A LOWER-BOUND APPROACH TO ACTIVE AND PASSIVE EARTH PRESSURE PROBLEMS

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ABSTRACT

Active and passive earth pressures acting on retaining walls are known to be a complicated function of many factors, for instance, the cohesion and friction angle of backfill material, wall friction, and geometries of wall and backfill. Conventional theories based on stability analysis, e.g., Coulomb and Rankine methods which are now quite widely used, cannot necessarily take all these factors into consideration. Firstly this paper develops a numerical procedure for calculating active and passive earth pressures against rigid retaining walls, which enables to take account all these factors affecting active and passive pressures, and which presents full information about the earth pressures. Secondly, by comparing the procedure with Coulomb and Rankine methods, this paper aims to clarify the mechanical and practical meanings of these classical methods. Based on the lower-bound theorem in the theory of plasticity, the procedure searches the minimum and maximum values respectively of active and passive thrusts within the limitations of satisfying the equilibrium equations and non-failure condition. To represent these conditions, the earth pressure and the stress field within backfill are discretized in a similar manner as in the finite element method. On the problem formulation, two methods are considered depending on whether a potential slip surface is assumed or not. When assuming a slip surface, arises a serious difficulty that the criterion for locating the critical slip surface is contrary to the criterion for determining the stress field. When assuming no slip surface, it is shown that the proposed procedure provides the reasonable solutions of active and passive pressures and that the boundary condition of wall and backfill movements has an essential meaning in the analysis. The procedure is applied to a number of case studies, where comparisons are made thoroughly with classical theories. Through the development of the procedure, many important aspects in the earth pressure problems are clarified.

Key words: active earth pressure, computer application, failure, passive earth pressure, retaining wall, safety factor, stability analysis, static, stress distribution (IGC: E 5/H 2)

INTRODUCTION

Although the determination of lateral earth

pressure exerted against retaining structures has been one of the most extensively studied subjects in geotechnical engineering, the con-

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ventional stability analysis is still the principal method for designing the practical retaining walls. It has been seen that the actual response of soil-retaining wall is a complicated function of many factors, i.e., the relative soil-structure displacements, structural rigidity, backfill soil properties, construction history, and so on. Especially earth pressure is known to depend largely on the movements of wall and backfill. Since in engineering practices it may be difficult to consider all of these factors affecting earth pressure, classical earth pressure theories, e.g., Coulomb and Rankine methods have been widely used from the standpoint of strength analysis and have been proved reliable. However these classical theories contain some defects from theoretical and practical points of view. For instance, Coulomb method cannot consider the cohesion of backfill material when employing Mohr-Coulomb failure criterion. The method is based on the limit equilibrium analysis in which the equations of equilibrium and the failure criterion are satisfied only along the potential slip surface. It provides little information about the soil pressure distribution. Rankine method can be applied only to the vertical standing smooth retaining wall. Furthermore the relationship between Coulomb and Rankine methods has not been sufficiently clarified yet. The practical difficulty lies on the fact that active and passive pressures are a function of many factors, and that conventional stability analyses have not provided the method for considering all of the factors. At the first step, by applying a lower-bound approach in the theory of plasticity, this paper aims to develop a numerical procedure for calculating the active and passive earth pressures which enables to take account of all the factors affecting these pressures. The final object of this paper is to clarify the mechanical meanings of Coulomb and Rankine methods by using the proposed procedure and to disclose many important aspects in active and passive earth pressure problems. Then we restrict our analysis to the static earth pressure acting on the rigid retaining wall behind which active and passive states of stress are assumed to

exist. The classical theories based on the stability analysis, do not explicitly consider soil or wall movements likely to exist but it is implicitly assumed that such movements are sufficient to produce a state of plastic equilibrium behind the wall and that soil and wall movements are restricted to acceptable level. For the sake of simplicity, it is further assumed throughout this paper that the wall is allowed to translate as a rigid body into or away from a backfill, though other types of structural displacements, rigid body rotation and flexure are possible. Even such an idealized stability problem of earth pressure has not been perfectly solved yet in spite of a number of publications on the subject of active and passive earth pressures, due to many factors affecting the earth pressure. The methods for stability analysis are divided into slip line, limit equilibrium and limit analysis methods (see Chen, 1975). The limit analysis method is based on the lower-and upper-bound theorems in the theory of plasticity. The lower-bound theorem which is employed in this paper, is described as: the loads determined from a distribution of stress alone, which satisfies the equilibrium equations, stress boundary conditions and nowhere violates the failure criterion, are not greater than the actual collapse load. The stress distribution which satisfies the conditions imposed in the lower-bound theorem, is termed a statically admissible stress field for the problem under consideration. Some of the recent theoretical analyses of active and passive earth pressures, are classified in to the three methods for stability analysis as follows. Slip line method: Sokolovskii (1965); Limit equilibrium method: Coulomb method, Janbu (1957), Shield and Tolunay (1973), Basudhar and Madhav (1980) and Bang (1985); Upper-bound approach; James and Bransby (1971) and Chen (1975); and Lower-bound approach: Rankine theory. The most difficult aspect in applying the lower-bound theorem to geotechnical problems, is to find a statically admissible stress field for a particular problem. To overcome this difficulty, Lysmer (1970), Basudhar, Valsangkar and Madhav (1979), Pastor and Tur-

