A SIMPLE GAUGE FOR LOCAL SMALL STRAIN MEASUREMENTS IN THE LABORATORY

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ABSTRACT

A simple device named "local deformation transducer (LDT)" was developed to investigate stiffness of soils that can be measured in the laboratory for strain levels ranging from 10^{-6} to 10^{-2} . The theoretical consideration into the non-linear, but uniquely determined, relationship between the gauge strain (i.e., output voltage) and the axial strain is first described. The accuracy of the strain measurement using LDT is then discussed based on the theory and the results of calibration tests. The stability of the output voltage was examined for a few days using a dummy. In isotropic consolidation tests performed on a fine clean sand in triaxial cells, the internal axial strains measured using LDT agreed well with those associated with the condition of no bedding error, which were extrapolated from the results of tests using the specimens with various heights. Besides, in plane strain compression, the internal axial strain measured using the LDT was close to that determined by means of a laser speckle method. The capability of the LDT has been demonstrated in triaxial tests in which the variations of Young's modulus of a gravel, a cement-treated sandy soil and a soft rock were successfully measured for the prescribed range of strains as the specimens were subjected to both monotonic and cyclic loadings.

Key words : bedding error, deformation, laboratory test, small strain, stress-strain relationship, test procedure, triaxial compression test (IGC : D0/D6)

INTRODUCTION

The small strain behaviour of soils is of great importance in various geotechnical engineering practices where the pre-failure

problems are considered. For instance, the predictions of settlements and stress distributions of ground subjected to loadings and excavations demand a proper understanding of the soil stiffness for several orders of

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strains, say, between 10^{-6} and 10^{-2} . The small strain behavior has also been highlighted in soil dynamics.

In triaxial specimens, it is crucial to measure "local" axial strain, by which definition bedding error is to be excluded in the measurements. The term 'local' strain herein implies a sort of averaged strain determined over a certain length of the soil specimen. For example, it has been shown by Jardine et al. (1984) that the stiffness of various stiff soil specimens measured outside the triaxial cell underestimated the true stiffness, to a great extent, for the local axial strains less than 10^{-4} (see also Burland, 1989). Similar observation has been made for soft rocks by Nakano et al. (1987). It has also been pointed out by many researchers that, when the specimens of various soils were monotonically or cyclically sheared, a marked loss in the stiffness was observed at an axial strain of 10^{-4} or less.

During the last decade or so, the accurate measurement of the soil stiffness at small strains has been one of the challenging topics in geotechnical engineering laboratories. In the early stage, miniature displacement transducers were often employed to measure the relative displacement between two reference points on the lateral surface of the triaxial specimen (Brown et al., 1980; Costa Filho, 1985 amongst others). Jardine, Symes and Burland (1984), using an electrolevel gauge (Burland and Symes, 1982), have successfully measured the local axial strain for a wide range between 10^{-5} and 10^{-1} . Clayton et al. (1989) have developed a local strain device which utilizes a Hall effect semiconductor. A similar attempt has been made to make use of proximity transducers (Hird et al., 1989).

In the geotechnical engineering laboratory at Institute of Industrial Science, University of Tokyo, a simple device named 'local deformation transducer (LDT)' has been, for a last few years, under intensive use for measuring local axial deformation of triaxial and plane strain specimens for a wide range of strain between 10^{-6} and 10^{-2} . The concept of the LDT is similar to that of a clip gauge.

ERROR INVOLVING EXTERNAL STRAIN MEASUREMENT

For a triaxial specimen, there are several sources of error involved in the external axial strain measurement, which are;

- (1) system compliance due to the deflection of load cell, top cap, cell, piston and so on,
- (2) tilting of the specimen,
- (3) bedding error on the top and bottom of the specimen,
- (4) strain non-uniformity of the specimen due to end restraints, leading to bulging of the specimen, and
- (5) shear banding (strain localization) of the specimen.

