe (1988) performed the undrained triaxial compression and extension tests of some soils classified by plasticity index PI. Nakase and Kamei (1986) investigated the influence of the strain rate upon various soils subjected to K_0 -consolidation empirically.

The objective of this paper is to present a constitutive model that is capable of describing the time-dependent behaviour of cohesive soil subjected to shear deformation subsequent to anisotropically consolidated stresses. The initial yield surface rotates from the isotropic stress line due to the previous anisotropic consolidation and the subsequent yield surface expands and translates as the shear stresses are applied to a sample. The soil behaviour is assumed to be simulated by a combination of isotropic and kinematic hardening. The expansion of yield surface is prescribed by conventional isotropic hardening and the translation of yield surface is specified by the evolution of kinematic hardening. The influence of time on mechanical behaviour of clay was not taken into account in the previous papers (Hirai, 1988 a, b). Although time effect may be incorporated into not only isotropic hardening but also kinematic hardening, the influence of time on kinematic hardening is left out of consideration in the present paper. The isotropic hardening is connected to the time history as well as the viscoplastic strain history. The model simulations are compared with some experimental data obtained by Nakase and Kamei (1986).

FAILURE FUNCTION

The general failure surface of soils has been proposed in a variety of stress forms. It is indicated from available experimental data that the failure surface takes a conical shape with

Fig. 1. Failure surface in the principal stress space

smoothly curved meridians in the principal stress space and roundly cornered hexagons of the Mohr-Coulomb type on the octahedral plane. The failure surface employed in the present paper is of a conical shape proposed by Willam and Warnke (1975). The failure surface is illustrated in Fig. 1 and is given by the following form:

$$F = J_2^{1/2} + \omega R(\theta) I_1 = 0 \quad (1)$$

where ω is a failure material constant and I_1 and J_2 are the first invariant of stress σ_{ij} and the second invariant of deviatoric stress s_{ij} , respectively defined as follows:

$$I_1 = \sigma_{kk} \quad (2)$$

$$J_2 = s_{ij}s_{ij}/2 \quad (3)$$

$$s_{ij} = \sigma_{ij} - I_1/3 \delta_{ij} \quad (4)$$

The variable θ denotes the angle similar to the Lode angle on the octahedral plane, defined by the following equation:

$$\cos(3\theta) = -3 \cdot 3^{1/2} J_3 / (2 J_2^{3/2}) \quad (5)$$

where J_3 is the third invariant of deviatoric stress:

$$J_3 = s_{ij}s_{jk}s_{ki}/3 \quad (6)$$

The function $R(\theta)$ determines the shape of the cross section of the failure surface on the octahedral plane and takes the form:

$$R(\theta) = U(\theta)/V(\theta) \quad (7)$$

where $U(\theta) = 2(1 - R_F^2) \cos \theta + (2R_F - 1)$

$$\times \{4(1 - R_F^2) \cos^2 \theta + 5R_F^2 - 4R_F\}^{1/2} \quad (8)$$

$$V(\theta) = 4(1 - R_F^2) \cos^2 \theta + (1 - 2R_F)^2 \quad (9)$$

in which R_F is a material constant related to the distance from the origin to failure surface on the octahedral plane and it is found that $R(0) = R_F$ and $R(\pi/3) = 1$.

YIELD SURFACE AND PLASTIC POTENTIAL

An important feature for K_0 -consolidation of soils is induced anisotropy due to plastic deformation. The deformation-induced plastic anisotropy may be described by the combination

Fig. 2. Yield surface in the principal stress space

of isotropic and kinematic hardening. The yield surface in general takes the following form

$$f(\sigma_{ij}, \alpha_{ij}, \beta_{ij}, k, t) = 0 \quad (10)$$

where α_{ij} denotes a kinematic hardening function relating to the translation of yield surface, β_{ij} indicates an anisotropic tensor to represent the rotation of yield surface, k is an isotropic hardening function, and t denotes time.