and Arai and Nakagawa (1988) introduced the discretization technique used in the finite element method. These researches gave the reasonable solutions to the problems of bearing capacity, slope stability and of passive earth pressure. Still now, however, the active earth pressure problem has not been solved sufficiently by the technique, though the problem arises more frequently in actual designing works.

PROBLEM FORMULATION

As schematically illustrated in Fig. 1, we consider the following two methods for lower-bound approach based on the discretization of stress field. Method I discretizes the whole region of the problem under consideration (see Arai and Tagyo, 1985 b). Method II assumes a potential slip surface passing through the toe of retaining wall and discretizes only the inside of the slip surface (see Arai and Nakagawa, 1988). The latter combines the lower-bound approach with the limit equilibrium method, which requires the search for the critical position for the slip surface. Method II has the advantages that it does not need to impose the artificial boundary condition and that the solution is easily compared with the conventional limit equilibrium solutions. The problem formulation for these two methods are outlined in the subsequent paragraph. The details of the problem formulation for the bearing capacity and slope stability analyses are given by Arai and Tagyo (1985 b) and Arai and Nakagawa (1988).

Method I

Modeling and Finite Element Meshing: For convenience of illustration, let us consider a homogeneous backfill with zero pore pressure, since it is easy to extend the proposed procedure to multiple layered backfill. As shown in Fig. 1 (a), the region of backfill soil is subdivided into a number of finite elements. The distribution of earth pressure is considered to be arbitrary and approximated by a series of straight lines as illustrated in Fig. 1. The active and passive earth pressures at the

Fig. 1. Modeling and definitions

nodal point n which are applied normal to the wall surface, are designated respectively as p_a^n and p_p^n . Throughout this paper, subscripts a and p indicate active and passive component respectively. The frictional resistance between wall surface and backfill soil is represented by friction angle δ according to the conventional expression. Active thrust P_a and passive thrust P_p are defined as the resultant force of active and passive pressures and wall friction. The backfill soil mass is assumed to move downward and upward respectively for active and passive earth pressures.

Equations of Equilibrium: A set of stresses are assumed to be constant within each finite element, because the stresses are the independent variable to be determined by the proposed procedure. Such a discretization technique of stress field is different from the technique used by Lysmer (1970) and the succeeding researchers in which stresses are assumed to vary linearly in each element. By employing the principle of virtual displacement, the following equation defines a set of equivalent nodal forces which is statically in equilibrium with the stress condition in the element.

$$\{F_{xi}^m, F_{yi}^m, \dots\} = [B]^T \{\sigma_x^m, \sigma_y^m, \tau_{xy}^m\}^T V^m \quad (1)$$

where F_{xi}^m, F_{yi}^m : respectively horizontal and vertical components of equivalent nodal forces, $\sigma_x^m, \sigma_y^m, \tau_{xy}^m$: horizontal and vertical normal stresses and shear stress in the element, $[B]$: matrix for calculating the strains from nodal displacements, V^m : volume of the element, m : finite element number, and n : node number. Since the quadrilateral element is considered to be composed of the four triangular

Fig. 2. Quadrilateral element

elements as seen in Fig. 2, the $[B]$ matrix in the quadrilateral element is constructed by the superposition of the $[B]$ matrix in each constant strain triangular element in classical FEM. The equations of equilibrium mean that the sum of the equivalent nodal forces and nodal loads must be equal to zero at each of the nodal points.

