SEEPAGE–FAILURE EXPERIMENTS ON MULTI–LAYERED
SAND COLUMNS

—Effects of Flow Conditions and Residual Effective Stress
on Seepage–Failure Phenomena—

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ABSTRACT

An apparatus was designed for one-dimensional seepage-failure tests. The apparatus consists of seepage cylinder, constant-head device, and open piezometers. Using the apparatus, a series of seepage-failure experiments on the one- and two-layered sand columns was carried out. It was found that the flow condition: (i) $\frac{k_D}{k} > 15$ (exerted-pressure type) or (ii) $1 < \frac{k_D}{k} < 5$ (uniform-flow type) is one of the most important factors in the seepage failure of sand columns, where $k_D$ and $k$ are the coefficients of permeability of the rectifying filter and the sand layer, respectively. The following results were then obtained:

(1) The modes of failure of sand columns are able to be predicted theoretically from the residual effective stress diagrams.
(2) The one-layered sand column (with no loaded filter) collapses in quite different ways according to the above two flow conditions.
(3) In the case of the flow condition of the exerted-pressure type, a multi-layered sand column collapses at the smaller hydraulic head difference than that in the uniform-flow type.
(4) When the relative density $D_r$ is small almost all the one-layered sand columns (with no loaded filter) collapse at the smaller hydraulic gradient than the theoretical critical one.
(5) As the residual effective stress within a sand column increases, the friction between the vertical cylindrical surface and sand particles becomes large, and thus the resistance to failure of the sand column increases. As a result the sand column collapses at the larger hydraulic head difference than the theoretical critical one.
(6) The effect of the cylindrical surface friction on the critical hydraulic head difference of a sand column is able to be estimated from a magnitude of the residual effective stress.

Key words: at rest pressure, failure, filter, heaving, laboratory test, layered system, sand, seepage, test equipment, test procedure (JGC: E7)

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INTRODUCTION

When an excavation is made in a water-bearing ground as shown in Fig. 1, the seepage failure of the ground behind a sheet pile wall caused by a vertically upward seepage flow is a problem. A soil column shown in Figs. 2 and 3 is often used to model such a ground (Tanaka et al., 1982). In this paper, the one-dimensional seepage failure problem is discussed.

Many experimental studies have been made on the seepage failure problem of a soil column. In the one-dimensional seepage failure problem, the following three approaches are mainly presented:

1) critical velocity method (Justin, 1923),
2) critical hydraulic gradient method (Terzaghi, 1925), and
3) internal effective stress method (Sawada et al., 1980a, 1980b; Tanaka and Hasegawa, 1981; Tanaka et al., 1982).

From the viewpoint of the critical velocity, Sugii et al. (1989) experimented on seepage failure of the one-layered sand columns with no loaded filter and discussed the failure process and the effects of engagement of sand particles, and proposed a new theoretical equation of the critical velocity, which has good agreement with experimental results.

As to the critical hydraulic gradient, Terzaghi (1925) conducted a series of tests on the same problem, and obtained the following results on a safe side:

$$\frac{i_{c_m}}{i_{c_t}} = 1.00 - 1.24,$$

where $i_{c_m}$ is the critical hydraulic gradient obtained by the experiment, and $i_{c_t}$ is the theoretical one. Herzog (1938) also made vigorously seepage-failure experiments on sand columns, and took into consideration the entrance head loss, the mode of failure of a sand column, the swelling of a sand stratum near the critical time, and other factors. In recent years, Saito and Miki (1973) and Inada...
and investigated the effects of void ratio, property of soil, etc., on the resistance to movement of soil particles. Källin (1977) emphasized, from the theoretical point of view, the importance of the minimum criterion in defining the safety factor for seepage failure by bulk heave of sand columns. He also made experiments on seepage failure of the three-layered sand column with no loaded filter and the continuous inhomogeneous sand column, and investigated the mode and process of failure of the sand columns. He concluded that the experiments on the sand columns support the theory regarding the depth at which failure initiates.

From the standpoint of the internal effective stress, in a piece-wise homogeneous sand column, an effective stress remains in the sand layers at the critical time, e.g., as shown in Figs. 4 (b) and (c). It is thought that the residual effective stress has an effect on the stability of the sand column at the critical time and the mode of failure of the sand column (Tanaka and Hasegawa, 1981). It is considered that the flow condition has also an effect on the mode of failure of the sand column, the critical hydraulic head difference, etc. The effects of the residual effective stress on the stability for seepage failure of the sand column have not been, however, clarified by experiments. Here we will discuss the following three factors which have an effect on the mode of failure and the resistance to seepage failure of the sand column from experimental results:

1) the flow conditions,
2) the residual effective stress,
3) the cylindrical surface friction.

Therefore, the authors made a series of seepage-failure experiments on the following three types of sand columns (shown in Fig. 4), and investigated the above-mentioned effects and other theoretical findings (Sawada et al., 1980a, 1980b; Tanaka et al., 1982):

(1) One-layered sand column \( p=0 \): the one-layered sand column (with no loaded filter) shown on the left hand side in Fig. 4(a).

Where \( p \) is the effective overburden pressure of the loaded filter which is placed on the top
of the sand column.

(2) One-layered sand column \((p>0)\): the one-layered sand column (with a loaded filter) shown on the left hand side in Fig. 4(b).

(3) Two-layered sand column \((k\gamma_r'>1, p=0)\): the two-layered sand column \((k\gamma_r'>1, \text{with no loaded filter})\) shown on the left hand side in Fig. 4(c).

A loaded filter is often referred to as an inverted filter, and is defined as a material being coarse enough to permit the free outflow of the seepage water, but fine enough to prevent the escape of base soil particles through its voids (Terzaghi and Peck, 1967).

The notations \(k_r\) and \(\gamma_r'\) are the ratios of the coefficient of permeability and the submerged unit weight of the upper layer to ones of the lower layer of the two-layered sand column, respectively. In Figs. 4(a), (b) and (c), the schematic sketch on the left hand side and the vertical effective normal stress diagrams in the initial state and the critical state on the right hand side are shown for the cases of the one-layered sand column \((p=0)\), the one-layered sand column \((p>0)\) and the two-layered sand column \((k\gamma_r'>1, p=0)\), respectively.

