OSCILLATORY PORE PRESSURE AND LIQUEFACTION IN SEABED INDUCED BY OCEAN WAVES

KOUKI ZEN¹⁾ and HIROYUKI YAMAZAKI¹¹⁾

ABSTRACT

The oscillatory pore pressure excited in porous seabed by ocean waves is theoretically and experimentally examined in the context of the wave-induced liquefaction. A governing equation on the oscillatory pore pressure is presented and verified with the model tests. The comparisons of the finite difference solutions with the measurement of the oscillatory pore pressure have shown a good agreement. The governing equation is also adopted to estimate the liquefaction potential of the model seabed and its applicability is confirmed. It is shown that the wave-induced liquefaction is a kind of quick sand caused by the upward seepage force induced repeatedly in the seabed by ocean waves.

Key words: liquefaction, model test, ocean soil, pore pressure, sand, seepage, water pressure, wave propagation (IGC: E 7/E 8)

INTRODUCTION

The interaction between propagating ocean waves and seabed has received an extensive attention in the field of offshore engineering. The wave-induced liquefaction in porous seabed is one of the significant research subjects since the liquefaction, if it happens, may cause severe damage of offshore structures.

Yamamoto (1977), Madsen (1978), Finn et al. (1983) and Okusa (1985) analyzed the wave-associated stress in the porous seabed, on the basis of Biot's poro-elastic theory, by coupling the pore pressure with the stress or strain in the seabed. May and Foda (1981) and Mynett and Mei (1982) applied Biot's type

equation to the interaction problems among waves, offshore structures and seabeds. Nago (1981) derived the same governing equation as those by Yamamoto (1977) and Madsen (1978) and interpreted the liquefaction phenomena in sand deposits reproduced by laboratory experiments. Biot's type of coupled equations are more attractive than others presented by Sleath (1970), Liu et al. (1979), Moshagen and Tørum (1975) and Inoue (1975), in a sense that they give not only the oscillatory pore pressure but also the stress in the seabed.

In these previous papers, however, the wave-induced liquefaction in the porous seabed is not clearly addressed in the geotechnical sense of the word. Namely, even if the tensile

¹⁾ Chief, Soil Dynamics Laboratory, Geotechnical Engineering Division, Port and Harbour Research Institute, Ministry of Transport, Nagase 3-1-1, Yokosuka, 239.

¹¹⁾ Member, ditto.

Manuscript was received for review on July 18, 1989.

Written discussion on this paper should be submitted before July 1, 1991, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

stress in the seabed is given by the elastic analysis, it does not necessarily mean the occurrence of the liquefaction unless the excess pore pressure is clarified. Zen and Yamazaki (1990) presented an liquefaction criterion in which the dominant factors on the liquefaction are the wave-associated bottom pressure, the oscillatory pore pressure in the seabed and the vertical effective stress at calm. When the wave-associated bottom pressure and the vertical effective stress at calm are given, the only factor to be known is the oscillatory pore pressure in the seabed. Therefore, in this paper, the characteristics of the oscillatory pore pressures are investigated in detail in the context of the wave-induced liquefaction in the porous seabed.

LIQUEFACTION CRITERIA

When ocean waves propagate over the seabed, they induce the oscillatory excess pore pressures and effective stress changes in the seabed. The effective stress, σ'_v , under one dimensional plane strain conditions, is generally expressed by the following equation (Zen and Yamazaki, 1990):

$$\sigma'_v = \sigma'_{v0} + (p_b - p_m) \quad (1)$$

where, σ'_{v0} ; the vertical effective stress at calm, p_b ; the wave-associated pressure at the surface of the seabed and p_m ; the oscillatory pore pressure induced in the seabed. Then, the oscillatory excess pore pressure, u_e , is defined by:

$$u_e = -(p_b - p_m) \quad (2)$$

The liquefaction criterion of the porous seabed is derived from Eq. (1):

$$\sigma'_{v0} \leq -(p_b - p_m) \quad (3)$$

Therefore, if the σ'_{v0} , p_b and p_m are known, the liquefaction potential can be easily evaluated. As the σ'_{v0} is obtained from the submerged unit weight of deposits and the p_b is estimated approximately by the linear wave theory, the only factor to be known is the oscillatory pore pressure, p_m .

OSCILLATORY PORE PRESSURE

Governing Equation to Irregular Waves

The propagation of ocean waves exerts the differential pressures on the surface of the seabed. When the wave length is large enough compared with the thickness of permeable seabed, it may be assumed that the wave-associated pressures distribute uniformly on the surface of the seabed. Then, the oscillation of pressures and the flow of pore water are treated under one dimensional conditions.