The specific shape of the yield surface used in the present paper is of a generalized form of the Modified Cam clay model (Roscoe and Burland, 1968). The yield surface which translates with expansion is illustrated in Fig. 2 and is given by the following equation:

$$f(t) = \bar{J}_2/r(\bar{\theta})^2 + m\bar{I}_1^2 + k\bar{I}_1 = 0 \quad (11)$$

where m is a material constant, k is an isotropic hardening function defined as $k = k_1 + k_2$, where k_1 depends on the viscoplastic work and k_2 designates the effect of time upon the viscoplastic deformation, as will be shown in later. The invariants are

$$\bar{I} = \bar{\sigma}_{kk} \quad (12)$$

$$\bar{J}_2 = \bar{s}_{ij}\bar{s}_{ij}/2 \quad (13)$$

$$\bar{J}_3 = \bar{s}_{ij}\bar{s}_{jk}\bar{s}_{ki}/3 \quad (14)$$

$$\cos(3\bar{\theta}) = -3 \cdot 3^{1/2} \bar{J}_3 / (2\bar{J}_2^{3/2}) \quad (15)$$

where

$$\bar{\sigma}_{ij} = \sigma_{ij} - \alpha_{ij} - \beta_{ij}I_1/3 \quad (16)$$

$$\bar{s}_{ij} = s_{ij} - \alpha_{ij}' - \beta_{ij}'I_1/3 \quad (17)$$

$$\alpha_{ij}' = \alpha_{ij} - \alpha_{kk}\delta_{ij}/3 \quad (18)$$

$$\beta_{ij}' = \beta_{ij} - \beta_{kk}\delta_{ij}/3 \quad (19)$$

The variable $r(\bar{\theta})$ in Eq. (11) is given by replacing θ and R_F in Eqs. (7) to (9) with $\bar{\theta}$ and r_y respectively. Since the tensor α_{ij} represents translation of yield surface, the yield surface given by Eq. (11) can expand with isotropic hardening and simultaneously translate with kinematic hardening in stress space, as shown in Fig. 3.

The cohesive soils exhibit a characteristic sensitive to the strain-rate of applied loadings. Fig. 4 schematically shows the yield surfaces and effective stress paths regarding a very fast, medium and a very slow strain histories which are called accelerated, general and retarded histories respectively (Gurtin and Herrera, 1965). In Fig. 4, a soil sample subjected to

Fig. 3. Translation and expansion of yield surface and plastic potential in p' - q plane

Fig. 4. Schematic relationships of the effective stress path for accelerated, general and retarded histories

K_0 -consolidation reaches the stress point A. When the triaxial compression test proceeds with the accelerated history, the corresponding yield surface represents the form approximated by $f(0)$, the initial yielding takes place at the

stress point B, and the effective stress traces a path from B to C. In the case of a general history, the yield surface shrinks from $f(0)$ toward $f(t)$ with the passage of time, the initial yielding occurs on the stress point D at certain time t and the effective stress path reaches the point E. For the retarded history, the initial yield surface can be considered to be represented by a limiting surface $f(\infty)$ and the effective stress path exhibits a curve from the point A to F. In two limiting strain histories, the behaviour of a material leads to a property independent of time. In other words, the response of elasto-viscoplastic materials approaches that of elasto-plastic materials for accelerated and retarded histories. The discussion concerning accelerated and retarded histories was made in detail by Gurtin and Herrera (1965) and Hirai, Satake and Yanagisawa (1983).

The direction of incremental viscoplastic strain is defined by specifying the shape of the plastic potential surface. As it has been suggested that plastic potential surface is usually different from yield surface, the non-associative flow rule is essentially to be employed in the incremental viscoplasticity theory. The plastic potential used in the present paper is a generalized form of the Cam clay model (Schofield and Wroth, 1968) and is given by the following form:

$$g = \hat{J}_2^{1/2} / (R(\hat{\theta}) \hat{I}_1) - \eta \ln(\hat{I}_1/h) = 0 \quad (20)$$

where η is a material constant, h is a hardening function unnecessary to define explicitly, and the invariants are

$$\hat{I}_1 = \hat{\sigma}_{kk} \quad (21)$$

$$\hat{J}_2 = \hat{s}_{ij} \hat{s}_{ij} / 2 \quad (22)$$

$$\hat{J}_3 = \hat{s}_{ij} \hat{s}_{jk} \hat{s}_{ki} / 3 \quad (23)$$

$$\cos 3\hat{\theta} = -3 \cdot 3^{1/2} \hat{J}_3 / (2 \hat{J}_2^{3/2}) \quad (24)$$