Amongst these, shear banding inevitably occurs even when ideal boundary conditions are applied to the soil element. The system compliance can be avoided by monitoring the movement of the top cap of the specimen (Kokusho, 1980). The tilting could also be avoided by using a non-tilting cap. However, in many cases, the bedding error cannot be perfectly avoided in the current configuration of the triaxial specimen. For example, the effect of disturbed zones formed at both ends of the specimen during the sample preparation may be serious for stiff materials in that the observed stiffness would be greatly underestimated. The use of lubrication layers at both ends of the specimen would lead to a low degree of non-uniformity of stresses and strains in the specimen at the expense of increasing the bedding error. Therefore, it is crucial to measure an averaged local strain over the central part of the soil specimen, which is relatively free from the end restraints.

DEVELOPMENT OF LOCAL DEFORMATION TRANSDUCER (LDT)

Fig.1 shows the dimensions of current version of the LTDs. Each LTD consists of a thin, hence flexible, strip of phosphor bronze and a couple of pseudo-hinged attachments. As shown in Fig.1, four strain gauges, two

Fig. 1. Dimensions of local deformation transducers

Fig. 2. Set-up of local deformation transducer (LDT) using a fixing device

on each side, are glued at the central part of the strip. Fig.2 shows how to set up the LDTs on a triaxial specimen. A couple of pseudo-hinged attachments, which are also made of phosphor bronze, are first adhered to the membrane using a powerful glue (e.g., Permabond #268 adhesive). With the aid of a fixing device, the attachments are vertically aligned. It takes approximately twenty seconds for the glue to mobilise the full strength. The strip is then gently pinched between the two hinges (see Fig.3). As described in the following section, the distance between the two attachments are to be slightly shorter than the length of the strip so that, when the strip is mounted, it balances itself, by its own elastic force, against the fixed attachments. The initial distance between the

two attachments should be determined by regarding the kind of the test to be performed; i.e., in the case of monotonic loading tests in compression and in extension, the distance is to be as close to largest and smallest within the range of operation, respectively.

The working principle of the LDT is such that, when the specimen is deformed, the change in distance between the two attachments triggers the increase (or decrease) of the gauge strain which in turn determines the local axial strain over the gauge length. The axial strain is the average of those measured using a couple of LDTs which are instrumented right opposite to each other (see Fig.2). Fig.3 shows a LDT mounted on a rectangular specimen in plane strain compression. The four square aluminium foils attached on the specimen wall are targets for the proximity transducers to measure the

Fig. 3. Local deformation transducer mounted on a rectangular specimen of 4cm wide, 18cm long and 10.5cm high, with a pair of confining platens set on the σ_2 -planes

Fig. 4. Connection of strain gauges

deformation of the specimen in the horizontal direction.

The strain gauge currently in use has a resistance of 120 ohm with the grid dimension of 0.84mm wide and 2.0mm long (Kyowa KFG-2N-120-C1-16). As shown in Fig. 4, the four strain gauges form a full Wheatstone bridge so that the temperature effect is theoretically minimized in the measurements. Each strain gauge is energized using a constant voltage of 2 volt. The output voltage, V , is magnified using an alternating current amplifier. In the present practice in the Authors' laboratory, the analogue signal is digitized by means of a 12 bits A/D converter installed in a 16 or a 32 bits microcomputer. A single output voltage is the average of twenty readings which are continuously logged at an interval of 30 microseconds.

The early version of the LDT incorporated only two strain gauges adhered on the strip, whereas the other two inactive gauges were attached to a dummy (Goto, 1986; Tatsuoka, 1986). The new version is obviously advan-

tageous, compared to the old version, in that the output gauge strain is doubled and the configuration is much simpler.

THEORY OF LOCAL DEFORMATION TRANSDUCER

The deformation of the LDT is, in sequence of a testing, illustrated in Fig. 5. The symbols of L , l and Δ represent the length of the LDT, the current distance and the relative displacement of the two hinges, respectively; hence, $l = L - \Delta$.