As shown in Fig. 3, layers of a piece-wise homogeneous sand column (the \(n\)-layer case) are named Stratum 1, Stratum 2, \(\ldots\ldots\), Stratum \(n\) upward from the bottom layer. The following notations are used here;

\[ h = \text{difference in total hydraulic head between the topmost level and the bottom level of the sand column} \]
\[ z = \text{depth from the topmost level of the sand column (excluding a loaded filter)} \]
\[ l_j = \text{thickness of Stratum } j \ (j=1, 2, \ldots, n) \]
\[ \gamma_j' = \text{buoyant unit weight of Stratum } j \ (j=1, 2, \ldots, n) \]
\[ k_j = \text{coefficient of permeability of Stratum } j \ (j=1, 2, \ldots, n) \]
\[ i_j = \text{hydraulic gradient of Stratum } j \ (j=1, 2, \ldots, n) \]
\[ p = \gamma_r'l_r, \text{ effective overburden pressure of a loaded filter} \]
\[ \gamma_r' = \text{buoyant unit weight of a loaded filter} \]
\[ l_r = \text{thickness of a loaded filter} \]

**Definition of the Critical State for Seepage Failure of a Sand Column**

The general definition of the “critical state for seepage failure of a sand column” is given as the state of the sand column at the critical time when an effective stress becomes, first and foremost, zero in a certain layer or at a certain level (but at the topmost level) within the sand column as \(h\) increases. The residual effective stress (diagram) is then referred to as the effective stress (diagram) at the critical time, and \(h_e\) is also referred to as the critical difference in total hydraulic head between the topmost level and the bottom level of the sand column. Where

\(\ldots\ldots\)|\(e\): value at the critical time,

the same notation will be used throughout the paper.

**DESCRIPTION OF APPARATUS, TEST MATERIALS, PREPARATION OF SPECIMENS AND TEST PROCEDURE**

**Test Apparatus**

An apparatus was designed for seepage failure tests shown in Fig. 5. The apparatus consists of the three parts: (1) seepage cylinder, (2) constant-head device, and (3) open piezometers as described as follows:

1. **Seepage cylinder**

   The permeameter is a vertical Acryl seepage cylinder, 385 mm high and 151 mm in inside diameter, with 8 piezometer holes of 5 mm in inside diameter spaced at 50 mm. Seepage water flows up from the bottom of the seepage cylinder. The water which percolates through a soil specimen flows out through a drain valve of 20 mm in inside diameter. The downstream water level is, therefore, kept constant at the valve.

2. **Constant-head device**

   The constant-head device consists of two Acryl cylinders. One of them is a constant head overflow tube, 300 mm long and 150 mm in inside diameter. The other is a constant head stabilization tube, 1000 mm long and 150 mm in inside diameter. The upstream water head is given to a specimen stepwise by raising up the former, and is kept constant stably by
the latter.

Seepage water flows through a specimen under the difference in water head between the downstream water level at the drain valve of the seepage cylinder and the upstream water level kept constant by the constant head stabilization tube, $H$.

(3) Open piezometers

The open piezometers are Acryl pipes, 1500 mm long and 5 mm in inside diameter. These are connected to the piezometer holes at the wall of the seepage cylinder.

**Table 1. Physical properties of test materials**

<table>
<thead>
<tr>
<th>Material</th>
<th>$G_s$</th>
<th>$c_{\text{max}}$</th>
<th>$c_{\text{min}}$</th>
<th>$U_e$</th>
<th>$D_{50}$ (mm)</th>
<th>$k_{15}$ (cm/s) at $D_r=50%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand 1 (Naka-Umi sand)</td>
<td>2.666</td>
<td>1.309</td>
<td>0.794</td>
<td>1.80</td>
<td>0.170</td>
<td>0.0276</td>
</tr>
<tr>
<td>Sand 2 (Lake-Biwa Sand)</td>
<td>2.652</td>
<td>1.102</td>
<td>0.688</td>
<td>1.64</td>
<td>0.310</td>
<td>0.1146</td>
</tr>
<tr>
<td>Sand 3 (River sand)</td>
<td>2.620</td>
<td>1.134</td>
<td>0.782</td>
<td>1.27</td>
<td>0.872</td>
<td>0.397$\sim$0.718 (average 0.521)</td>
</tr>
</tbody>
</table>

**Test Materials**

Three uniform sands used in a series of experiments are following:

1. Naka-Umi Sand: Sand 1 is the sand used for an embankment (the reclamation dike at Iya district), a part of the Naka-Umi Polder Reclamation Project, Shimane Prefecture.

2. Lake-Biwa Sand: Sand 2 is the sand deposited outside the Dainakano-Ko Reclamation dike in the east of Lake Biwa, Shiga Prefecture.

3. River Sand: Sand 3 is the river granitic sand with a grain size range of 590 $\mu$m to 2000
The physical properties and the grain-size distribution curves of these test materials are shown in Table 1 and Fig. 6, where the maximum and minimum void ratios $e_{\text{max}}$ and $e_{\text{min}}$ are obtained using the method proposed by JSSMFE (1980). $k_{15}$ is the coefficient of permeability at the relative density $D_r = 50\%$ which is calculated into the value at 15°C.

**Preparation of Specimens**

The seepage cylinder was filled with water. The three brass mesh screens (openings 74 $\mu$m, 420 $\mu$m and 840 $\mu$m from the bottom) were then laid on the bottom percolated plate in the seepage cylinder, and a boiled and deaired filter sand about 80 mm thick was placed on them to rectify the seepage flow. The sand filter is referred to as the "rectifying filter", through which water flows uniformly into a specimen. A brass mesh screen (an opening 420 $\mu$m or 840 $\mu$m) was laid on the rectifying filter as a protective filter for the upper two test sands. Sand 3 also fills the requirements as a loaded filter for Sand 1.

![Fig. 6. Grain size distribution curves of test sands](image)

![Fig. 7. Test specimens. (a) A one-layered sand column (with no loaded filter); (b) A one-layered sand column (with a loaded filter), or a two-layered sand column ($k_r \gamma_r > 1$, with no loaded filter)](image)
filter, and a boiled and desired test sand was poured into the seepage cylinder little by little. The test sand was put into by 5~10 layers, and each layer was compacted by 0~100 drops of an aluminium compacting rod to a desired density. (The compacting rod has an outside diameter of 10 mm, a length of 494 mm, a mass of 100 g and its drop height of 3~5 cm.) The lengths of the sand column were measured at 4 portions around the seepage cylinder and averaged. In this way a specimen of a sand column (with no loaded filter) was completed as shown in Fig. 7(a). The seepage failure test of the one-layered sand column \((p=0)\) was started from this state.