Considering a soil element at depth, z , in the seabed and assuming the flow of pore water in the soil element is governed by the steady state form of Darcy's law, Eq. (4) is derived from the conservation of mass of pore water:

$$\frac{k}{\gamma_w} \frac{\partial^2 p_m}{\partial z^2} = -m_v \frac{\partial \sigma'_v}{\partial t} + nm_w \frac{\partial p_m}{\partial t} \quad (4)$$

where, k ; the coefficient of permeability, m_v ; the coefficient of volume compressibility, n ; the porosity, m_w ; the compressibility of pore water including air, γ_w ; the unit weight of pore water and t ; time. The k and m_v are assumed constant irrespective of time and space. As the effective vertical stress is expressed by Eq. (1) and $\partial \sigma'_{v0} / \partial t$ is equal to 0, we obtain,

$$\frac{\partial \sigma'_v}{\partial t} = \frac{\partial (p_b - p_m)}{\partial t} \quad (5)$$

which may be introduced into Eq. (4) to yield:

$$\frac{k}{\gamma_w m_v} \frac{\partial^2 p_m}{\partial z^2} = \left(1 + \frac{nm_w}{m_v}\right) \frac{\partial p_m}{\partial t} - \frac{\partial p_b}{\partial t} \quad (6)$$

Replacing,

$$C_v = \frac{k}{\gamma_w m_v} \quad (7)$$

$$\alpha = 1 + \frac{nm_w}{m_v} \quad (8)$$

Eq. (6) may be written in the form:

$$C_v \frac{\partial^2 p_m}{\partial z^2} = \alpha \frac{\partial p_m}{\partial t} - \frac{\partial p_b}{\partial t} \quad (9)$$

Eq. (9) represents the governing equation on the oscillatory pore pressure in the seabed, applicable to any complex wave conditions such as random waves in real ocean.

As the oscillatory pore pressure is equal to the wave-associated pressure at the surface of the seabed, the boundary condition is written:

$$p_m = p_b \quad \text{at } z=0 \quad (10)$$

If the porous seabed is of finite thickness, l , and the underlying soil is assumed impermeable, no water flow normal to the horizontal boundary comes rise to. Thus, we obtain:

$$\frac{\partial p_m}{\partial z} = 0 \quad \text{at } z=l \quad (11)$$

Initial condition is written as:

$$p_m = 0 \quad \text{at } t=0 \quad (12)$$

It is noted that the p_m does not mean the 'excess' pore pressure but denotes the oscillatory nature of pore pressure. The governing equation on the excess pore pressure is derived from Eq. (9) by using Eq. (2):

$$C_v \frac{\partial^2 u_e}{\partial z^2} = \alpha \frac{\partial u_e}{\partial t} + (\alpha - 1) \frac{\partial p_b}{\partial t} \quad (13)$$

When the compressibility of pore water is negligible, say the coefficient, α , is equal to 1.0, Eq. (13) becomes identical with the consolidation equation.

Governing Equation to Linear Waves

When a small amplitude wave is assumed, the wave-associated pressure on the surface of the seabed, p_b , is expressed by the linear wave theory:

$$p_b = p_0 \sin\left(\frac{2\pi t}{T}\right) \quad (14)$$

$$p_0 = \frac{\gamma_w H}{2} \frac{1}{\cosh(2\pi h/L)} \quad (15)$$

where, H ; the wave height, T ; the wave period, L ; the wave length and h ; the water depth. Eq. (9) is rewritten by replacing,

$$\bar{P} = \frac{p_m}{p_0} \quad (16)$$

$$\bar{Z} = \frac{z}{l} \quad (17)$$

$$\bar{T} = \frac{t}{T} \quad (18)$$

$$C = \frac{C_v T}{l^2} \quad (19)$$

in the form:

$$C \frac{\partial^2 \bar{P}}{\partial \bar{Z}^2} = \alpha \frac{\partial \bar{P}}{\partial \bar{T}} + 2\pi \cos(2\pi \bar{T}) \quad (20)$$

The boundary conditions are:

$$\bar{P} = \sin(2\pi \bar{T}) \quad \text{at } \bar{Z}=0 \quad (21)$$

$$\frac{\partial \bar{P}}{\partial \bar{Z}} = 0 \quad \text{at } \bar{Z}=1 \quad (22)$$

The initial condition is:

$$\bar{P} = 0 \quad \text{at } \bar{T}=0 \quad (23)$$

With the same manner, Eq. (13) is expressed in terms of the non-dimensional excess pore pressure, $\bar{U}$:

$$C \frac{\partial^2 \bar{U}}{\partial \bar{Z}^2} = \alpha \frac{\partial \bar{U}}{\partial \bar{T}} + 2\pi(\alpha - 1) \cos(2\pi \bar{T}) \quad (24)$$

where,

$$\bar{U} = \frac{u_e}{p_0} \quad (25)$$

Coefficient: C

The meanings of the coefficient, C , can be found by setting the coefficient, α , is equal to 1.0 in Eq. (24). Since Eq. (24) becomes identical with the consolidation equation, the C defined by Eq. (19) is considered to represent the drainage effect of the excess pore pressure. As the coefficient of consolidation, C_v , and the wave period, T , become larger and as the thickness of permeable layer, l , becomes smaller, the excess pore pressure will dissipate more quickly from the permeable seabed. In this sense, the non-dimensional coefficient, C , is referred to the coefficient of drainage, hereafter. The coefficient of drainage has a similar form to the time factor in which the time is replaced by the wave period.