Referring to a x - y coordinate shown in Fig. 5, the bending strain on the surface of a thin elastic strip, ϵ_0 , is obtained in the following form (see Appendix).

$$\epsilon_0 = (\pi t/l) \sqrt{(\Delta/l)} \sin \{\pi x_0/l\} \quad (1)$$

where ' t ' is the thickness of the strip. In the case of the LDT, the value of ϵ_0 corresponds to that at $x_0 = l/2$, so that Δ is related to the measured gauge strain in the following equation.

$$\Delta = \{l^3/(\pi t)^2\} \epsilon_0^2 = \{(L-\Delta)^3/(\pi t)^2\} \epsilon_0^2 \quad (2)$$

Note that, if the strain gauges are adhered slightly off the center of the strip (i.e., $x_0 \neq l/2$), the equation (2) is then only approximate. However, the form of the relation between Δ and ϵ_0 is still precise. From Eq. (2), the sensitivity of the axial strain measurement using the LDT, compared to the local axial strain $d(\Delta/l)$, is obtained as;

$$d(\epsilon_0)/d(\Delta/l) = \{\pi t/(2l)\} / \sqrt{\Delta/l} \quad (3)$$

Fig. 5. Modes of deformation of local deformation transducer

In the present practice in the Authors' laboratory, the value of Δ/l at the commencement of shearing is in the order of 10^{-2} . For the dimensions of LDTs shown in Fig.1, the sensitivity of the four-gauge LDTs (see Fig. 4), $4 \times d(\epsilon_0)/d(\Delta/l)$, is about 0.14. This means that the sensitivity of the axial strain measurement using LDT is worse by less than an order than that of the axial of the specimen. Such a minor loss of the sensitivity may be tolerated when considering that the use of strain gauges for a direct measurement of the axial strain of the specimen is restricted only to hard rocks. Note also in Eq. (3) that the sensitivity of the axial strain measurement using LDT increases as the value of Δ/l decreases. Therefore, it is recommended to have the value of Δ/l to be smallest possible as the LDT is set-up.

An appropriate thickness of the strip is to be determined as a result of compromise between the sensitivity and the possible creep of the hinges. Namely, the sensitivity is better for the larger ' t '. On the other hand, the increase in the thickness inevitably involves the increase of the contact force acting at the hinges, together with the increase in the weight of the LDT itself, which in turn may encourage the creep of the hinges. For the dimensions of LDTs shown in Fig.1, the contact force was about 280gf (2.9N) and 118gf (1.2N) for the LDTs with $L=500$ mm and $L=90$ mm, respectively. The contact force measured was more or less constant with the increasing value of Δ/L , say, up to 10^{-1} .

Eq. (2) indicates that the value of Δ is proportional to the square of ϵ_0 only when Δ/L is small. As described later on, such an intrinsic nonlinearity between Δ and ϵ_0 as expressed in Eq. (3) can easily be overcome by having different calibrations for different scopes of Δ .

DISCUSSION

Accuracy of Strain Measurement : The accuracy of the strain measurements using the LDT depends on the following several sources of error; i.e.,

Fig. 6. A method for calibration of LDTs using a triaxial cell

- (1) resolution of the transducer system,
- (2) resolution of the A/D converter,
- (3) calibration characteristics (e.g., hysteresis)
- (4) environmental disturbance (temperature change, vibration and etc.),
- (5) adequacy of the gauge length,
- (6) performance of the glue used,
- (7) variation of the elastic property of the membrane, and
- (8) slip between the membrane and the soil specimen.

The items from (1) to (4) are sources of error common in any transducers, however the rest is rather specific to the LDT. The majority of these are discussed in the follow-

Fig. 7. A calibration characteristic of a LDT with 500 mm long and 1.2mm thick

ings.

Fig. 6 is a schematic view of calibrating the LDTs in a conventional triaxial cell. The relative displacement of the attachments (Δ) is measured using a proximity transducer or a micrometer with a resolution of 1 micrometer. Typical calibration characteristic of a LDT is shown in Fig. 7, in which Δ' represents the value of Δ at the start of calibration, and the symbol V_0 denotes the output voltage corresponding to the state of no gauge strain ($\epsilon_0=0$, see the upper sketch in Fig. 5). As suggested by Eq. (2), the observed correlation between $(V-V_0)^2$ and Δ is virtually linear. It should also be noted that the hysteresis is negligible. This is a crucial characteristic so as to measure the stiffness and damping ratio of soils when subjected to cyclic loadings.