In the case of the one-layered sand column \((p>0)\) or the two-layered sand column \((k_i\gamma'_i>1, p=0)\), a loaded filter or another sand layer was directly settled on the one-layered sand. Pouring the upper sand material into the seepage cylinder, compacting it by the compacting rod, and measuring its length, a one-layered sand column (with a loaded filter) or a two-layered sand column \((k_i\gamma'_i>1, p=0)\) was completed as shown in Fig. 7(b). The seepage failure test of the one-layered sand column \((p>0)\) or the two-layered sand column \((k_i\gamma'_i>1, p=0)\) was started from this state.

In the one-layered sand column \((p=0)\), Sand 1 was used as a sand layer, and was compacted to various densities. Test sands of the one-layered sand column \((p>0)\) and the two-layered sand column \((k_i\gamma'_i>1, p=0)\) were compacted to \(D_r\approx 50\%\). In the one-layered sand column \((p>0)\), Sand 1 and Sand 3 were used as a sand layer and a loaded filter, respectively, in which the ratio of the coefficient of permeability of the loaded filter \(k_r\) to one of the sand layer \(k\); \(\frac{k_r}{k}\) was 20~30. In the two-layered sand column \((k_i\gamma'_i>1, p=0)\), Sand 1 and Sand 2 were used as the lower sand layer (Stratum 1) and the upper sand layer (Stratum 2), respectively, in which \(k_r=k_2/k_1\approx 4.15\), where \(k_1\) and \(k_2\) are the coefficients of permeability of Stratum 1 and Stratum 2 of the two-layered sand column, respectively.

**Test Procedure**

The specimen prepared shown in Fig. 7(a) or Fig. 7(b) was subjected to the greater hy-

| Table 2. Sequence of seepage-failure tests of the one- and two-layered sand columns |
|-------------------------------------|------------|-------------|---------------|---------------------|--------------|
| **Test name** | **Flow condition** | **l** | **Cases** | **Specimen** | **Rectifying filter** |
| Test a | Exerted-pressure type | 1 ≤ 15 cm | 27 | Sand 1 | Sand 3 |
| Test b | Uniform-flow type | 1 ≤ 15 cm | 29 | Sand 1 | Sand 2 |

| **One-layered sand column \((p>0)\)** |
|-------------------------------------|------------|-------------|---------------|---------------------|--------------|
| **Test name** | **Flow condition** | **l** | **Cases** | **Sand layer** | **Loaded filter** | **Rectifying filter** |
| Test (a) | Exerted-pressure type | 1 ≤ 10 cm | 19 | Sand 1 | Sand 3 | Sand 3 |
| Test (b) | Uniform-flow type | 1 ≤ 10 cm | 22 | Sand 1 | Sand 3 | Sand 2 |
| Test (c) | Uniform-flow type | 1 ≥ 15 cm | 18 | Sand 1 | Sand 3 | Sand 2 |

| **Two-layered sand column \((k_i\gamma'_i>1, p=0)\)** |
|-------------------------------------|------------|-------------|---------------|---------------------|--------------|
| **Test name** | **Flow condition** | **l** | **Cases** | **Upper layer** | **Lower layer** | **Rectifying filter** |
| Test (a) | Exerted-pressure type | 1 ≤ 10 cm | 25 | Sand 1 | Sand 2 | Sand 3 |
| Test (b) | Uniform-flow type | 1 ≤ 10 cm | 25 | Sand 1 | Sand 2 | Sand 2 |
| Test (c) | Uniform-flow type | 1 ≥ 15 cm | 21 | Sand 1 | Sand 2 | Sand 2 |
drainage head difference stepwise using the constant head overflow tube, and was finally led to collapse. An increment of hydraulic gradient \( i \); \( \Delta i \) was given at each step as follows: \( \Delta i = 0.1 \sim 0.2 \) in the beginning (for \( i < 0.85i_{c} \)), and \( \Delta i = 0.008 \sim 0.015 \) near the critical state (for \( i > 0.85i_{c} \)) which is about 1% of the critical hydraulic gradient \( i_{c} \). At each step, the constant-head permeability test was carried out, i.e., the discharge velocity at 15°C; \( q_{15} \) and the hydraulic gradient \( i \) were measured, and the coefficient of permeability of the specimen \( k_{15} \) was then calculated.

A series of tests shown in Table 2 was carried out here. In Test a and Test b of the one-layered sand column (\( p = 0 \)), experiments on specimens with various values of void ratio \( e \) were carried out, in Test (a), Test (b) and Test (c) of the one-layered sand column (\( p > 0 \)) with various values of \( p \), and in Test (a), Test (b) and Test (c) of the two-layered sand column (\( k_{r} > 1, p = 0 \)) with various values of \( l \).

**MODE OF FAILURE OF SAND COLUMNS**

*Theoretical Prediction*

Changes in the vertical effective normal stress diagram in the one-layered sand column (\( p = 0 \)), the one-layered sand column (\( p > 0 \)) and the two-layered sand column (\( k_{r} > 1, p = 0 \)) are shown on the right hand side in Figs. 4(a), (b) and (c), respectively. In Fig. 4(a), the values of the vertical effective normal stresses at the top and the bottom of the sand column are shown, and in Figs. 4(b) and (c), the values of the vertical effective normal stresses at the top, the boundary point of the two layers, and the bottom of the sand columns are shown. From the residual effective stress diagrams shown in Figs. 4(a), (b) and (c), modes of failure of these sand columns at a time when \( h \) exceeds its critical value \( h_{c} \), were predicted theoretically as follows (Tanaka et al. 1982):

1. **One-layered sand column (\( p = 0 \))**

   Changes in the vertical effective normal stress in the case of the one-layered sand column (\( p = 0 \)) up to its critical state are shown in Fig. 4(a), where

\[
\gamma_{w} = \text{unit weight of water} \\
\gamma' = \text{unit weight of the one-layered sand column} \\
l = \text{length of the one-layered sand column} \\
i = \text{hydraulic gradient.}
\]

Fig. 4(a) shows that at the time when \( h = \frac{\gamma'}{\gamma_{w}}I \), the vertical effective normal stress \( \sigma_{v}' \) becomes zero entirely in the sand layer. This state is defined as the critical state in this case. At the critical time, the effect of gravity on sand particles in the sand column has just been consumed by the seepage force (i.e., \( \gamma' - i_{r} = 0 \)), and the sand particles are considered to be in a "quick" state. Taking such a state of the sand particles in the sand column (\( p = 0 \)) into consideration, it is predicted that the sand layer boils when \( h \) increases beyond \( h_{c} \).