Coefficient: α

Provided that the seabed is under undrained conditions, the left term in Eq. (9) is omitted. Then, Eq. (9) is rewritten as:

$$\frac{1}{\alpha} = \frac{\partial p_m / \partial t}{\partial p_b / \partial t} \quad (26)$$

Since the right term in Eq. (26) denotes the increase rate of the oscillatory pore pressure to the wave-associated bottom pressure, the $(1/\alpha)$ appears equivalent to the Skempton's B -value. It is, however, true only under the steady states of the pressures of p_b and p_m . Under the transient states of the pressures, the $(1/\alpha)$ is, in general, a function of time, closely related to the compressibility of pore water.

The compressibility of pore water is affected

by the amount of the dissolved air in water and the attached air to the soil particles. It is known that the effect of such air is represented by the degree of saturation, S_r and the pore pressure by means of absolute value, p_{mg} . The p_{mg} is represented by the sum of the hydrostatic pressure and the wave-associated dynamic pressure. Okusa (1985) presented the compressibility of pore water by the following equation:

$$m_w = m_{w0} S_r + \frac{1 - S_r}{p_{mg}} \quad (27)$$

where, m_{w0} ; the compressibility of water at fully saturated state. Introducing Eq. (27) into Eq. (8), we obtain:

$$\alpha = 1 + \frac{n}{m_v} \left(m_{w0} S_r + \frac{1 - S_r}{p_{mg}} \right) \quad (28)$$

The α is function of time because the p_{mg} varies according to the wave-associated bottom pressure. It is, however, shown by Inoue (1975) that the rapid variation of the dynamic component to the p_{mg} , usually met during storm waves, affects the compressibility of pore water very little.

If the degree of saturation is equal to 1.0 and the compressibility of pore water, m_{w0} , is small enough to be ignored, the α becomes equal to 1.0. Then, the wave-associated pressure on the surface of the seabed, p_b , is fully propagated into the seabed. Thus, the oscillatory pore pressure, p_m , becomes identical with the p_b . This means that no excess pore pressure is excited in the seabed. Whereas, when the S_r is less than 1.0, say the α is greater than 1.0, the p_b is not completely propagated into the seabed and the p_m will be less than p_b . Therefore, the α is considered a coefficient representing the propagation characteristics of the water pressure and is referred to the coefficient of propagation, hereafter.

Finite Difference Solution

In order to solve Eq. (20), the finite difference analysis is adopted. The differential meshes in space and time used in the computation are respectively $\Delta Z = 0.1$ and $\Delta T = 0.001$. These mesh intervals are determined from the ones that give the stable and con-

Fig. 1. Response of pore pressure

Fig. 2. Vertical effective stress change

vergent solutions. The computation is continued for more than 3 cycles of waves, until the steady state of the oscillatory pore pressures are obtained. An example of the solution is illustrated in Figs. 1 and 2. It is clearly shown in Fig. 1 that the wave-associated pressure, p_b , propagates into the seabed with damping and phase lag. The changes in the effective stresses, $(p_b - p_m)$, are presented in Fig. 2, in which the $(p_b - p_m)$ is normalized by the amplitude of the wave-associated pressure, p_0 . In Fig. 2, the vertical effective stresses oscillate continuously on both negative and positive sides even in one period of the wave loading. According to the liquefaction criterion, it is understood that the liquefact-

ion will take place where the wave-associated stresses exceed the vertical effective stresses at calm.

Influential Factors on Oscillatory Pore Pressure

Referring to Eqs. (6), (14) and (15), the influential factors on the oscillatory pore pressure, p_m , are expressed by a function, F :

$$p_m = F(H, L, T, m_v, k, n, \gamma_w, m_w, h, z, l, t, S_r, N_c) \quad (29)$$

Among these factors, the H , L , T and N_c are the factors depending on the wave characteristics. The m_v and n are related to the soil properties, and the γ_w and m_w depend on the properties of pore water. The h , l and z are the geometrical factors. The k and S_r are related to the properties both of soils and pore water. Although the number of waves, N_c , is not included in Eq. (6), it is an important factor to be considered.

When a small amplitude wave is assumed and the non-dimensional pressure ratio, $\bar{P}$, is introduced, Eq. (29) can be expressed simply by the following function, G :

$$\bar{P} = G(\bar{T}, \bar{Z}, C, \alpha, N_c) \quad (30)$$

When the $\bar{T}$ and $\bar{Z}$ are given, the influential factors on the $\bar{P}$ are represented by the C , α and N_c .