In order to measure small strains at the commencement of shearing, the output voltage is usually amplified by 5 times, or even more. Fig. 8 shows an example for another LDT with the length (L) equal to 90.0 mm. Since Δ is not exactly proportional to $(V-V_0)^2$, the calibrations obtained using different degrees of amplification as shown in Figs. 8(a) and (b) are not identical to each other.

The present 12 bits A/D converter has a capacity of input voltage of ± 5 volt so that the resolution of the logging system is $\pm 2.44 \times 10^{-3}$ volt. Therefore, the resolutions of the LDT with the length of 90 mm is $\pm 0.6 \mu\text{m}$ and $\pm 0.12 \mu\text{m}$ for two cases shown in Fig. 8 (a) and (b), respectively. These correspond to the resolutions of the axial strain measurement of 7×10^{-6} and 1.3×10^{-6} , respectively. In the present logging system, the electrical noise from the amplifier and the others is minimized by having a filter in the circuit. Further, the analogue signal, when converted to digital signal, is averaged by means of stacking. Therefore, the fluctuation of the output voltage observed in a temperature controlled room was not worse than the resolution of the logging system.

Fig. 9 is a sketch of the contact between the LDT and a pseudo-hinged attachment.

Fig. 8. Calibration curves of a LDT (90 mm long and 0.2 mm thick); (a) the overall range of Δ and (b) an initial small range of Δ

Fig. 9. Gauge length of LDT

The axial strain, ϵ_a , of the specimen is obtained as

$$\epsilon_a = -\ln \{1 - (\Delta - \Delta_1)/GL\} \quad (4)$$

in which Δ_1 is the value of Δ at the instant

Fig. 10. Long-term stability of the measurements of a LDT with $L=90$ mm

when $\varepsilon_a=0$ is defined. The gauge length, GL , is equal to $L-A_1-B$ in which ' B ' denotes the length of the attachment.

One may consider that the gauge length defined at the center of each attachment is inappropriate. In an extreme case when the gauge length is defined as GL' equal to $GL+B$ (see Fig.9), for the configurations of LDTs with the values of GL'/GL equal to 1.02 and 1.04 (see Fig.1), the axial strain underestimates those calculated using the gauge length of GL by 2 and 4 in percent points, respectively. In spite of these small values, it is obvious that the shorter the length of the attachment the better to avoid the ambiguity for the definition of axial strain.

The long-term stability of the output reading of a LDT has been examined in a couple of calibration tests. In these tests, a 90 mm long LDT was adhered to a steel dummy in a pressurized triaxial cell with a constant pressure of 1 kgf/cm^2 ($=98.1 \text{ kN/m}^2$). The output voltage of the LDT, together with temperature of the cell water, was continuously monitored over a period of about three days. In Fig.10, the deformation of the LDT, together with the variation in temperature is plotted against time. As sketched in the figure, the details of the contact between the dummy and the hinges were dif-

ferent between the two tests. 'Type 1' refers to the case in which the hinged attachment was directly glued to a steel dummy, whereas it was adhered on a flake of 0.3 mm thick membrane in the case of 'Type 2'. It should be noted that i) in both cases, the distance between the ends of the LDT steadily increased with time, ii) the rate of creep was higher for 'Type 2' (about $1 \mu\text{m}$ per day) than for 'Type 1' (about $0.3 \mu\text{m}$ per day) and iii) the output voltage was scarcely affected by the change in temperature as observed in the beginning of 'Type 1' test. In the 'Type 1' test, the change in distance between the ends of the LDT could possibly be due to the creep of the glue, whereas it may be attributed to the creep of both glue and the membrane in the other test. Considering the period required for a standard soil testing, it may be pointed out that the time effect is negligible.