2. **One-layered sand column (\( p > 0 \))**

   Changes in the vertical effective normal stress in the case of the one-layered sand column (\( p > 0 \)) up to its critical state are shown in Fig. 4(b). Fig. 4(b) shows that at the time when

\[
h = \frac{\gamma'}{\gamma_{w}}I + \frac{\beta}{\gamma_{w}}, \sigma_{v}' \text{ becomes zero at the bottom of the sand column.} \\
\text{This state is defined as the critical state in this case. At the critical time, sand particles in a loaded filter is subjected to the effect of gravity (i.e., } \gamma' - i_{r} = \gamma' > 0) \text{ and are in a "stable" state, where } i_{r} \text{ is the hydraulic gradient in a loaded filter and is regarded as } i_{r} > 0 \text{ from the definition of a loaded filter.} \\
\text{On the other hand, the effect of gravity on sand particles in the sand layer has already been consumed by the seepage force (i.e., } \gamma' - i_{r} = 0) \text{, so the sand particles are considered to be in a state of "being pushed upwards". Taking such a state of the sand particles into consideration, it is predicted that the sand column is separated from its bottom and is lifted up when } h \text{ exceeds its critical value } h_{c}.
\]

3. **Two-layered sand column (\( k_{r} > 1, p = 0 \))**

   Changes in the vertical effective normal stress in the case of the two-layered sand column (\( k_{r} > 1, p = 0 \)) up to its critical state are shown in Fig. 4(c). Fig. 4(c) shows that at the time when

\[
h = (1 + \gamma' I_{r}) \gamma' I_{r} \frac{1}{\gamma_{w}}, \sigma_{v}' \text{ becomes zero}
\]
at the bottom of the sand column. This state is defined as the critical state in this case. At the critical time, sand particles in the upper stratum of the sand column are still subjected to the effect of gravity (i.e. \( \gamma' - i \gamma_m > 0 \)) and are in a "stable" state. On the other hand, the effect of gravity on sand particles in the lower stratum of the sand column has already been consumed by the seepage force (i.e. \( \gamma' - i \gamma_m < 0 \)), so the sand particles are considered to be in a state of "being pushed upwards". Taking such a state of the sand particles in each stratum into consideration, it is predicted that the sand column is separated from its bottom and is lifted up when \( h \) exceeds its critical value \( h_c \).

**Experimental Observation**

(1) One-layered sand column (\( p = 0 \)) : Flow conditions

From a series of seepage-failure experiments on the one-layered sand column (\( p = 0 \)), it is observed that the sand columns collapse in quite different ways according to the ratio of the coefficient of permeability of the "rectifying filter" \( \frac{k_D}{k} \) to one of the sand layer \( k \) : (i) \( \frac{k_D}{k} > 15 \) and (ii) \( 1 < \frac{k_D}{k} < 5 \). In this paper, (i) and (ii) are referred to as the flow conditions of the exerted-pressure type and the uniform-flow type, respectively. In a series of seepage-failure experiments (Table 2), Sand 3 and Sand 2 (Table 1 and Fig. 6) are made use of as the rectifying filters in the exerted-pressure type and the uniform-flow type, respectively. The coefficients of permeability of the rectifying filters which are compacted to \( D_r = 50\% \) as stated before are \( k_D \approx 0.521 \text{ cm/s} \) in the exerted pressure type and \( k_D \approx 0.1146 \text{ cm/s} \) in the uniform-flow type (Table 1). On the other hand, the coefficient of permeability of the sand layer (Sand 1) \( k \) depends on the void ratio \( e \) and is represented as the following Kozeny–Carman estimation which is obtained from experimental \( k_{15} \sim e \) data with a standard regression analysis:

\[
k_{15} = 0.0482 \frac{e^4}{1 + e} \text{ (cm/s), for Sand 1.}
\]

**Fig. 8. Mode of failure of the one-layered sand column \( (p = 0) \).** (a) Flow condition of the exerted-pressure type \( (k_D/k > 15) \); (b) Flow condition of the uniform-flow type \( (1 < k_D/k < 5) \)

Figs. 8(a) and (b) show the modes of failure of the one-layered sand column \( (p = 0) \) observed in experiments of the exerted-pressure type and the uniform-flow type, respectively. Figs. 9(a) and (b) also show typical examples of the relationships between the discharge velocity \( q_{15} \) and the hydraulic gradient \( i \) measured in the experiments of the exerted-pressure type and the uniform-flow type, respectively. On each flow condition, the mode of failure of the one-layered sand column \( (p = 0) \) is as follows:

(i) In the case of the exerted-pressure type \( (k_D/k > 15) \)

Sand particles move partially, i.e., piping occurs locally as shown in Fig. 8(a) about the critical time. Discharge increases rapidly
Fig. 9. Relationship between discharge velocity \( q_{15} \) and hydraulic gradient \( i \). (a) Exerted-pressure type \( (k_0/k = 21.5) \); (b) Uniform-flow type \( (k_0/k = 2.40) \)

Fig. 10. Mode of failure of the one-layered sand column \( (p > 0) \).
(a) \( h < h_c \); (b) Small horizontal cracks occurring at the bottom
(c) Small pipes occurring under cracks; (d) Cracks rising; (e) Cracks joining; (f) Large horizontal crack opening (i.e. upper layer rising)

After the critical time and \( q_{15} \sim i \) curve shows "rapid change" as shown in Fig. 9(a). It is considered that, near or after the critical time, the flowing water concentrates into a part of the sand column (e.g., a location with a larger coefficient of permeability or a feeblers part than the other). Thus, for \( \frac{k_0}{k} > 15 \), the sand column is in a state of "being conditioned by constant hydraulic head difference between the topmost level and the bottom level of the sand column" about the critical time.

(ii) In the case of the uniform-flow type \( (1 < \frac{k_0}{k} < 5) \)

The one-layered sand column, in which sand particles are in a state of "flotation" as a whole and finer particles move to an upper part about the critical time, collapses gradually as shown in Fig. 8(b). Discharge increases gradually after the critical time and \( q_{15} \sim i \) curve shows "smoothly down" as shown in Fig. 9(b). It is considered that, near and after the critical time, the flow is kept uniform entirely in the cross section of the seepage cylinder by the rectifying filter and the flowing water does not
concentrate into a part of the sand column. Thus, for $1 < \frac{k_F}{k} < 5$, the sand column is in a state of "being conditioned by uniform flow in the total cross section of the seepage cylinder" about the critical time.