EXPERIMENTS

Model tests were performed, under one dimensional wave loading conditions, with a newly developed apparatus shown in Fig. 3. The apparatus is made up of 1) a cylinder for constituting the model seabed, which is composed of sixteen cells, 2) a pneumatic sine loading unit to apply the hydro-static and dynamic pressures, 3) measuring sensors and 4) a recording system. The air-dried Toyoura sand, the physical properties of which were

shown in Table 1, was poured into the cells in the process of the fabrication of the cylinder. The thickness of deposit is 1.90 m and the initial relative density is 50 %. The carbon dioxide gas was circulated before filling the water. The hydro-static pressure of 1.0 kgf/cm² (98 kN/m²) was applied to the deposit. Then, sinusoidal wave forms of the dynamic pressures were repeated up to 500 cycles in each loading stage. The double amplitudes of the hydro-dynamic pressures were between 0.197 kgf/cm² (19.3 kN/m²) and 0.818 kgf/cm² (80.2 kN/m²). The wave periods were between 3 s and 15 s. The applied wave loading conditions are tabulated in Table 2. The oscillatory pore pressures were measured at the depth shown by P1, P3, P5, P7, P9, P11, P13 and PL in Fig. 3. After the wave loading has been completed, the density of the deposit and the degree of saturation were measured by cutting out the deposit in each cell of the cylinder. More details of the experiments are presented elsewhere (Zen et al., 1987).

The permeability tests were performed using Toyoura sand for several kinds of the relative density. One dimensional cyclic compression tests were separately executed with a steel mould. The inner diameter and the height of

Table 1. Physical properties

Specific gravity	G_s	2.674
Maximum void ratio	e_{max}	0.988
Minimum void ratio	e_{min}	0.615
Mean grain size	D_{50} (mm)	0.181
Uniformity coefficient	U_c	1.79

Fig. 3. Test apparatus

Table 2. Test conditions

Test No.	Stage No.	Applied pressure $2\bar{p}_0$ (kgf/cm ²)	Applied wave period T (s)	Thickness l (m)	Relative density D_{r0} (%)	Remarks
1	1	0.197	7	1.90	50	—
	2	0.325	9			
	3	0.473	11			
	4	0.722	13			
	5	0.555	11			
	6	0.390	9			
	7	0.243	7			
2	1	0.507	15	1.90	50	$2\bar{p}_0 \div \text{constant}$
	2	0.509	11			
	3	0.515	7			
	4	0.497	3			
	5	0.791	15			$2\bar{p}_0 \div \text{constant}$
	6	0.793	13			
	7	0.813	11			
	8	0.818	9			
3	1	0.247	7	1.90	50	$T = \text{constant}$
	2	0.439	7			
	3	0.595	7			
	4	0.756	7			
	5	0.604	7			
	6	0.443	7			
	7	0.253	7			
4	1	0.258	7	1.90	50	$q_s = 0.0716 \text{ kgf/cm}^2$
	2	0.445	9			
	3	0.618	11			
	4	0.804	13			
	5	0.633	11			
	6	0.446	9			
	7	0.266	7			

1) the number of waves, N_c , in each stage; 500 cycles.2) q_s ; overburden pressure.3) $1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$.

the mould are 305 mm and 350 mm, respectively. The loading and unloading and reloading were repeated up to 5 cycles by applying the weight step by step. The vertical settlements were measured at each loading step. The coefficient of volume compressibility was estimated from the vertical stress and strain relationships. The coefficient of permeability and the coefficient of volume compressibility were used to calculate the coefficient of consolidation.

The coefficient of propagation was measured with a conventional cyclic triaxial test apparatus. The specimen, the diameter and the height of which were respectively 50 mm and 125 mm, was constructed in the cell. The degree of saturation of specimens was roughly

controlled in the process of the construction of specimens. The specimens were isotropically consolidated with the cell pressures between 1.5 kgf/cm^2 (147 kN/m^2) and 2.5 kgf/cm^2 (245 kN/m^2) under the back pressure of 1.0 kgf/cm^2 (98 kN/m^2). The sinusoidal forms of the cyclic cell pressures whose amplitudes were between 0.1 kgf/cm^2 (9.8 kN/m^2) and 0.3 kgf/cm^2 (29.4 kN/m^2) were repeated up to 500 cycles in maximum. The responses of the pore pressures in the specimen as well as the cell pressures were recorded. The B -value was measured before and after the cyclic loading. The exact value of the degree of saturation was measured immediately after the tests. The coefficient of propagation was evaluated by using Eqs. (26) and (28).

TEST RESULTS

Wave Characteristics affecting Oscillatory Pore Pressure

a) Wave Height, Wave Length and Water Depth

Fig. 4 shows the relationships between the amplitude of wave-associated bottom pressure, p_0 , and the pressure ratio, p_m/p_0 , at the non-dimensional depths, $z/l=0.13$ and 0.84 . It is found in Fig. 4 that the pressure ratio is

Fig. 4. Pressure ratio and wave-associated bottom pressure ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

constant irrespective of the p_0 . As the p_0 is a function of the wave height, H , the wave length, L , and the water depth, h , as shown in Eq. (15), the oscillatory pore pressure in terms of the non-dimensional pressure ratio, p_m/p_0 , is considered independent of the H , L and h .

b) Wave Period

Fig. 5 shows the pressure ratio and the wave period, T , at the non-dimensional depth, $z/l=0.45$. The ranges of the existing data by other investigators which are collected and presented by Zen and Yamazaki (1990), are also drawn by the dotted lines in Fig. 5. The