Horizontal direction:

$$P_1^n = F_{xn}^m - R^n \sin \alpha - S^n \cos \alpha = 0 \quad (2)$$

Vertical direction:

$$P_2^n = F_{yn}^m + R^n \cos \alpha - S^n \sin \alpha - G_y^n = 0 \quad (3)$$

where

$$R^n = H^n p_a^n, \quad S^n = R^n \tan \delta$$

R^n : normal load due to active earth pressure, S^n : frictional load, H^n : coefficient to replace active earth pressure p_a^n with the nodal load, and G_y^n : vertical component of the nodal load which contains the weight of soil mass. The second and third terms both in Eqs. (2) and (3) correspond respectively to the components of normal earth pressure and wall friction. For the passive earth pressure problem, the equations of equilibrium are quite similar to the active case except that the friction angle δ in Eqs. (2) and (3) is replaced with the negative value $(-\delta)$.

Non-failure Condition: When employing the Coulomb failure criterion, the following non-failure condition must be satisfied for each finite element.

$$P_3^m = \{(\sigma_x^m + \sigma_y^m) \sin \phi + 2c \cos \phi\} - \{(\sigma_x^m - \sigma_y^m)^2 + (2\tau_{xy}^m)^2\} > 0 \quad (4)$$

Furthermore assuming that backfill soil has no tensile strength,

$$\sigma_x^m \geq 0, \quad \sigma_y^m \geq 0 \quad (5)$$

Boundary Conditions: When applying Method I, some displacement constraints as seen in Fig. 1 (a) must be imposed along the artificial boundary surface as well as in the ordinary finite element displacement approach. When a directional displacement is restricted at a certain node, the equilibrium equation for the corresponding direction is ignored at the node (see Lysmer, 1970). The boundary condition applied for the backfill has a highly important meaning in the earth pressure problem as explained later. The question of great concern is how deep behind the wall the backfill is in motion. Since our main concern in this paper is to clarify the mechanical meanings of Coulomb and Rankine methods, we define the following two types of boundary condition at the base of backfill soil stratum, which correspond respectively to Rankine and Coulomb methods. Boundary condition I (see Fig. 3 (a), Rankine method): the boundary AB in Fig. 3 (a) is assumed free to move horizontally but is restricted vertically. When comparing strictly Method I with Rankine method, we cannot find other type of boundary condition but Fig. 3 (a), because Rankine method assumes that backfill soil expands or is compressed laterally without any restriction. Boundary condition II (see Fig. 3 (b), Coulomb method): only node A in Fig. 3 (b), which represents the toe of retaining wall and the backfill base adjacent to the wall, is horizontally movable. When comparing the Method I with Coulomb method, we can give no boundary condition other than Fig. 3 (b), because Coulomb method assumes a potential slip surface passing through the

(a) Boundary condition I (b) Boundary condition II

Fig. 3. Displacement boundary conditions

toe of retaining wall. Due to the artificial condition as shown in Fig. 3(b), the solutions of active and passive thrusts are slightly influenced by the mesh size of finite element, but the difference between the solutions is limited within 1 percent, unless employing the extreme meshing of finite element. The lateral boundary BC shown both in Figs. 3 (a) and 3 (b) is completely restrained from movement, because it gives little effect to the earth pressures. The case studies will certify that these boundary conditions are valid to Rankine and Coulomb methods.

Problem Formulation: The problem of finding the active and passive earth pressure distributions is formulated as an optimization problem which isolates a particular stress field in the backfill soil stratum. The problem is to find the optimum solution of decision variable x^* solving

$$\text{minimize } J=f(x) \quad (6)$$

subject to the equality and inequality constraints

$$h(x)=0, \quad g(x) \geq 0 \quad (7)$$

where J is called as an objective function. The decision variable to be found consists of active or passive earth pressure p_a^n , p_p^n , and the stresses in each finite element σ_x^m , σ_y^m , and τ_{xy}^m . The constraints to be satisfied are Eqs. (2) through (5). Referring to Fig. 4, the minimization of active thrust provides a suitable lower-bound solution of active pressure, while the maximization of passive force is required for passive pressure problem. Thus the objective function to be minimized is given as

$$\begin{aligned} \text{minimize } J &= P_a \quad (\text{active}) \\ &= -P_p \quad (\text{passive}) \end{aligned} \quad (8)$$

Method II

Modeling and Finite Element Meshing: As shown in Fig. 1 (b), the location of a potential slip surface is assumed. The potential slip surface may be of any physically reasonable shape. The potential sliding body is divided into a number of finite slices by vertical lines, and the slip surface is approximated by a series of straight lines. Each of the slices is

Fig. 4. Load-displacement relationship

further subdivided into several triangular and quadrilateral elements.