where

$$\hat{\sigma}_{ij} = \sigma_{ij} - \beta_{ij} \hat{I}_1 / 3 \quad (25)$$

$$\hat{s}_{ij} = s_{ij} - \beta_{ij}' \hat{I}_1 / 3 \quad (26)$$

If putting that $R(\hat{\theta})=1$ and $\beta_{ij}=0$, Eq. (20) is reduced to the yield function of the Cam clay model. The variable $R(\hat{\theta})$ in Eq. (20) is given by replacing θ and R_F in Eqs. (7) to (9) with $\hat{\theta}$ and R_P respectively. The plastic

potential surface g given by Eq. (20), which is illustrated in Fig. 3, is connected with the yield function of Eq. (11) to set up a non-associative stress-strain relationship. The phase transformation line (Ishihara, Tatsuoka and Yasuda, 1975) shown in Fig. 3 can be specified as a locus where the viscoplastic volumetric strain derived from a plastic potential function changes from contraction to expansion in the stress space.

HARDENING FUNCTIONS

It is generally considered that the hardening process is composed of isotropic and kinematic hardening due to viscoplastic deformation. It is necessary to provide the evolutionary rules of each hardening. First of all, let us focus our attention on the isotropic hardening. For the viscoplastic behaviour, it may be considered that both the viscoplastic strain rate and time contribute to the evolution of the isotropic hardening. If the isotropic hardening may depend not only on the viscoplastic work related with the change in volume but also on the viscoplastic work related with the change in shape, the evolution of isotropic hardening is expressed in the following forms (Hirai, 1988 a, b):

$$\begin{aligned} \dot{k}_1 = & \phi \{1 + J_2^{1/2} / (\omega R(\theta) I_1)\} (\sigma_{ii} \dot{e}_{jj}^{(vp)} / \\ & 3 + \hat{s}_{ij} \dot{e}_{ij}'^{(vp)}) \end{aligned} \quad (27)$$

where $\dot{k}_1 = \partial k_1 / \partial t$ means the derivative of k with respect to t , ϕ denotes a material constant and $\dot{e}_{ij}^{(vp)}$ and $\dot{e}_{ij}'^{(vp)}$ are the viscoplastic strain rate and the deviatoric viscoplastic strain rate respectively. The time effect upon the isotropic hardening may be taken into account in the following form:

$$\dot{k}_2 = \phi e^{-\xi t} \quad (28a)$$

or

$$k_2 = k_0 + \frac{\phi}{\xi} (1 - e^{-\xi t}) \quad (28b)$$

where ϕ and ξ are material constants and k_0 denotes the value of isotropic hardening at $t=0$. The influence of time upon viscoplastic deformation may be incorporated into the kinematic hardening as well as the isotropic hardening. The time-dependent kinematic hardening,

however, will be left out of consideration in the present paper for simplicity.

The kinematic hardening used in the present paper is a modified form of the Ziegler's law (1959). This evolution is expressed as

$$\dot{\alpha}_{ij} = D\{1 + J_2^{1/2}/\omega R(\theta)I_1\} \bar{s}_{ij} \dot{e}_{vp} \quad (29)$$

where D is a material constant and

$$\dot{e}_{vp} = (\dot{e}_{mn}'^{(vp)} \dot{e}_{mn}'^{(vp)})^{1/2} \quad (30)$$

STRESS-STRAIN RELATIONSHIPS

A general elasto-viscoplastic formulation is based on the assumption that the total strain response consists of elastic and viscoplastic strain responses. This is represented as follows:

$$\dot{e}_{ij} = \dot{e}_{ij}^{(e)} + \dot{e}_{ij}^{(vp)} \quad (31)$$

where $\dot{e}_{ij}$ and $\dot{e}_{ij}^{(e)}$ are the total strain and elastic strain rates respectively.

If the direction of viscoplastic strain rate tensor coincides with the normal of a plastic potential surface, this condition is satisfied by the following equation:

$$\dot{e}_{ij}^{(vp)} = \Lambda \partial g / \partial \sigma_{ij} \quad (32)$$

where Λ is a scalar function dependent on stress, strain and time, and g is a plastic potential.