(2) One-layered sand column ($p > 0$)

The typical mode of failure of the one-layered sand column ($p > 0$) is shown in Fig. 10. As $h$ is increasing, near the critical time, small horizontal cracks and/or small movements of sand particles (small pipes) occur at the bottom of a lower part of the sand column (Figs. 10(b) and (c)). As $h$ is gradually increasing, the entire sand column above the horizontal crack begins to rise at a certain point of time. Once separated, the column starts to lose sand grains at its lower surface by grain precipitation (Fig. 10(e)). In this way, the sand column is separated from its bottom, lifted up and collapses, in other words, a horizontal crack grows larger and is rising.

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**Fig. 11. Views of a seepage-failure test of a one-layered sand column ($p > 0$)**

(Test (a): $l = 9.95 \text{ cm}$, $l_F = 4.05 \text{ cm}$, $k_F/k = 18.9$).

(a) Initial ($h = 0$); (b) $h < h_c$; (c) Cracks rising; (d) Cracks joining;
(e) Large horizontal crack opening (i.e. upper layer rising)
up (Fig. 10(f)). On the other hand, the sand particles separated from the upper sand column continue boiling (Fig. 10(f)). Views of a typical example of seepage failure of a one-layered sand column ($p>0$) are shown in Fig. 11. In the case that $p$ is small, cracks and small pipings hardly occur at the bottom of the sand column before the sand column collapses. On the other hand, in the case that $p$ is large, a horizontal crack which occurs at the bottom of the sand column stays for a long time in a lower part of the sand column, and can not rise up easily. It is observed that the sand column and sand particles are very stable in this case. This is because as $p$ increases, the residual vertical effective normal stress $\sigma'_{z,\text{res.}}$ within the one-layered sand column ($p>0$) in its critical state grows larger as shown in Fig. 4(b) and as a result the residual horizontal effective normal stress $\sigma'_{x,\text{res.}}$ becomes larger, so the friction between the vertical cylindrical surface and sand particles and the friction between sand particles increase.

(3) Two-layered sand column ($k_1\gamma_1'>1$, $p=0$)

The mode of failure of the two-layered sand column ($k_1\gamma_1'>1$, $p=0$) is similar to one of the one-layered sand column ($p>0$). In the case of the two-layered sand column ($k_1\gamma_1'>1$, $p=0$), as $\lambda_r$ increases, the residual effective stress within the sand column in its critical state ($\sigma'_{z,\text{res.}}$ and $\sigma'_{x,\text{res.}}$) grows larger.

Thus, from a series of seepage-failure experiments, the modes of failure of the one-layered sand column ($p=0$), the one-layered sand column ($p>0$) and the two-layered sand column ($k_1\gamma_1'>1$, $p=0$) are, respectively, considered to be similar to the modes of failure of the sand columns predicted theoretically from the residual effective stress diagrams in the critical states.

**COMPARISON WITH THEORETICAL FINDINGS**

(1) One-layered sand column ($p=0$)

Fig. 12 shows the relationship between the void ratio $e$ and the critical hydraulic gradient $i_*$ for the seepage failure test of the one-layered sand column ($p=0$). The maximum hydraulic gradient which the sand column is subjected to up to failure $i_*$ is taken as the critical hydraulic gradient obtained by experiment as shown in Fig. 9, that is to say, $i_*$ is about the hydraulic gradient at the time when or just before the discharge is rapidly increasing. The hydraulic gradient at the time when $i\sim q_{15}$ curve deviates from the Darcy's linearity law is considered to become another critical hydraulic gradient obtained by the experiment. In some cases, $i\sim q_{15}$ curve deviates from the Darcy's law just before the sand column collapses as shown in Fig. 9. In some cases, however, $i\sim q_{15}$ curve deviates from the Darcy's law at the early stage of the experiment. The sand column whose $i\sim q_{15}$ curve has deviated from the Darcy's law at the early stage of the experiment is stable and is not considered to be critical. For that reason,
here, the hydraulic gradient at the time when $i = q_{15}$ curve deviates from the Darcy’s linearity law is not taken as the critical hydraulic gradient obtained by the experiment.

In Fig. 12, the solid lines show the theoretical equation:

$$i_{9, t} = \frac{h}{l} \Rightarrow \frac{1}{1+e} = \frac{r'}{r_{w}}$$  \hspace{1cm} (1a)

$$h_{9, t} = \frac{r'}{r_{w}}$$  \hspace{1cm} (1b)

An increase in the hydraulic gradient $i$; $di$ is given as mentioned above. At each step, a constant-head permeability test was carried out, and lengths of the sand column were also measured at 4 portions around the seepage cylinder and averaged. The “average” void ratio $e$ was then calculated at each step. The range changing in the “average” void ratio $e$ of the sand column up to failure is shown in Fig. 12 according to the notation on the upper right hand side in Fig. 12(a), where

\begin{align*}
(.....)_{e, t} &= \text{theoretical value of (.....)$_e$} \\
(.....)_{e, m} &= \text{measured value of (.....)$_e$ in experiment.}
\end{align*}

The same notations will be used throughout the paper.

In the case of the poorly-graded sand (Sand 1), the sand layer shrinks for $D_{90}<50\%$ and swells for $D_{90}>50\%$ due to the flowing water. Where $D_{90}$ is the initial value of the relative density of the one-layered sand column ($p=0$). This is because while water is flowing through the sand layer, the sand layer shrinks due to the rearrangement of sand particles (for $D_{90}<50\%$) and swells due to the decrease in an effective stress (for $D_{90}>50\%$). It is also observed many times for $D_{90}<50\%$ that the once-shrunked sand layer swells again near the critical state.

In the case of the flow condition of the uniform-flow type, as shown in Fig. 12(b), for $D_{90}>60\%$ the sand layer collapses at the larger hydraulic gradient than the theoretical critical hydraulic gradient $i_{9, t}$ (Eq. (1a)), whereas for $D_{90}<50\%$ the sand layer collapses at the smaller hydraulic gradient than $i_{9, t}$. This is because when $D_{90}$ is large, the void ratio $e$ is small, and the sand particles engage each other, which gives the sand particles the resistance to movement, i.e., the resistance to failure of the sand layer (quicksand). On the other hand, when $D_{90}$ is small, the relative distance between sand particles is large, and the sand particles are in an unstable state, which gives the sand particles the smaller resistance to movement, i.e., the smaller resistance to failure of the sand layer. It should be noted that for $D_{90}<50\%$ almost all the test specimens collapse at the smaller hydraulic gradient than the theoretical critical hydraulic gradient $i_{9, t}$.

In the case of the flow condition of the exerted-pressure type, it should be also noted that for $D_{90}<80\%$ almost all the specimens collapse at the smaller hydraulic gradient than $i_{9, t}$.