Problem Formulation: The earth pressure distribution and wall friction are represented by the same manner as in Method I. In this Method II, the normal and shear stresses on the slip surface element σ_a^m , τ_a^m must be introduced in addition to the stresses in the plane finite element. Since the failure criterion is satisfied on the slip surface, the shear resistance available τ_a^m is uniquely specified as

$$\tau_a^m = c + \sigma_a^m \tan \phi \quad (9)$$

Thus only σ_a^m is treated as a decision variable. By use of these discretized stresses, the equilibrium equations and non-failure condition as the constraints are expressed by the similar manner as in Method I. Such a system of representing the equilibrium equations, however, may make the stress field unstable in the plane finite element neighboring to slip surface, when both the stress field in sliding mass and the stress distribution along slip surface are unknown. To avoid this instability, all the plane finite elements in contact with the slip surface, which are shaded in Fig. 1 (b), are assumed to fail (see Arai and Nakagawa, 1988).

$$\begin{aligned} P_4^m &= \{(\sigma_x^m + \sigma_y^m) \sin \phi + 2c \cos \phi\} \\ &\quad - \{(\sigma_x^m - \sigma_y^m)^2 + (2\tau_{xy}^m)^2\} = 0 \end{aligned} \quad (10)$$

In this Method II, we must locate the critical slip surface which gives the minimum or maximum values of respectively active and passive thrusts. The procedure employed here is essentially founded due to the formulation

used in Arai and Tagyo (1985 a). All the interslice abscissas shown in Fig. 1 (b) are stipulated and a potential slip surface is assumed to intersect the backfill surface only on the interslice boundary. Such a formulation requires some trials on which the boundary condition concerning the point of intersection between a potential slip surface and the ground surface is varied methodically. Thus the search for the critical slip surface having an arbitrary shape, is reduced to a problem of determining the interslice elevation of slip surface y_i on a specified boundary condition adopted to the intersection between the slip and backfill surfaces. This problem is formulated by slightly modifying the previous problem formulation for Method I, such that a series of interslice elevations of slip surface y_i is also considered as the independent variable to be found. When handling a series of y_i as independent variable, there are cases where the proposed procedure does not yield a physically reasonable shape of slip surface. To locate the slip surface as a downward convex function of the lateral coordinate, the following constraint must be added to the problem formulation in Method II.

$$P_5^i = (y_{i-1} + y_{i+1})/2 > y_i \quad (11)$$

As a result, by adding the decision variables σ_a^m and y_i , and the constraint Eqs. (10) and (11) to the previous formulation for Method I, the problem formulation for Method II is completed. In this formulation, the objective function defined by Eq. (8) plays the role of searching the position of critical slip surface; while in Method I, Eq. (8) isolates merely a particular stress field. It is noted that a serious difficulty lies in this aspect as described in the following case studies.

Numerical Analysis

Since the formulated optimization problem cannot be solved analytically due to the inequality constraints and the non-linearity in non-failure condition, the numerical procedure is employed here. The formulated problem can be effectively solved by SUMT (Sequential Unconstrained Minimization Technique) interior point method developed by Fiacco and McCormick (1968), which converts the con-

strained optimization problem to a series of unconstrained optimization problem. In the search for an unconstrained minimum, the proposed procedure employs the conjugate gradient technique developed by Fletcher and Reeves (1964). This technique, at first, calculates the gradient of the objective function with respect to the decision variables to be determined. Starting from the trial values of the decision variables, the technique repeats an iteration procedure to search the optimum solution. The detailed introduction of these techniques is given by Arai and Tagyo (1985 b). The minimum calculated by the combination of SUMT and conjugate gradient technique, is a local minimum. Repeated use of the proposed procedure from some different trial solutions has provided more or less the same optimal solution. This result appears to exclude the possibility that several other local minima are present.

Accuracy of Solution

In the proposed procedure, the conditions of equilibrium are not necessarily satisfied at the interfaces between two finite elements, because the discretization technique employed is essentially founded on the finite element displacement approach, and because a set of stresses are assumed constant in each element. When giving external loads, the finite element displacement approach is known to produce solutions both of displacement and of stress smaller than the correct solutions, in the particular problem in which no initial strains and no initial stresses exist (see Zienkiwicz, 1971). In contrast with this case, when optimizing the active and passive thrusts (external load) with changing the element stresses, the active and passive thrusts obtained by the proposed procedure may be greater than the precise solution of the problem. This means that the proposed procedure tends to overestimate the active and passive thrusts comparing with the correct solution. Referring to Fig. 4, such a tendency leads to certify that the procedure provides an appropriate lower-bound solution of active thrust, while this is not the case for passive thrust. Since the solution by

the procedure contains some numerical errors due to SUMT and iterative technique, this problem must be investigated further after getting the numerical solutions for some particular problems.