Fig. 5. Pressure ratio and wave period

pressure ratio decreases from about 1.0 to 0.2~0.3 as the wave period becomes smaller. The wave period is considered one of the dominant factors to the oscillatory pore pressure in the seabed. Assuming the linear waves, this effect can be taken into account by the coefficient of drainage defined by Eq. (19).

c) Number of Waves

The oscillatory pore pressures may vary depending on the number of waves. As this effect is not included in the governing equation shown in Eqs. (9) and (20), it was experimentally examined. Figs. 6 indicates the effect of the number of waves, N_c , on the pressure ratio, p_m/p_0 . As can be seen in Fig. 6, quite little difference of the pressure ratio at each number of waves is observed. Fig. 7 shows the variance of the pressure ratio between at arbitrary number of waves, N_c , and at 10 cycles, $(p_m/p_0)_{N_c}/(p_m/p_0)_{N_{c10}}$, against the non-dimensional cycle, N_c/N_{c10} . The N_{c10} in Fig. 7 denotes the number of waves, 10 cycles. It is shown that the $(p_m/p_0)_{N_c}/(p_m/p_0)_{N_{c10}}$ lies within 5% of a narrow range, regardless of the non-dimensional cycles. Therefore,

Fig. 6. Pressure ratio and number of waves ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

Fig. 7. Change in pressure ratio ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

Fig. 8. Coefficient of permeability

the effect of the number of waves on the oscillatory pore pressures can be considered so small to be ignored.

Coefficient of Drainage

The coefficient of drainage, C , is defined by the coefficient of consolidation, C_v , the wave period, T , and the thickness of permeable layer, l . Since the T and l are dependent of the wave characteristics and the seabed profile, respectively, the C_v is examined experimentally in the laboratory. Fig. 8 shows the relationship between the coefficient of permeability, k , and the initial relative density,

Fig. 9. Coefficient of volume compressibility
(1 kgf/cm² = 98 kN/m²)

Fig. 10. Coefficient of consolidation

D_{r0} . The k decreases linearly on the semi-log scale in accordance with the increase of the D_{r0} . Fig. 9 is the coefficient of volume compressibility, m_v , calculated from the stress-strain relationships measured from more than four specimens with the same relative density. The plots corresponding to the rebound process are illustrated in Fig. 9 because the m_v in such process is considered more dominant to the liquefaction. It is found from Fig. 9 that the m_v as well as the k shown in Fig. 8 tends to decrease as the D_{r0} increases. The open and solid circles in Fig. 9 denote the average value of the m_v in 5 cycles and the value at the 5th cycle, respectively. The differences of these plots seem rather small, so that the average value of the m_v in 5 cycles is adopted to estimate the C_v . Fig. 10 shows the relationships between the C_v and the D_{r0} calculated from the k and the m_v . The plots in Fig. 10 appear to be slightly dependent of the relative density. The C_v is, however, regarded approximately constant without reference to the relative density in this study. This assumption will be acceptable, when it is taken into account that the change of the relative density of sand deposits is considerably little unless the liquefaction and successive densification occur in deposits (Zen and Yamazaki, 1990).

Coefficient of Propagation

Fig. 11. An example of pressure response ($1 \text{ kgf/m}^2 = 98 \text{ kN/m}^2$)

The coefficient of propagation, α , is expressed by Eq. (26). Thus, if the pore pressure responses, $\partial p_m / \partial t$, to the applied pressure increases, $\partial p_b / \partial t$, is measured under undrained conditions, the α is easily determined. Fig. 11 is an example of the pressure responses measured with a conventional cyclic triaxial test apparatus. The p_{bc} and the p_{ms} in Fig. 11 indicate the isotropically applied cyclic cell pressure and the oscillatory pore pressure in the specimen, respectively. It is found from 11 that the p_m is less than the p_{bc} . This suggests that the α will become more than 1.0. So, the α is calculated with such observed data as shown in Fig. 11 by applying Eq. (26). The $\partial p_m / \partial t$ and the $\partial p_b / \partial t$ are obtained from the inclinations of lines like as ab , and $a'b'$, bc and $b'c'$ in Fig. 11 and the average values of the four are plotted in Fig. 12 in relation to the degree of saturation, S_r .

Fig. 12. Coefficient of propagation and degree of saturation ($1 \text{ kg/cm}^2 = 98 \text{ kN/m}^2$)

The S_r is measured immediately after the test. The plots which correspond to the number of waves, 10 cycles, are conveniently drawn in Fig. 12, since little effect of the number of waves on the α is observed. In Fig. 12, the α increases drastically even by a small amount of the reduction of the S_r . It should be noted that the α is very sensitive to the S_r .

The solid line indicates the relationship between the α and the S_r drawn by the method of least squares to the open circles, in which Eq. (28) is adopted omitting the m_{w0} , and the 2.0 kgf/cm^2 (196 kN/m^2) of the hydro-static cell pressure is used for the p_{mg} . The n/m_v of 295 kgf/cm^2 (28.9 kN/m^2) is obtained for the solid line. The dotted lines in Fig. 12 are drawn from Eq. (28) for various assumed values of the n/m_v . As the values of the n/m_v separately measured with the one dimensional loading-unloading test are between 252 kgf/cm^2 (24.7 MN/m^2) and 312 kgf/cm^2 (30.6 MN/m^2) for the compression process, and between 493 kgf/cm^2 (48.3 MN/m^2) and 722 kgf/cm^2 (70.8 MN/m^2) for the rebound process, then n/m_v estimated from the solid line is in between the two. Taking account of the scatter of the data, the relationship estimated from Eq. (28) may be considered to represent the measured α .