Features of Local Deformation Transducer : The LDT is simple and inexpensive to manufacture, and compact enough to be used in a conventional triaxial cell. Phosphor bronze is rustless even in water and it has a high degree of elasticity (c.f., Young's modulus is $1.2 \times 10^6 \text{ kgf/cm}^2$). By coating the strain gauges with Silicone rubber, the LDT can be used in a water filled cell. Since the LDT

measurement using the LTDs, two series of comparative tests have been carried out on dense Toyoura sand specimens, prepared using an air-pluviation method. In all the tests, lubrication was made at the top and bottom of the specimen by means of a latex membrane with $200\text{ }\mu\text{m}$ thick, together with a layer of Dow grease each having a controlled initial thickness of $50\text{ }\mu\text{m}$ (Tatsuoka et al., 1984). An isotropic consolidation that was performed by increasing the cell pressure σ'_c from 0.1 to 0.8 kgf/cm^2 using a small size specimen with the nominal dimension of 7 cm in diameter and 15 cm high. The axial strain was measured both internally and externally using a set of LTDs and a couple of proximity transducers mounted above the top cap. Fig. 11 shows another set up of a large size triaxial specimen with the nominal dimension of 30 cm in diameter and 63 cm in height (Goto, 1986). The overall (=external) axial strain was measured using two proximity transducers, No.1 and No.2, which were instrumented above the top platen. Each of the proximity transducers of No.1, 2, 3 and 4 measured the vertical displacement at the corresponding point of the specimen. In the test using a large triaxial specimen, the cell pressure σ'_c was increased in steps from 0.1 up to 0.8 kgf/cm^2 . In Fig.12(a), the internal axial strain of a large specimen is assumed to be equal to the extrapolated one at $\sigma'_c =$

0.2 kgf/cm^2 (i.e., at point 'A' in the figure).

The test results are shown in Fig.12(a), in which the axial strains obtained from the two tests are plotted against σ'_c . Owing to bedding error, the axial strain measured externally was almost twice higher than those of the local axial strains throughout testing. It may be noted that, irrespective of σ'_c , the changes of the local axial strains determined using the proximity transducers (No.3, 4, 5 and 6) were slightly smaller than those determined using the LTDs for a small specimen.

In addition to these tests, a series of isotropic compression tests were performed on various sizes of specimens (Fig.12(b)). From the test results, the correct axial strain which is free from the bedding error, was estimated by extrapolating the results to the infinite specimen height. These values are also plotted in Fig.12(a). Note that, except for the low pressure levels, the changes of the correct axial strains are remarkably close to those measured using the LTDs.

By having both the external and internal axial strains, $(\epsilon_a)_e$ and $(\epsilon_a)_i$ in a test, the overall bedding error at the top and bottom of the specimen, ΔBE , was estimated as $\{(\epsilon_a)_e - (\epsilon_a)_i\}H$, where 'H' is the height of the specimen. In a plane strain compression test performed on a dense specimen of Toyoura sand, the value of ΔBE was calculated

Fig. 13. Bedding error estimated based on the axial strain measurements of LTD and laser speckle method

(b)

(c)

(d)

Fig. 14. Stress-strain relationship of a sedimentary soft rock (mudstone) in a drained triaxial compression test; (a) Maximum scale for ϵ_a equal to 3%, (b) 0.1%, (c) 0.01% and (d) 0.001%

from the internal axial strain measured on the lateral surface of the specimen by means of laser speckle method (Tatsuoka, et al., 1990a). The accuracy of the displacement measurement using this method was approxi-

mately 5 micrometers. The variation of ΔBE , using similar end conditions, has also been estimated in another plane strain compression test in which local axial strain was measured using a set of LDT. The comparison of the test results is made in Fig. 13. A good agreement was observed amongst the three tests in the magnitude of bedding error estimated using the two different techniques to determine $(\epsilon_a)_t$.

Stiffness of Various Geotechnical Materials Measured Using LDT: The performance of

(a)

(b)

Fig. 15. Soil response to cyclic loadings at small strains; (a) Hime gravel in a drained triaxial compression test and (b) cement-treated sandy soil in an undrained triaxial compression test (Tatsuoka et al., 1990b and Shibuya et al., 1990b)

the LDTs with L equal to 500 mm and 90 mm has been demonstrated in the stiffness measurements of Hime gravel ($D_{50}=1.85$ mm) and a cement-treated sandy soil when subjected to monotonic loadings (Tatsuoka et al., 1990b, Shibuya et al., 1990a).