CASE STUDIES

To verify the validity of the proposed procedure, its performance is demonstrated here in several case studies, which will clarify a lot of unknown aspects in the active and passive earth pressure problems. Comparisons are made mainly with Coulomb and Rankine methods according to our primary object. Throughout the case studies, subscript C and R indicate the solutions respectively by Coulomb and Rankine methods. When employing Coulomb method, the triangular distribution of earth pressure is assumed, since the method produce little information about the distribution of earth pressure.

Example 1: The first example (see Figs. 5 through 8) considers the simplest problem of active earth pressure against a vertical standing smooth retaining wall having the horizontal and cohesionless backfill. In this problem it is well known than the theoretical solutions by Coulomb and Rankine methods agree with each other.

[Method I] Fig. 5 shows the results calculated by the proposed procedure (Method I) according to the boundary condition at the backfill base (see Fig. 3). The proposed procedure furnishes full information on the earth pressure, i.e., the earth pressure distribution, active thrust P_a , principal stress field, and safety factor distribution as illustrated in Fig. 5. The safety factor within each finite element is defined as schematically shown in Fig. 6. Considering the numerical errors, a finite element is assumed to attain the state of plastic equilibrium when the safety factor does not exceed 1.03. On the boundary condition I or in Fig. 5 (a), both the active thrust and earth pressure distribution calculated by Method I are in good agreement with Rankine's solution. Note that Rankine's solution is a lower-bound solution supported by a statically

(a) Boundary condition I

(b) Boundary condition II

Fig. 5. Example 1: Method I (smooth wall)

Fig. 6. Safety factor F_s in each finite element

admissible stress field. The slight discrepancy of earth pressure at the upper and lower ends is due to the limitation of discretization technique in which a set of stresses are assumed to be constant within each finite element. The active thrust given by Method I is slightly greater than Rankine's lower-bound solution (see Fig. 5(a)). Referring to Fig. 4, this means that Method I furnishes an appropriate lower-

bound solution of active thrust, despite of the numerical errors due to SUMT and iterative technique. As stated previously, this is the tendency resulting from the finite element displacement approach on which Method I is founded. Clearly the safety factor distribution shown in Fig. 5 (a) verifies that the boundary condition I defined in Fig. 3 (a) corresponds to Rankine method in which the whole region of backfill is assumed to reach the state of plastic equilibrium. In Fig. 5 (b), the boundary condition II defined in Fig. 3 (b) produces a definite failure region which is coincident with the slip surface assumed by Coulomb method (Due to the assumption that a set of stresses are constant within each finite element, Method I enables to define the safety factor in each finite element, resulting in the clear representation of failure region which links a number of failure elements. This is one of the advantages of Method I over the discretization of stress field employed by Lysmer, 1970). The result shown in Fig. 5 (b) proves that Coulomb's failure mechanism expressed by a potential slip surface corresponds to the boundary condition II defined in Fig. 3 (b), because the condition assumes implicitly that the active earth pressure is created only by the sliding soil mass. This aspect is supported more strongly by the following case studies. Thus the agreement of active thrust between Rankine and Coulomb methods cannot be necessarily expected except in this particular Example where the respective slip surface happens to coincide with each other, though the displacement boundary condition assumed is quite different from each other. The computation time required for the total iteration steps in this Example is less than 1 minute when using a combination of the personal computer NEC PC 9801 RA and AWS 20 (CPU: MC 68020, 16 MHz) and softwares SPARK-68 K and Absoft FORTRAN 77 compiler.

[Method II] Fig. 7 (a) shows the result calculated by the proposed procedure (Method II) in which the plane slip surface following Coulomb method is located and restrained. The active pressure obtained by Method II

Fig. 7. Example 1: Method II (plane slip surface)

seems to agree well with the one by Method I (see Fig. 5 (a)). Thus in this particular case, Coulomb's solution, which is based on the limit equilibrium method and which considers neither equations of equilibrium nor failure condition inside or outside of the slip surface, agrees with the solution by the proposed procedure. Fig. 7 (b) compares the active pressure in Fig. 7 (a) with the pressure obtained by using other position of slip surface. In Fig. 7 (b), the notation of earth pressure distribution corresponds to that of slip surface. Note that the slip surface except in Coulomb method produces the conservative value of active pressure at the bottom of wall, comparing with the active pressure in Fig. 7 (a) which was founded on the slip surface specified in Coulomb method. And that the slip surface other than in Coulomb method gives the active thrust smaller than Rankine's solution. Method II may not provide an appropriate lower-bound solution, unless assuming an adequate slip surface. This is because the inadequately located slip surface leads to overestimate the shear resistance mobilized along the slip surface, resulting in the decrease in the active pressure at the toe of retaining wall. Figure 8 shows the active earth pressure and the position of slip surface when considering also the location of slip surface as unknown variable. In Fig. 8 the point of intersection between slip and ground surfaces is restricted to the same point as in Fig. 7 (a). The decrease in earth pressure at the toe of