Fig. 13 is the α plotted against the relative density, D_{r0} . The α is measured in the cell of the cyclic triaxial test apparatus. As can be seen from Fig. 13, the effect of the D_{r0} on the α is quite little, compared with that of B-value.

Fig. 13. Coefficient of propagation and relative density

Fig. 14. Distribution of degree of saturation

Degree of Saturation

Fig. 14 denotes the degree of saturation measured after the tests by cutting out the deposits in each cell with a thin plate. Before cutting the cell, the water was drained from the bottom down to the surface level of the deposit and the water in the deposit in the uppermost cell was drawn by a spoon. Then, the unsaturated state of the deposit was cut out by the cell. The weight of the deposit

taken out of the cell was measured together with the water drawn beforehand by the spoon. The degree of saturation thus measured lies between 97 % and 101 %. More than 100 % of the S_r around the surface of the deposit may be attributed to the technical reason that

Test No.3, Stage No.1, $\alpha=2.3$, $C=5.57$

• Observed — Calculated

(a)

(b)

(c)

Fig. 15. Comparisons between theory and measurements ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

(a) Pressure ratio, (b) Excess pore pressure, (c) Vertical effective stress

the water above the deposit in the uppermost cell couldn't be separated from the pore water in the deposit in the process of the drainage because of the slight inclination of the deposit surface after the wave loading.

DISCUSSIONS

Oscillatory Pore Pressure and Liquefaction

The observed oscillatory pore pressures are presented in Fig. 15, together with the theoretical ones. The observed data are obtained from the tests performed under the double amplitude of the applied water pressure, $2\bar{p}_0 = 0.247 \text{ kgf/cm}^2$ (24.2 kN/m^2), and the wave period, $T=7 \text{ s}$. The thickness of the deposit is 1.90 m .

Fig. 15 (a) shows the pressure ratios at the non-dimensional depths, $z/l=0.45$ and $z/l=1.0$. The damping and phase lag of the p_m against the p_b are clearly shown in Fig. 15. (a) As previously mentioned, these damping and phase lag lead to the excess pore pressures in the deposit. The oscillatory excess pore pressure, u_e , which is equivalent to the wave-associated vertical effective stress, $(p_b - p_m)$, is illustrated at the typical phases, ωt , between 0° to 360° in Fig. 15 (b). It is quite interesting that the u_e indicates not only the positive values but also the negative ones. The oscillation of the u_e brings about the periodical changes of the vertical effective stresses as demonstrated in Fig. 15 (c).

In the theoretical analysis, the coefficient of drainage, $C=5.57$, is determined from Eq. (19) by using the C_v , T , and l . The coefficient of propagation, $\alpha=2.3$, is determined from Eq. (28) with the S_r , p_{mq} and n/m_v . The sinusoidal form of the wave-associated bottom pressure, p_b , is given as the boundary condition to the surface of the deposit, $z=0$. It is found from Figs. 15 (a) to 15 (c) that the theoretically calculated results compare very well with the observed ones.

Referring to Fig. 15 (b), the u_e does not exceed the initial vertical effective stresses, σ'_{v0} , at all. This means that the liquefaction is not induced in the deposit. It is, however, noted that the vertical effective stresses vary periodically as shown in Fig. 15 (c). Espe-

(a)

(b)

Fig. 16. Comparisons between theory and measurements ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

(a) Excess pore pressure

(b) Vertical effective stress

cially, the vertical effective stresses become less than the initial vertical effective stresses, σ'_{v0} , near under wave trough, say at the phase, $\omega t=250^\circ$. This fact is very important since the decrease of the vertical effective stress will lead to the reduction of the shear strength of sand deposits.

The excess pore pressures, u_e , observed for the $2\bar{p}_0$ of 0.756 kgf/cm^2 (74 kN/m^2) are pre-

sented in Fig. 16 (a). In Fig. 16 (a), it is observed that the excess pore pressures exceed the initial vertical effective stresses, σ'_{v0} , above some depths of the deposit. According to the liquefaction criterion expressed by Eq. (3), it is considered that the liquefaction is certainly brought about there. The liquefied depth is more clearly illustrated in Fig. 16 (b), in which the maximum depth of the liquefaction, 0.75 m, is given at the phase of 230° .

The observed distributions of the oscillatory excess pore pressures in Fig. 16 (a) indicate some peculiar shapes above the depth of 0.75 m after the liquefaction occurs, and they are quite different from the analytical ones. The reason is considered that the suspended soil particles after liquefaction behave like a dynamic fluid and the assumptions introduced in the governing equation such as Darcy's law become impertinent. Post liquefaction phenomena should be treated with some other approaches.