The function Λ is determined by use of the consistency condition which is expressed as

$$\partial f / \partial \sigma_{ij} \dot{\sigma}_{ij} + \partial f / \partial \alpha_{ij} \dot{\alpha}_{ij} + \partial f / \partial k \dot{k} = 0 \quad (33)$$

Substituting Eqs. (27) through (30) and (32) into Eq. (33) and solving for Λ leads to

$$\Lambda = \frac{\frac{\sigma f}{\partial \dot{\sigma}_{mn}} \dot{\sigma}_{mn} + \frac{\partial f}{\partial k} \phi e^{-\epsilon t}}{\phi \left(1 + \frac{\sqrt{J_2}}{\omega I_1 R(\theta)} \right)} \cdot \left(\eta I_1 + b \frac{\sqrt{2} \bar{J}_2}{r(\bar{\theta})^2 R(\bar{\theta}) I_1} \sqrt{1 + \frac{W(\bar{\theta})^2}{R(\bar{\theta})^2}} \right) \quad (34)$$

where $b = D/\phi$ and $W(\bar{\theta}) = \partial R(\bar{\theta}) / \partial \bar{\theta}$. It is found that $W(\bar{\theta}) = 0$ for triaxial compression and extension tests.

The constitutive relation in the elastic range is given by the following equation:

$$\dot{e}_{ij}^{(e)} = (\dot{\sigma}_{ij} - \dot{p}' \delta_{ij}) / (2\mu p') + 0.435 C_s \dot{p}' \delta_{ij} / \{3(1 + e_0) p'\} \quad (35)$$

where μ is an elastic material constant, e_0 is the initial void ratio, C_s is the swelling index of clay, $p' = I_1/3$ and μ is given as follows:

$$\mu = \{3(1 - 2\nu)(1 + e_0)\} / \{2(1 + \nu)0.435 C_s\} \quad (36)$$

where ν is the Poisson's ratio.

SIMULATION OF UNDRAINED TRIAXIAL TEST OF ANISOTROPICALLY CONSOLIDATED COHESIVE SOILS

In order to investigate the time-dependency of strength and deformation of cohesive soils, undrained triaxial compression and extension tests have been carried out after isotropic consolidation or K_0 -consolidation. The proposed model can be applied to the simulation of the time-dependent behaviour of isotropically consolidated and K_0 -consolidated cohesive soils. In the present paper, the predicted results are compared with the test results obtained for normally consolidated cohesive soils (Nakase and Kamei, 1986). These experimental data were provided regarding Kawasaki clay-mixture series, which are classified into three groups: M-10, M-15, and K-30, according to their values of the plasticity index (PI). The notations, M and K, stand for clay-sand mixture and Kawasaki clay respectively. The preparation method of the soil samples has been reported elsewhere (Nakase and Kamei, 1983, 1988; Kamei, 1985; Nakase, Kamei and Kusakabe, 1988). The physical property and material parameters used in prediction of undrained triaxial tests are shown in Table 1 and 2. Further detailed information regarding determination of material parameters will be

Table 1. Index properties of soils studied

	K-30	M-15	M-10
G_s	2.69	2.67	2.67
$w_L(\%)$	55.3	34.8	27.6
$w_p(\%)$	25.9	20.2	16.9
PI	29.4	14.6	10.7
Sand(%)	16.1	47.4	60.4
Silt(%)	61.6	39.2	28.9
Clay(%)	22.3	13.4	9.7

Table 2. Material parameters used in prediction of undrained triaxial tests

Samples	M-10	M-15	K-30
Elastic Parameter			
e_0	0.70	0.79	1.07
ν	0.27	0.27	0.27
C_s	0.0115	0.0184	0.0483
Failure Parameter			
ω	-0.319	-0.319	-0.319
R_F	0.861	0.861	0.861
Yield Parameter			
m	0.015	0.017	0.012
r_y	1.0	1.0	1.0
β^*	0.917	0.883	0.812
P_0'/σ'_{vc}	0.904	1.07	1.50
Plastic Potential Parameter			
η	-0.308	-0.308	-0.308
R_p	0.875	0.875	0.875
Isotropic Hardening			
λ	0.0382	0.0503	0.0773
Kinematic Hardening			
D	384	333	161
Viscous Parameter			
ξ	0.591	0.448	0.197
ϕ	0.00696	0.00786	0.00461

$\beta^* = \beta_{11} - \beta_{33}$, $\beta_{22} = \beta_{33}$, $\beta_{ii} = 0$ in Triaxial Tests
 $\sigma'_{vc} = 392 \text{ kPa}$ (Vertical Effective Consolidation Pressure)

available in appendix. The stresses employed in triaxial tests are effective mean principal stress p' and principal stress difference q .