retaining wall is attributed to the same reason described as in Fig. 7 (b). The result obtained both in Figs. 7 (b) and 8 indicates that the critical slip surface must be located so that it maximizes the active thrust, because the incorrect position of slip surface leads to underestimate the active thrust. This criterion for locating the critical slip surface is identical with the one employed by Coulomb method. Due to this conclusion, a serious difficulty arises when applying Method II. The criterion for locating the critical slip surface is to maximize the active thrust, whereas the criterion for isolating the stress distribution is to minimize the thrust (see Eq. 8). This is one of the mechanical properties in the active earth pressure problem quite different from other stability problems. Thus Method II can be applied only to the case where the location of slip surface is correctly stipulated and restrained. From this point of view, it is reasonable that Coulomb method assumes a plane slip surface, because the assumption restricts the shape of slip surface and excludes the possibility to select the inadequate position of slip surface which underestimates the active thrust. Hereafter, the application of Method II is abandoned, since our main object is to get more general and practical method for calculating active and passive earth pressures. In the subsequent case studies, only Method I

is discussed. In addition, when applying Method II to the passive earth pressure problem, the inadequate position of slip surface tends to overestimate the passive earth pressure at the base of retaining wall. Also in this case, the criterion to locate the slip surface disagrees with the one to isolate a particular stress field within the backfill.

Although the lower-bound approach rationally considers no velocity field, the stress distribution obtained by the approach should be supported by a reasonable velocity field. Both the horizontal and vertical stresses estimated by Method I (see Fig. 5) seem to distribute quite smoothly and to increase proportionally to the depth from the backfill surface. When assuming the elastic behavior of backfill material, we get the similar distribution of strain rate. Such a strain rate field is accompanied with the velocity field where both horizontal and vertical velocities increase proportionally to the square of the depth. Thus it may not be difficult to find a velocity field which backs up the stress field obtained by Method 1. On the other hand it is not easy to specify a velocity field accompanied with the stress distribution provided by Method II, because Method II assumes a slip surface on which the discontinuous change in velocity field may occur. However we do not go deep into this problem, because Method II has been already abandoned due to the contradiction in the objective function to be employed.

Example 2: By using the same geometry model as in Example 1, the second example (see Fig. 9) discusses the effect of wall roughness on the active pressure. Fig. 9 presents the results calculated by Method I. The relationship between the angle of wall friction and active thrust is summarized in Fig. 10 and compared with Coulomb's solution, since Rankine method cannot take the wall friction into consideration. Referring to Fig. 10, the angle of wall friction causes the increase in active thrust on the boundary condition I, while it reduces the active thrust on the condition II in this vertical standing wall. This result is understood as follows. On the

Fig. 9. Example 2: Method I (rough wall)

boundary condition I, the horizontal component of active thrust P_a is held nearly constant despite of the increase in wall friction, as seen in Figs. 5 (a) and 9 (a). In other words, the lateral component of active pressure is not governed by the stress state just behind the retaining wall but is controlled by the active failure zone distant from the wall (see Fig. 9 (a)). Then the addition of vertical component due to wall friction causes the increase in active force P_a as shown in Fig. 10. In Fig. 9 (b) or on the boundary condition II, a definite failure region develops passing through the toe of retaining wall and the directional change of active force which is due to wall friction, reduces the active thrust P_a , because a partial weight of sliding soil mass is supported by the frictional resistance along the wall. The findings is consistent with the tendency estimated by Coulomb

Fig. 10. Comparison with Coulomb method

method as shown in Fig. 10. This result supports the aspect that Coulomb method assumes implicitly the boundary condition II as pointed out previously. The discrimination between the boundary conditions I and II will be shown more clearly in the subsequent Example 4. In this Example, Method I based on the boundary condition II produces somewhat greater active thrust than Coulomb method (see Fig. 10). However the Coulomb's solution is not always smaller than the lower-bound solution obtained by Method I as seen in the following Examples, because Coulomb's solution is a limit equilibrium solution, which is not necessarily a lower- or upper-bound solution.