It is also observed from Fig. 12 that the initial void ratio $e_0$ vs. the critical hydraulic gradient $i_{9, m}$ curves which are obtained by experiment have stronger inclination than the theoretical one (Eq. (1a)).

Thus the relative density $D_{90}$ is considered to be one of the parameters which represent resistance to quicksand of the one-layered sand column ($p=0$).

(2) One-layered sand column ($p>0$)

Figs. 13(a), (b) and (c) show the relationships between the nondimensional value of $p$; $\frac{p}{r'/l}$ and the normalized nondimensional value of $h_{9}; \left(\frac{h}{l}\right)_{9/9_{w}}/\left(\frac{r'}{r_{w}}\right)$ for Test (a), Test (b) and Test (c), respectively. The difference in total hydraulic head at the time when the sand column is separated from its bottom, lifted up and collapses is taken as the critical hydraulic head difference obtained by experiment. In Fig. 13, the solid lines show the theoretical equation (Sawada et al., 1980a):

$$\left(\frac{h}{l}\right)_{9, t} = 1 + \frac{p}{r'/l}$$  \hspace{1cm} (2a)

$$h_{9, t} = \frac{r'}{r_{w}} + \frac{p}{r_{w}}$$  \hspace{1cm} (2b)
In Eqs. (2a) and (2b), the friction between the vertical cylindrical surface and sand particles is not taken into consideration.

In the case of the one-layered sand column \((p>0)\), it is observed from Fig. 13 that for \(0.0<\frac{p}{\gamma' l}<0.1\) the sand column collapses at the smaller hydraulic head difference than the theoretical critical hydraulic head difference \(h_{c,t}\) (Eq. (2b)), for \(0.1<\frac{p}{\gamma' l}<0.3\) at the same hydraulic head difference as \(h_{c,t}\) and for \(0.3<\frac{p}{\gamma' l}\) at a larger hydraulic head difference than \(h_{c,t}\). This is because, in the case of \(0.0<\frac{p}{\gamma' l}<0.1\), the residual effective stress is small and has no effect on the stability of the sand column, and in the case of \(0.3<\frac{p}{\gamma' l}\), the residual effective stress is large and the friction between the vertical cylindrical surface

---

Fig. 13. Relationship between \(\frac{p}{\gamma' l}\) and 
\[
\left(\frac{h}{l}\right)_c / \left(\frac{y'}{\gamma_{wo}}\right)
\]
(a) Test (a) (Exerted-pressure type)
(b) Test (b) (Uniform-flow type)
(c) Test (c) (Uniform-flow type)
and sand particles is large and the resistance to failure (heaving) of the sand column increases.

(3) Two-layered sand column \((k, \gamma' > 1, p=0)\)

Figs. 14(a), (b) and (c) show the relationships between the ratio of \(l_2 \) to \(l_1 \); \(l_r \) and the normalized value of \((i_1)_{c'} \); \((i_1)_{c'} \left( \frac{\gamma'_r}{\gamma_w} \right)\) for Test \(\oplus\), Test \(\oplus\) and Test \(\oplus\), respectively. Where \((i_1)_{c'}\) is the hydraulic gradient within the lower layer of the two-layered sand column at the critical time, and is the nondimensional value of \(h_{c'}\) and \(l_1 \) and \(l_2 \) are the lengths of the lower and the upper layers of the two-layered sand column, respectively. The difference in total hydraulic head at the time when the sand column is separated from its bottom, lifted up and collapses is taken as the critical difference obtained by experiment, similarly as stated in (2). In Fig. 14, the solid lines show the theoretical equation (Sawada et al., 1980b):

\[
\frac{(i_1)_{c',t}}{\left( \frac{\gamma'_r}{\gamma_w} \right)} = \frac{k_r(1+\gamma'_r/l_r)}{k_r+l_r} \tag{3a}
\]

\[
h_{c',t} = (1+\gamma'_r/l_r) \frac{\gamma'_r}{\gamma_w} l_1 \tag{3b}
\]

In Eqs. (3a) and (3b), the friction between the vertical cylindrical surface and sand particles is not taken into consideration. The solid lines shown in Figs. 14(a), (b) and (c) are obtained using Eq. (3a) from the following average values of \(k_r\) and \(\gamma'_r\) in each series of tests, respectively:

Test \(\oplus\) : \(k_r = 4.62, \gamma'_r = 1.05\)
Test $\text{(a)}$: $k_r = 4.53$, $r' = 1.06$
Test $\text{(b)}$: $k_r = 4.72$, $r' = 1.05$.

In the case of the two-layered sand column $(k_r r' > 1, p = 0)$, it is observed from Fig. 14 that for $0.0 < L < 0.1$ the sand column collapses at the smaller hydraulic head difference than the theoretical critical hydraulic head difference $h_{e, t}$ (Eq. (3b)), for $0.1 < L < 0.3$ at the same hydraulic head difference as $h_{e, t}$, and for $0.3 < L$, at the larger hydraulic head difference than $h_{e, t}$. This is because, in the case of $0.0 < L < 0.1$, the residual effective stress is small and has no effect on the stability of the sand column, and in the case of $0.3 < L$, the residual effective stress is large and the friction between the vertical cylindrical surface and sand particles is large and the resistance to failure (heaving) of the sand column increases.

**EFFECTS OF A LOADED FILTER, FLOW CONDITIONS AND SURFACE FRICTION**

*Effects of a Loaded Filter*

From the experimental results, the following three effects of a loaded filter are clarified:

(i) Increase in $h_{e, m}$

As shown in Fig. 13, as $\frac{p}{\gamma' l}$ is increasing, $\left(\frac{h}{l}\right)_{e, m} / \left(\frac{\gamma'}{\gamma_w}\right)$ increases, that is to say, as $p$ is increasing, $h_{e, m}$ increases.

(ii) Increase in stability of a sand column in its critical state

It is observed from Fig. 13 that for $\frac{p}{\gamma' l} > 0.3$

the sand column collapses at the larger difference in total hydraulic head $h_{e, m}$ than the theoretical one $h_{e, t}$ (Eq. (2b)) and the difference $h_{e, m} - h_{e, t}$ becomes larger with $\frac{p}{\gamma' l}$ increasing (i.e., with $p$ increasing), that is to
say, the resistance (stability) to rising (heaving) of the sand column increases with $p$ increasing. This is because as $p$ increases, the residual effective stress (in the critical state) within the sand column grows larger as shown in Fig. 4(b), and the friction between the vertical cylindrical surface and sand particles becomes larger.

It is also observed in the seepage-failure test on the one-layered sand column ($p > 0$) that when $p$ is small, the sand particles are in a very unstable state, on the other hand as $p$ increases, the sand particles become more stable, i.e., the resistance (stability) to movement of sand particles increases.