Another example is shown in Fig.14. in which the results of a consolidated undrained triaxial compression test performed on a natural sedimentary rock (mudstone) are presented using various maximum scales for ϵ_a . In this test, a sheet of nylon mesh was used as a drain layer at each end of the specimen. The difference in the stress-strain relationship was remarkable between the internal and external measurements. A large degree of divergence may be seen for this stiff material due mostly to the deformation of the disturbed (or loosened) layers formed at the ends of the specimen during the process of sample trimming. Similar degree of divergence was observed even in an unconfined compression test in which the specimen was capped with gypsum. As can be seen in Fig.14(d), the resolution of the LDT is better than 10^{-6} . The initial linearity in the stress-strain relation can be rather objectively determined for the axial strain less than 10^{-5} .

The test results suggest that the LDT is a useful tool to measure the stiffness of soils, when subjected to both cyclic and monotonic loadings. Therefore, the elastic property of soil can be investigated in a monotonic loading test by applying the small unloading and reloading cycles at various strain levels. Fig.15 shows the stress-strain curves in the small strain regions observed in triaxial compression tests on Hime gravel and a cement-treated sandy soil (Tatsuoka et al., 1990b). The small unloading and reloading cycles were applied rather statically (i.e., a minute for a cycle). Shibuya et al. (1990b) also showed that the stiffness and damping of Hime gravel determined using the LDT yielded little scatter with the increasing number of cycles for the axial strains ranging from 10^{-5} to 10^{-4} . They also showed that the maximum Young's modulus E_{max} obtained in the monotonic loading test was very close

to that of the corresponding cyclic loading test. Further, the value of the maximum Young's modulus observed in the tests on mudstone and a cement-treated sandy soil (see Figs.14(d) and 15(b)) was very close to that obtained from the in-situ seismic survey.

CONCLUSIONS

A simple device named 'local deformation transducer (LDT)' has been developed to measure the average local strain of soil specimens when sheared in the laboratory. The overall accuracy of the strain measurement has been found to be approximately 10^{-6} . By using the LDT, it is possible to measure the local strain for a wide range between 10^{-6} and 10^{-2} , which enables the determinations of both the initial stiffness and the peak strength of soils in a test. The LDT is also capable of measuring altering stiffness of soils when subjected to both monotonic and cyclic loadings.

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APPENDIX : Theory of Local Deformation Transducer; Derivation of Eq. (1)

The theoretical consideration is based on a small strain theory of elasticity for a hinged thin column subjected to the axial force. For the x - y coordinate shown in the middle sketch in Fig.5, the length of the column is obtained from the integration from $x=0$ to $x=l$;

$$L = \int \sqrt{1 + (dy/dx)^2} dx \quad (A-1)$$

For a small deformation of the column, Δ is given as

$$\Delta = L - l = \int \sqrt{1 + (dy/dx)^2} dx - l$$

$$\sim \int \{1 + (1/2)(dy/dx)^2\} dx - l$$

$$= (1/2) \int (dy/dx)^2 dx \quad (\text{A-2})$$

The value of $(dy/dx)^2$ is zero at $x=0$ and $x=l$. A mode of deformation is assumed in the following sine function.

$$y(x) = a \sin(\pi x/l) \quad (\text{A-3})$$

where 'a' is a coefficient. By differentiating Eq. (A-3),

$$dy/dx = a(\pi/l) \cos(\pi x/l) \quad (\text{A-3})'$$

$$d^2y/dx^2 = -a(\pi/l)^2 \sin(\pi x/l) \quad (\text{A-3})''$$

By substituting dy/dx shown in Eq. (A-3)' into Eq. (A-2), Δ or a constant 'a' is obtained as

$$\Delta = (a\pi)^2/(4l) \text{ or } a = 2\sqrt{\Delta l}/\pi \quad (\text{A-4})$$

The bending moment, M , in the column is

$$M = -EI(d^2y/dx^2) \quad (\text{A-5})$$

The bending moment at a point of x_0 is

$$M_0 = EI(\pi/l)^2 a \sin(\pi x_0/l) \quad (\text{A-6})$$

On the other hand, the tangential stress at a point of x_0 on the surface of the column is

$$\sigma_0 = (M_0 t)/(2I) \quad (\text{A-7})$$

where 't' is the thickness of the thin column. Equations (A-4), (A-6) and (A-7), together with Hook's law, give the tangential strain, $\epsilon_0 = \sigma_0/E$, to be finally in the following form.