Example 3: Based on the same geometry model as in Example 1, the third example (see Figs. 11 and 12) examines the effect of cohesion of backfill soil. Figs. 11 and 12 show the results by Method I under the combination of boundary condition and wall roughness. In Fig. 11 (a), Method I gives an appropriate lower-bound solution larger than Rankine's solution. The active pressure distribution illustrated both in Fig. 11 (b), shows a fairly good agreement with the distribution predicted by Rankine method, regardless of

Fig. 11. Example 3: Method I, boundary condition I (cohesive backfill)

Fig. 12. Example 3: Method I, boundary condition II (cohesive backfill)

wall friction. On the boundary condition I, wall friction gives little effect to the active earth pressure by the same mechanism as stated in Example 2. As shown in Fig. 12, the boundary condition II gives a slightly different distribution of active pressure from Rankine's solution and tends to provide an active thrust a little smaller than the condition I. It is important to note that on the boundary condition II, wall friction reduces the active thrust. Such tendencies may make it sense to apply Rankine method to the actual design of vertical standing wall, though Rankine method considers neither wall friction nor practical boundary condition of wall and backfill movements.

Example 4: In order to investigate the effect

of more general geometry both of retaining wall and backfill, the fourth example (see Figs. 13 and 14) studies the active earth pressure acting on the wall inclining inward against the backfill. Figures 13 and 14 illustrate the results by Method I under many combinations of the boundary condition, wall roughness and the cohesion of backfill material. The comparison with Coulomb method is summarized in Fig. 10. As seen in Fig. 13, the boundary condition I yields the value of active thrust too much larger than Coulomb method, whereas in Figs. 14 (a) and 14 (b) the condition II provides nearly the same value as Coulomb's solution. This is because of the boundary condition I the active thrust against the wall is determined by the active failure region far from the retaining wall as indicated in Ex-

Fig. 13. Example 4: Method I, boundary condition I

ample 2, while on the condition II the active thrust is specified by the sliding soil mass as well as the failure mechanism assumed in Coulomb method. In Figs. 14 (a) and 14 (b), the slip surface assumed in Coulomb method

corresponds closely to the failure region estimated on the boundary condition II. These results also verify the aspect that the boundary condition II corresponds to Coulomb method. Coulomb method has been quite widely applied to this type of retaining structures and has been proved reliable. Our intuitive feelings may not accept the assumption that too wide region of backfill is in free motion at the bottom such as seen on the boundary condition I. Thus the boundary condition II or Coulomb method seems to be more suitable for most practical cases. Referring to Figs. 14 (a) through 14 (d), the cohesion of backfill soil appears to reduce substantially the active pressure against the wall inclining toward the inside of backfill. Concerning this limited Example on the boundary condition II, we cannot find the passive failure zone in contact

(a)

(b)

(c)

(d)

Fig. 14. Example 4: Method I, boundary condition II

Fig. 15. Example 5: Method I, boundary condition I

(a)

(b)

(c)

(d)

Fig. 16. Example 5: Method I, boundary condition II

with the retaining wall which was predicted by Sokolovskii (1965).

Example 5: The fifth example (see Figs. 15 and 16) investigates the earth pressure acting on the retaining wall inclining outward against the backfill. Figs. 15 and 16 illustrate the results by Method I together with Coulomb's solution. The comparison with Coulomb method is also presented in Fig. 10. As plotted in Fig. 10, in this particular case, both the boundary conditions I and II tend to provide almost the same value of active thrust. This is due to the failure mechanism as shown in Figs. 15 and 16 where a similar region of active failure develops behind the wall irrespective of the boundary condition at the bot-

(a)

(b)

Fig. 17. Example 6: Method I, boundary condition II (layered backfill)

tom of backfill. Such an active failure region may be created by the geometries of wall and backfill, where the active earth pressure is mainly controlled by the weight of sliding soil body. Also in this Example, the slip surface assumed in Coulomb method appears to be in good agreement with the failure region estimated on the boundary condition. II In this case, the active pressure is not so sensitive to the cohesion of backfill soil and it becomes considerably large at the upper part of retaining wall. These tendencies are clearly different from the results in Example 4. In Fig. 16 (b), note that the high wall friction appears to make the active pressure close to the triangular distribution which is usually assumed in the application of Coulomb method.

Example 6: Fig. 17 shows the results by Method I when the backfill consists of multiple soil layers. In spite of a marked similarity in the active thrust amount, there exists a notable difference of earth pressure distribution between the two cases shown in Figs. 17 (a) and 17 (b).