(iii) Descent of a critical location

When $p$ is zero, the critical location is in the whole layer of the one-layered sand column ($p > 0$). When $p$ is larger than zero, the critical location is at the bottom of the one-layered sand column ($p > 0$). That is to say, a critical location goes down with $p$ increasing.

**Effects of Flow Conditions**

Now we consider the effect of flow conditions on resistance to seepage failure of sand columns precisely. Figs. 15(a), (b) and (c) show the average curves between the initial void ratio $e_0$ and the critical hydraulic gradient $i_c$ for Test a and Test b, the average curves between the non-dimensional value of $p; \frac{p}{\gamma' l}$ and the normalized nondimensional value of $h_e; \left( \frac{h}{l} \right) e \left( \frac{\gamma'}{\gamma_w} \right)$ for Test (a) and Test (b), and the average curves between the ratio of $l_2$ to $l_1$; $l_r$ and the normalized value of $(i_c) e; \left( \frac{i_c}{l_r} \right)$ for Test 5 and Test 6, respectively. It is observed from Figs. 15(a), (b) and (c) that the sand column in the case of the flow condition of the exerted-pressure type collapses at the smaller hydraulic head difference than that in the uniform-flow type. This is because in the case of the flow condition of the exerted-pressure type the flowing water concentrates into a feeble part of the sand column near the critical time and the sand column collapses rapidly. (The concentration of the flowing water is conspicuous in the case that the residual effective stress in the critical state is small.) Whereas in the case of the flow condition of the uniform-flow type, the flow is not able to concentrate locally and the sand layer collapses gradually.

Moreover, the one-layered sand columns ($p = 0$) collapse in quite different ways according to the flow conditions as stated before.

Thus it is thought that, in the seepage failure problem of a sand column subjected to an upward seepage flow, the effects of the flow conditions should be taken into consideration.

**Effect of Surface Friction**

Let us consider the results of seepage-failure experiments on the one-layered sand column ($p > 0$) and the two-layered sand column ($k_2 > 1, p = 0$). It is observed from Fig. 13 or Fig. 14 that for $\frac{p}{\gamma' l} > 0.3$ or $l_r > 0.3$ each sand column collapses at the larger difference in total hydraulic head than the theoretical critical difference $h_{e, r}$ (Eq. (2b) or (3b)). This is because as $\frac{p}{\gamma' l}$ or $l_r$ increases, the residual effective stress becomes large, i.e. the friction between the vertical cylindrical surface and sand particles becomes large and the resistance to failure of the sand column increases.

In this paper, it is assumed that the residual horizontal effective normal stress in the critical state relates to the resistance to failure of a sand column, which is an approximate calculation method for practical use. Let us consider the special case that a multi-layered sand column becomes critical at the bottom as shown on the right hand side in Fig. 16). The critical

\[ \gamma' = \gamma' (j = 1, 2, \ldots, n), \] so at the time when $h = 0$, the vertical effective normal stress in the $n$-layered sand column changes linearly. It should be noted that, at the time when $h = 0$, the vertical effective normal stress diagram is not linear in a general case. In this paper, the only case will be investigated that the layered sand columns become critical at their bottoms in the critical states. So Fig. 16 shows the case that a multi-layered sand column becomes critical at the bottom. If it is not the case, the further investigation is required for the estimation of the following "critical equilibrium of forces".

\[ nI=-I_{-I} \]
equilibrium of forces at the bottom of the sand column is written as follows:

\[ \langle h \rho_w S \rangle_e = \gamma' T L_T S + R_f \]  

(4)

in which \( R_f \) is a term representing the frictional resistance acted on the cylindrical surface. Using the following notations:

\[ \chi = \frac{R_f}{\gamma' T L_T S} \]  

(5)

\[ \eta_e = \frac{h}{L_T \gamma_w} \]  

(6)

Eq. (4) is rewritten as follows:

\[ \eta_e = 1 + \chi \]  

(7)

Where the following notations are used here:

\[ S = \text{cross-sectional area of the seepage cylinder} \]
\[ L_T = L + L_F \] ; total length of the \( n \)-layered sand column with a loaded filter

\[ L = \sum_{j=1}^{n} l_j, \text{ length of the } n \text{-layered sand column} \]

\[ \gamma' T = \frac{\gamma'_{av} L + \gamma' F L_F}{L_T}, \text{ average buoyant unit weight of the } n \text{-layered sand column with a loaded filter} \]

\[ \gamma'_{av} = \frac{\sum_{j=1}^{n} \gamma'_{j} l_j}{L}, \text{ average unit weight of the } n \text{-layered sand column} \]

\[ R_f \] = surface-frictional resistance to seepage failure of the sand column caused by the
residual horizontal effective normal stress in its critical state

\( \chi \) is a nondimensional value of the surface-frictional resistance caused by the residual horizontal effective normal stress \( R_f \), and \( \eta \) is a normalized nondimensional value of the critical hydraulic head difference \( h_c \). In the cases of the one-layered sand column \(( p > 0 )\) and the two-layered sand column \(( k_{11} > 1, p = 0 )\), Eq. (5) leads to Eqs. (8) and (9), respectively.

For the one-layered sand column \(( p > 0 )\):

\[
R_f = \left( \frac{1}{2} l K_0 \tan \delta + \frac{1}{2} l_f K_{0,F} \tan \delta_F \right) 2\pi R
= (plK_0 \tan \delta + pl_f K_{0,F} \tan \delta_F) \pi R
\]

\[
\chi = \frac{pl}{R_f^2 7L_T} \left( K_0 \tan \delta + \frac{l_f}{l} K_{0,F} \tan \delta_F \right)
\]  

(8)

For the two-layered sand column \(( k_{11} > 1, p = 0 ):\)

\[
R_f = \left( \frac{1}{2} l_1 K_{0,1} \tan \delta_1 + \frac{1}{2} l_2 K_{0,2} \tan \delta_2 \right) (\sigma'_{e1=0})_2^\varepsilon \times 2\pi R
= (l_1 K_{0,1} \tan \delta_1 + l_2 K_{0,2} \tan \delta_2) (\sigma'_{e1=0})_2^\varepsilon \pi R
\]

\[
\chi = \frac{(\sigma'_{e1=0})_2^\varepsilon l_1}{R_f^2 7L_T} (K_{0,1} \tan \delta_1 + l_2 K_{0,2} \tan \delta_2)
\]  

(9)