$$\epsilon_0 = (\pi t/l) \sqrt{\Delta/l} \sin(\pi x_0/l) \quad (1)$$

REFERENCES

- 1) Brown, S. F., Austin, G. and Overy, R. F. (1980): "An instrumented triaxial cell for cyclic loading of clays," *Geotechnical Testing Journal*, ASTM, Vol. 3, No. 4, pp. 145-152.
- 2) Burland, J. B. and Symes, M. J. (1982): "A simple axial displacement gauge for use in the triaxial apparatus," *Geotechnique*, Vol. 32, No. 1, pp. 62-65.
- 3) Burland, J. B. (1989): "Small is beautiful-the stiffness of soils at small strains," Ninth Laurits Bjerrum Memorial Lecture, *Canadian Geotechnical Journal*, Vol. 26, pp. 499-516.
- 4) Clayton, C. R. I., Khatrush, S. A., Bica, A. and Siddique, A. (1989): "The use of hall effect semiconductors in geotechnical instrumentation," *Geotechnical Testing Journal*, ASTM, Vol. 12, No. 1, pp. 69-76.
- 5) Costa Filho, L. de M. (1985): "Measurement of axial strains in triaxial tests," *Geotechnical Testing Journal*, ASTM, Vol. 8, No. 1, pp. 3-13.
- 6) Goto, S. (1986): "Strength and deformation characteristics of granular materials in triaxial tests," Doctoral thesis, University of Tokyo.
- 7) Hird, C. C. and Yung, C. Y. (1989): "The use of proximity transducers for local strain measurements in triaxial tests," *Geotechnical Testing Journal*, ASTM, Vol. 12, No. 4, pp. 292-296.
- 8) Jardine, R. J., Symes, M. J. and Burland, J. B. (1984): "The measurement of soil stiffness in the triaxial apparatus," *Geotechnique*, Vol. 34, No. 3, pp. 323-340.
- 9) Kokusho, T. (1980): "Cyclic triaxial test of dynamic soil properties for wide strain range," *Soils and Foundations*, Vol. 20, No. 2, pp. 45-60.
- 10) Nakano, R., Shimazaki, F. and Shimizu, H. (1987): "Anisotropic mechanical properties of diatomaceous mudstone-with special reference to tensile Young's modulus and mechanical properties of fault clay of tuffaceous mudstone in relation to tunneling," *Proc. of 7th Japan Symposium on rock mechanics*, pp. 67-72.
- 11) Shibuya, S., Tatsuoka, F. and Kong, X. J. (1990a): Discussion, *Proc. of 12th. Int. Conf. on SMFE*, Rio de Janeiro, Vol. 4 (to be published).
- 12) Shibuya, S., Kong, X. J. and Tatsuoka, F. (1990b): "Deformation characteristics of gravels subjected to monotonic and cyclic loadings-with particular reference to their small strain behaviour," *Proc. of 8th Symposium of Japanese Society of Earthquake Engineering* Vol. 2, pp. 771-776.
- 13) Tatsuoka, F., Molenkamp, F., Torii, T. and Hino, T. (1984): "Behavior of lubrication layers of platens in element tests," *Soils and Foundations*, Vol. 25, No. 1, pp. 113-128.
- 14) Tatsuoka, F. (1986): "Some recent development in triaxial testing systems for cohesionless soils," *ASTM STP 977*, pp. 7-67.
- 15) Tatsuoka, F., Nakamura, S., Huang, C.-C. and Tani, K. (1990a): "Strength anisotropy and shear band direction of sand in plane strain," *Soils and Foundations*, Vol. 30, No. 1, pp. 35-54.
- 16) Tatsuoka, F., Shibuya, S., Goto, S., Sato, T. and Kong, X. J. (1990b): Discussion on the Paper by C. R. I. Clayton, S. A. Khatrush, B. V. D. Bica and A. Siddique, *Geotechnical Testing Journal*, Vol. 13, No. 1, pp. 63-65.