In the model simulation, it is assumed that in the undrained tests, the volumetric strain composed of elastic and viscoplastic parts vanishes. For the kinematic hardening parameter α_{ij} and the anisotropic tensor β_{ij} , α_{11} , α_{33} ($\alpha_{22} = \alpha_{33}$), β_{11} and β_{33} ($\beta_{22} = \beta_{33}$) are employed in triaxial compression and extension tests. In order to investigate the influence of the loading rate upon strength and deformation, the strain controlled tests were performed and the prescribed strain rates per minute are three kinds of $7 \times 10^{-1}\%$, $7 \times 10^{-2}\%$ and $7 \times 10^{-3}\%$.

Figs. 5 through 7 show the comparison between simulated and experimental results of effective stress paths, stress-strain relationships and excess pore pressure-strain relationships for the normally consolidated soil when the sample named M-10 is subjected to three kinds of strain rates after the K_0 -consolidation was complete at the stress point A.

(a) Model Simulation **(b) Experimental Results (Nakase and Kamei, 1986)****Fig. 5. Comparison of the computed and observed effective stress paths for M-10**

(a) Model Simulation

(b) Experimental Results (Nakase and Kamei, 1986)

Fig. 6. Comparison of the computed and observed stress-strain behaviour for M-10

(a) Model Simulation

(b) Experimental Results (Nakase and Kamei, 1986)

Fig. 7. Comparison of the computed and observed excess pore pressure-strain behaviour for M-10

First of all, an attention is focused on triaxial compression tests. For the strain rate $7 \times 10^{-1} \%$ /min., the simulation indicates that the initial yielding takes place at the passage of time 0.18 min. on the stress point B. When a sample is subject to the strain rates $7 \times 10^{-2} \%$ /min. and $7 \times 10^{-3} \%$ /min., the initial yielding is simulated to be observed on the stress points C and D at times 1.29 min. and 6.43 min. respectively. It is found that as the strain rate increases, the initial yield stress increases.

As the viscoplastic deformation proceeds, the softening appears initially and the hardening resumes after the effective stress path has arrived at the phase transformation line.

In triaxial extension tests, the model simulation indicates that the initial yielding occurs on the stress points E, F and G at times 0.266 min., 2.14 min. and 15.0 min. respectively, according as the strain rates are $7 \times 10^{-1} \%$ /min., $7 \times 10^{-2} \%$ /min. and $7 \times 10^{-3} \%$ /min. The effective stress path is straight line vertical to p'

axis because the elastic state is kept until the stress point reaches the initial yield surface. As the viscoplastic deformation continues with the decrease of the principal stress difference q , the effective mean principal stress p' decreases with the approach toward the phase transformation line. After the stress point reaches the phase transformation line, the effective mean principal stress p' increases and the axial strain tends to increase rapidly with the increase of the principal stress difference q . For the excess pore pressure-strain behaviour shown in Fig. 7, the model simulation designates a trend that the excess pore pressure increases with the increase of the axial strain in triaxial compression tests. On the other hand, the simulated results for triaxial extension tests indicate that after the excess pore pressure decreases initially, the excess pore pressure changes to increase, culminates on the phase transformation line and decreases gradually with the increase of axial strain.

Although details of the simulated results are different from the experimental results, the model simulation is capable of predicting qualitatively the salient features of time-dependent behaviour for the anisotropically consolidated soils.

Figs. 8 through 10 and Figs. 11 through 13 show the comparison between the simulation and experimental data of effective stress paths, stress-strain relationships and excess pore pressure-strain relationships for the normally consolidated soils named M-15 and K-30 respectively when three kinds of the strain rate are applied to the sample subject to K_0 -consolidation. Since the initial yield surfaces of M-15 and K-30 take a distorted elliptical form larger than M-10, the initial yield stresses for the soils of M-15 and K-30 are larger than M-10.