It is demonstrated in Fig. 16 (b) that the water pressure oscillation induces the compressional stresses to possibly densify the deposit. Namely, the vertical effective stresses which exceed the initial vertical effective stresses, σ'_{v0} , are observed at the phases beyond 300° . As already presented elsewhere (Zen and Yamazaki, 1990), the densification in sand deposits is attributed to these stresses repeatedly and alternately excited by waves.

Seepage Force and Liquefaction

The spacial differences of the oscillatory excess pore pressure in the seabed will exert seepage forces on the soil skeleton. As the increment of the vertical effective stress is represented by,

$$\frac{\partial \sigma'_v}{\partial z} = j + \gamma' \quad (33)$$

the vertical effective stress is derived by integrating Eq. (33):

$$\sigma'_v = \int_0^z j dz + \gamma' z \quad (34)$$

where j : the seepage force and γ' : the submerged unit weight of deposits. Eq. (34) is equivalent to Eq. (1) when the σ'_{v0} is identi-

Fig. 17. Seepage force and vertical effective stress ($1 \text{ kgf/cm}^2 = 98 \text{ kN/m}^2$)

cal with $\gamma'z$.

The seepage force, j , is derived from Eq. (1) by taking $\partial \sigma'_{v0} / \partial z = \gamma'$ and $\partial p_b / \partial z = 0$ into account:

$$j = -\frac{\partial p_m}{\partial z} \quad (35)$$

Fig. 17 shows the seepage forces at the phase of 270° calculated from the measured distribution of the p_m by using Eq. (35). The negative value of the j means the upward flow of pore water. It is found that the seepage force under wave trough becomes maximum at the surface of the deposit in this test case. The vertical effective stresses obtained from Eq. (34) and Eq. (1) are also illustrated respectively with the open circles and the solid curve in Fig. 17. Both the open circles and the solid curve show a good agreement with each other, indicating that the liquefaction, in other word a kind of quick sand, is caused at the depth shallower than 0.54 m. Madsen (1974) concluded that the flow induced in a porous seabed by waves generally is of minor importance in the context of transport mechanics of soils. Carstens et al. (1976) demonstrated that the effects of pressure gradients, except the critical gradient, may be ignored in considering the scouring. As indicated in the previous section, however, the liquefaction will easily occur in real ocean environment. Once the liquefaction happens, the suspended state of soil particles will be transported as a fluid together with the bottom current. Furthermore, even if the liquefaction depth is

limited to very near the surface of the seabed, the repeat of the liquefaction by a large number of cyclic waves may result in a critical scouring which causes a significant damages of offshore structures.

Comparison between One and Two Dimensional Analyses

The proposed theory is based on the assumption of one dimensional plane strain conditions. In order to examine the applicability of this assumption, one and two dimensional analyses are compared. The two dimensional elastic analysis presented by Okusa (1985) is used for the comparison. Since the liquefaction area is not clearly addressed in his analysis, the area which denotes the negative vertical effective stresses is compared with the liquefaction depth, d_l , calculated from the present analysis. The input data are listed in Table 3. The α in Table 3 is determined by assuming the elastic seabed. Under this assumption, the $(1/\alpha)$ becomes equal to the B-value. Namely, the B-value of 0.5, which is used in Okusa's analysis, corresponds to the α of 2.0. The results are illustrated in Fig. 18. It is clearly shown that both results compare well. Only the difference near the surface of the seabed may be attributed to the tensile strength of the soil expected in the two dimensional elastic analysis. Thus, it may be concluded that the assumption of one dimensional plane strain conditions of the seabed is reasonable when the wave length is large enough as shown in Table 3 and Fig. 18.

CONCLUSIONS

The response of the oscillatory pore pressure in the seabed associated with ocean wave propagation are theoretically and experimentally examined in the context of the wave-induced liquefaction. A governing equation of the oscillatory pore pressure is proposed and verified with the model tests. The following conclusions may be drawn from the study:

(1) The oscillatory pore pressure induced by ocean waves is generally expressed by Eq. (29), which is a function of the wave char-

Table 3. Input data

		Loose sand
Water depth	h (m)	20
Wave height	$H (=2\eta)$ (m)	5
Wave period	T (s)	15
Wavelength	L (m)	197
Wave-associated bottom pressure at peak	p_0 (kgf/cm ²)	0.207
Submerged unit weight of deposits	γ' (gf/cm ³)	0.5
Coefficient of permeability	k (m/s)	0.01
Coefficient of volume compressibility	m_v (cm ² /kgf)	9.0×10^{-3}
Poisson's ratio		0.3
B-value		0.5
Coefficient of drainage	C	4.16×10^{-3}
Coefficient of propagation	α	2.0

1) 1 kgf/cm² = 98 kN/m²

Fig. 18. Comparisons between one and two dimensional analyses (1 kgf/cm² = 98 kN/m²)

acteristics, the properties of soils and pore water, and the geometrical factors of the seabed.