Fig. 18. Example 7: Method I, boundary condition I (passive pressure)

Example 7: The seventh example (see Figs. 18 and 19) investigates the passive earth pressure problem by using Method I. Figs. 18 and 19 show the results respectively on the boundary conditions I and II. Fig. 20 compares the passive thrust calculated by Method I with the existing solutions. In Fig. 19 (a) the passive thrust by Method I happens to exceed the upper-bound solution by Chen (1975) which agrees with the Coulomb solution in this particular case. This inadequacy is attributed to the numerical error due to the finite element displacement approach which tends to provide larger value of external load under the condition. Except only this case, in Fig. 20, Method I produces the conservative value of passive thrust comparing with the upper-bound and slip line solutions respectively by Chen (1975) and Sokolovskii (1965). Thus in most cases Method I appears to provide an approximate solution of passive pressure. On the boundary condition I in Fig. 20, the passive thrust hardly increases with increasing wall friction, because as seen in Fig. 18 the passive pressure is determined by the Rankine passive failure zone distant from the retaining wall, as well as the active pressure on the condition I (see Example 2). Thus also in the passive earth pressure problem, we must employ the boundary condition II when considering the wall roughness. As having been pointed out, the limit equilibrium calculation by Coulomb method tends to produce much greater value of passive thrust with increasing the wall friction, because the method assumes a plane slip surface even for the rough wall. In Figs. 19 (b) and 19 (d), the proposed procedure provides smaller passive

Fig. 19. Example 7: Method I, boundary condition II (passive pressure)

pressure at the toe of retaining wall when considering the high degree of wall friction. Such a distribution of passive pressure may be attributed to the insufficient shear resistance along a passive failure region or a potential slip surface. That is, the curved slip surface resulting from rough wall causes the decrease in shear resistance comparing with the plane ship surface. In Figs. 19 (a) through

Fig. 20. Comparison with existing solutions

19(d), we can observe the change of passive failure region due to wall roughness. For instance, when assuming perfectly smooth wall in Fig. 19 (a), the slip surface assumed in Coulomb method corresponds closely to the passive failure region predicted by Method I, while the former is much longer than the latter when considering wall friction in Fig. 19 (b).

CONCLUSIONS

This paper has developed a numerical procedure for calculating active and passive earth pressures, by combining the lower-bound approach and the discretization technique in the finite element method. Though active and passive earth pressures are a complicated function of many factors, conventional theories based on stability analysis could not necessarily take all these factors into consideration. The proposed procedure has enabled to consider all the factors and to provide the solution of active and passive pressures which contains the earth pressure distribution and stress field within backfill. The problem of calculating earth pressure is formulated as an optimization problem which isolates a particu-

lar stress field within backfill. On the problem formulation, two methods are discussed depending on whether a potential slip surface is assumed or not. When assuming a potential slip surface, it has been found that the criterion for isolating the stress field is contrary to the criterion for locating the critical slip surface. When assuming no slip surface, we must locate an artificial displacement boundary which separates the region of analysis from infinite backfill domain, and which has an essential meaning in the earth pressure problem. Here two types of boundary condition are investigated. One type assumes that the whole base of backfill is free to move horizontally. The other assumes that only the backfill base adjacent to the wall is horizontally movable. The case studies have verified that these two conditions correspond respectively to classical Rankine and Coulomb methods. Because of the numerical error due to the finite element displacement approach on which the proposed procedure is founded, the procedure tends to provide somewhat greater value of active and passive thrusts than Rankine's lower-bound solution. Thus the proposed procedure gives an appropriate lower-bound solution of active thrust, while the passive thrust by the procedure should be considered as an approximate solution. The procedure has been applied to many case studies where most of the factors affecting active and passive earth pressures are considered and where the provided full information about the pressure are examined. Some of the practical findings are summarized as follows. Though Rankine method considers neither wall friction nor actual boundary condition of wall and backfill movements, it is reasonable to apply the method to the design of vertical standing walls. This is because Rankine's solution for perfectly smooth wall gives somewhat greater value of active thrust than the lower-bound solution by the proposed procedure which considers the wall roughness and practical boundary condition. For the cohesionless backfill, Coulomb method produces the value of active thrust close to the lower-bound solution by the proposed pro-

cedure, regardless of wall friction, wall inclination and backfill slope. And that the slip surface assumed in Coulomb method corresponds closely to the active failure region estimated by the proposed procedure. For the inclining smooth wall, the active pressure distribution obtained by the proposed procedure is considerably different from the triangular distribution assumed usually in the application of Coulomb method, while for the rough wall the difference decreases. The cohesion of backfill material, which is neglected in Coulomb method, reduces substantially the active thrust against some geometries of retaining wall. These results prove the validity of Rankine and Coulomb methods from the mechanical and practical viewpoints.

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