Although there are many qualitative similarities regarding time-dependent behaviour among the three soils M-10, M-15 and K-30, the quantitative differences are found in ex-

(a) Model Simulation

(b) Experimental Results (Nakase and Kamei, 1986)

Fig. 8. Comparison of the computed and observed effective stress paths for M-15

(a) Model Simulation

(b) Experimental Results (Nakase and Kamei, 1986)

Fig. 9. Comparison of the computed and observed stress-strain behaviour for M-15

(a) Model Simulation

(b) Experimental Results (Nakase and Kamei, 1986)

Fig. 10. Comparison of the computed and observed excess pore pressure-strain behaviour for M-15

perimental results. It is found that as the plasticity index PI increases, the softening after the initial yielding in triaxial compression tests has a trend not to plummet rapidly with the decrease of the principal stress difference q and the increase of the axial strain. In the range of the small axial strain, the slope of stress-strain relationships tends to become steep

in triaxial extension tests as the plasticity index PI decreases. For the three kinds of soils, although the excess pore pressure in the triaxial compression tests tends to approach a similar maximum value, the excess pore pressure in the triaxial extension tests has a tendency to take the different maximum value. It is seen that the model simulation gives the good

Fig. 11. Comparison of the computed and observed effective stress paths for K-30

Fig. 12. Comparison of the computed and observed stress-strain behaviour for K-30

Fig. 13. Comparison of the computed and observed excess pore pressure-strain behaviour for K-30

correspondence and the proposed model is able to reproduce the anisotropic property and time-dependence of some soils.

CONCLUDING REMARKS

In the model presented in this paper, an attempt is made to describe a time-dependent behaviour and an anisotropy of soils with a combination of isotropic and kinematic hardening theory. For anisotropically consolidated soils, the initial yield surface is presented in terms of a distorted and rotated ellipsoid in stress space. The subsequent yield surface expands and translates due to isotropic and kinematic hardening respectively. The isotropic hardening is related to the time history as well as to the viscoplastic strain history. The yield function can be expressed by means of three invariants which are based on a combination of the stress tensor and anisotropic tensors. The anisotropic tensors are connected to the translation and rotation of yield surface. The combined hardening model provides the capability to predict the time-dependent stress-strain behaviour of anisotropically normally consolidated clays where the strain rate exerts great influence on deformation.

Comparisons are made between experimental data and model simulations concerning the triaxial undrained compression and extension tests controlled with the strain rate. Simulated results provide the comparable agreement with some experimental data of cohesive soils.

Particularly, the softening behaviour appearing in anisotropically normally consolidated soils is predicted properly by the proposed model. Although the model proposed in this paper includes 15 material parameters, these parameters can be easily determined from standard triaxial compression and extension tests. It has been emphasized that the proposed model has versatility and flexibility to describe the characteristics of induced anisotropic time-dependent behaviour of the various kinds of soils regarding isotropically and anisotropically normally consolidated states.

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APPENDIX

The procedure for determining material parameters in the proposed model is mentioned briefly below:

(i) The elastic parameter C_s is determined with the relation $C_s = (1 + e_0)\kappa/0.435$ from isotropic compressive loading-unloading tests of soil where e_0 is the initial void ratio and the parameter κ is defined as $\dot{\nu}^{(e)} = \kappa \dot{p}'/p'$, and where $\dot{\nu}^{(e)}$ is the elastic volumetric strain rate. Poisson's ratio ν is assumed to be a constant value for convenience.

(ii) Yield parameters m , r_y , β^* and p_0' can be determined through a yield locus obtained from results of undrained triaxial tests of isotropically and anisotropically normally

consolidated and overconsolidated clays (Figs. 2, 3 and 4).

(iii) Plastic potential parameters η and R_p can be obtained from the phase transformation line where the viscoplastic volumetric strain changes from contraction to expansion (Fig. 3).

(iv) Failure parameter ω and R_F can be obtained from the failure plots of triaxial compression and extension tests (Fig. 3).

(v) The isotropic hardening parameter ϕ can be determined with the relation $\phi = -3m/(\lambda - \kappa)$, where the parameter λ is defined as $\dot{\nu} = \lambda \dot{p}'/p'$, and where $\dot{\nu}$ is the volumetric strain rate. The parameter λ is determined from the slope of the relationship between the volumetric strain and the effective mean principal stress in isotropic compression tests consorted with a very slow strain rate. The kinematic hardening parameter D have to be determined by a trial-and-error process through experimental results of undrained triaxial compression and extension tests.

(vi) The viscous parameters ξ and ψ are shown in Eqs. (28a) and (28b). These values can be obtained from the relationship between the strain rate and the initial yield point (Fig. 4).