(2) Assuming the linear waves, the oscillatory pore pressure in terms of the pressure ratio is represented by two non-dimensional parameters. These parameters are the coefficient of drainage and the coefficient of propagation. Little effect of the number of waves on the oscillatory pore pressure is observed.

(3) The comparisons of the oscillatory pore

pressure between the proposed theory and the measurement have shown a good agreement. The proposed governing equation is applicable to estimate the liquefaction potential in the porous seabed.

(4) The wave-induced liquefaction is caused by the upward seepage force repeatedly induced in the seabed, responding to the fluctuation of ocean waves.

(5) It is found that the oscillatory pore pressure may be analyzed under one dimensional plane strain conditions, when the wave length is large enough as treated herein.

ACKNOWLEDGEMENTS

The authors wish to express their sincere thanks to Mr. Atsushi Watanabe, JDC corporation, for his cooperation in performing the laboratory tests.

NOTATIONS

- C = coefficient of drainage
 H = wave height
 h = water depth
 j = seepage force
 L = wave length
 l = thickness of permeable layer
 m_w = compressibility of pore water
 m_{w0} = compressibility of pore water at fully saturated state
 N_e = number of waves
 n = porosity
 $\bar{P}$ = pressure ratio ($=p_m/p_0$)
 p_b = wave-associated water pressure at the surface of seabed
 p_m = oscillatory pore pressure in seabed
 p_{mq} = oscillatory pore pressure in seabed in terms of absolute pressure
 p_0 = amplitude of p_b
 $\bar{p}_0$ = amplitude of applied dynamic pressure
 T = wave period
 $\bar{T}$ = non-dimensional time ($=t/T$)
 $\bar{U}$ = non-dimensional excess pore pressure ($=u_e/p_0$)
 u_e = excess pore pressure
 $\bar{Z}$ = non-dimensional depth ($=z/L$)
 α = coefficient of propagation
 σ'_v = vertical effective stress
 σ'_{v0} = vertical effective stress at calm
 λ = wave number ($=2\pi/L$)

ω = angular frequency ($=2\pi/T$)

REFERENCES

- 1) Carstens, T., Brebner, A. and Kamphuis, J. W. (1976): "Seabed mobility under vertical pressure gradients", Proceedings of the International Conference on the Behaviour of Off-Shore Structures, Vol. 1, pp. 423-438.
- 2) Finn, W.D.L., Siddharthan, R. and Martin, G. R. (1983): "Response of seafloor to ocean waves", Journal of the Geotechnical Engineering Division, ASCE, Vol. 109, No. 4, pp. 556-572.
- 3) Inoue, R. (1975): "Propagation of pore water pressure in sand layer of high degree of saturation", Proceedings of the JSCE, Vol. 236, pp. 81-92 (in Japanese).
- 4) Liu, P.L.F., Timothy, P. and O'donnell (1979): "Wave induced forces on buried pipelines in permeable seabeds", Proceedings of the 4th Conference on Civil Engineering in the Oceans, pp. 111-121.
- 5) Madsen, O. S. (1974): "The stability of a sand bed under breaking waves", Proceedings of the 14th Coastal Engineering Conference, ASCE, Vol. 2, pp. 776-794.
- 6) Madsen, O. S. (1978): "Wave-induced pore pressures and effective stresses in a porous bed", Geotechnique, Vol. 28, No. 4, pp. 377-393.
- 7) May, C. and Foda, M. A. (1981): "Wave-induced stresses around a pipe laid on a pore-elastic sea bed", Geotechnique, Vol. 31, No. 4, pp. 509-517.
- 8) Moshagen, H. and Tørum, A. (1975): "Wave induced pressures in permeable seabeds", Journal of the Waterways, Harbour and Coastal Engineering, ASCE, Vol. 101, No. 1, pp. 49-58.
- 9) Mynett, A. E. and Mei, C. C. (1982): "Wave-induced stresses in a saturated pore-elastic sea bed beneath a rectangular caisson", Geotechnique, Vol. 32, No. 3, pp. 235-247.
- 10) Nago, H. (1981): "Liquefaction of highly saturated sand layer under oscillating water pressure", Memories of School of Engineering, Okayama University, Vol. 16, No. 1, pp. 91-104.
- 11) Okusa, S. (1985): "Wave-induced stresses in unsaturated submarine sediment", Geotechnique, Vol. 35, No. 4, pp. 517-532.
- 12) Sleath, J. F. A. (1970): "Wave-induced pressures in beds of sand", Journal of the Hydraulics Division, ASCE, Vol. 96, No. 2, pp. 367-378.

- 13) Yamamoto, T. (1977): "Wave induced instability in seabed", proceedings of the ASCE Special Conference, Coastal Sediments '77, pp. 898-913.
- 14) Zen, K., Yamazaki, H. and Watanabe, A. (1987): "Wave-induced liquefaction and densification in seabed", Report of the Port and Harbour Research Institute, Vol. 26, No. 4, pp. 125-180 (in Japanese).
- 15) Zen, K. and Yamazaki, H. (1990): "Mechanism of wave-induced liquefaction and densification in seabed", Soils and foundations, Vol. 30, No. 4, pp. 90-104.