Where

- \( R \): radius of the seepage cylinder
- \( K_0, K_{0,F} \): coefficients of earth pressure at rest in the sand layer and the loaded filter of the one-layered sand column \(( p > 0 )\), respectively
- \( K_{0,1}, K_{0,2} \): coefficients of earth pressure at rest in the lower layer and the upper layer of the two-layered sand column \(( k_{11} > 1, p = 0 )\), respectively
- \( \delta, \delta_F \): angles of friction between the vertical cylindrical surface and sand particles in the layer and the loaded filter of the one-layered sand column \(( p > 0 )\), respectively
- \( \delta_1, \delta_2 \): angles of friction between the vertical cylindrical surface and sand particles in the lower layer and the upper layer of the two-layered sand column \(( k_{11} > 1, p = 0 )\), respectively

\[
(\sigma'_{e1=0})_2^\varepsilon = \frac{k_{11} - 1}{k_{11} + l_2} \gamma_1 l_2 ; \text{ residual vertical effective normal stress in the critical state at the boundary point of the lower layer and the upper layer of the two-layered sand column } \( k_{11} > 1, p = 0 \).
\]

The equation \( \eta = 1.0 \) corresponds to the theoretical equation in which the surface-frictional resistance is not taken into consideration. Therefore, from Eq. (7), a value \( \chi \) represents the effect of surface friction on the resistance to seepage failure.

In the experiments, for the case of \( \frac{p}{R_f^2 7L_T} > 0.3 \) in the one-layered sand column \(( p > 0 )\) and \( l_1 > 0.3 \) in the two-layered sand column \(( k_{11} > 1, p = 0 )\) on the uniform-flow type flow condition, the surface friction is considered to have an effect on the stability of the sand columns. Using all these test results, \( K_0, K_{0,F} \), \( \tan \delta_1, K_{0,1} \tan \delta_1, K_{0,2} \tan \delta_2 \), are computed as the least square adjusted parameters with a standard regression analysis as follows:

For the one-layered sand column \(( p > 0 )\) (uniform-flow type):

\[
K_{0,1} \tan \delta_1 = 0.308
\]

\[
K_{0,F} \tan \delta_F = 0.571
\]  

(10)  

(11)

For the two-layered sand column \(( k_{11} > 1, p = 0 )\) (uniform-flow type):

\[
K_{0,1} \tan \delta_1 = 0.314
\]

\[
K_{0,2} \tan \delta_2 = 0.305
\]  

(12)  

(13)

The coefficient of earth pressure at rest in a medium-compacted sandy soil ground \( K_0 \) is considered to be about 0.5~0.6 (JSSMFE ed., 1982; Yamaguchi, 1984; JSSMFE ed., 1985). As all the sand strata in the cases of the one-layered sand column \(( p > 0 )\) and the two-layered sand column \(( k_{11} > 1, p = 0 )\) are compacted to about \( D_c = 50\% \) in the experiments, the coefficients of earth pressure at rest in sand strata \( K_0, K_{0,F}, K_{0,1} \), and \( K_{0,2} \) are then considered to be about 0.5~0.6. Assuming that all the values of \( K_0, K_{0,F}, K_{0,1} \), and \( K_{0,2} \) are equal to 0.55 (as an average value), the angles of friction between the cylindrical surface and sand particles \( \delta, \delta_F, \delta_1 \), and \( \delta_2 \), are calculated as shown in Table 3. Table 3 shows
Fig. 17. Relationship between $\chi$ and $\gamma_c$. (a) Exerted-pressure type; (b) Uniform-flow type
Table 3. Angles of friction between the cylindrical surface and sand particles

<table>
<thead>
<tr>
<th>Layer</th>
<th>Surface frictional angle</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand layer</td>
<td>$\delta = 29.2$</td>
<td>Sand 1</td>
</tr>
<tr>
<td>Loaded filter</td>
<td>$\delta_F = 46.1$</td>
<td>Sand 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Layer</th>
<th>Surface frictional angle</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower layer</td>
<td>$\delta_{1} = 29.7$</td>
<td>Sand 1</td>
</tr>
<tr>
<td>Upper layer</td>
<td>$\delta_{2} = 29.0$</td>
<td>Sand 2</td>
</tr>
</tbody>
</table>

that the angles of friction between the cylindrical surface and sand particles are about 30 degrees for Sand 1 and Sand 2, and about 46 degrees for Sand 3.

The lower layer of the one-layered sand column ($p > 0$) and Stratum 1 (the lower layer) of the two-layered sand column ($k_0 r' > 1$, $p = 0$) are the same material: Sand 1, which are compacted to about $D_r = 50\%$. Since $\delta \simeq \delta_1$ as shown in Table 3, it is considered that the proposed simple method of taking the effect of surface friction into consideration is appropriate.

Fig. 17 shows the relationship between $\chi$ and $\eta_e$ which are calculated from Eqs. (6), (8) and (9) using Eqs. (10) ~ (13). Figs. 17 (a) and (b) are the cases of the exerted-pressure type and the uniform-flow type, respectively.

In the case of the uniform-flow type, the test results are classified from Fig. 17 (b) as follows:

(i) For $0.0 < \chi < 0.06$, the sand column collapses at the smaller hydraulic head difference than the theoretical critical hydraulic head difference neglecting the effect of surface friction ($\eta_e = 1$). It is considered that, for this extent of $\chi$ ($0.0 < \chi < 0.06$), the residual effective stress is small and the surface friction due to it cannot function as the resistance to seepage failure of the sand column.

(ii) For $0.06 < \chi < 0.14$, the sand column collapses at the hydraulic head difference between the theoretical critical hydraulic head differences neglecting the effect of surface friction ($\eta_e = 1$) and considering the effect of surface friction ($\eta_e = 1 + \chi$).
(iii) For $0.14 < \chi < 0.50$, the sand column collapses at the same hydraulic head difference as the theoretical critical hydraulic head difference considering the effect of surface friction ($\eta_e = 1 + \chi$). It is considered that, for this extent of $\chi$ ($0.14 < \chi < 0.50$), the residual effective stress is large and the surface friction due to it can function as the resistance to seepage failure of the sand column.

Fig. 18 shows the average curves between $\chi$ and $\eta_e$ for the exerted-pressure type and the uniform-flow type. Fig. 18 shows that the sand column in the case of the flow condition of the exerted-pressure type collapses at the smaller hydraulic head difference than that in the uniform-flow type. The average relationships between $\chi$ and $\eta_e$ obtained from experimental results are as follows:

(i) Exerted-pressure type
For $0.0 < \chi < 0.13$: $\eta_e = 0.850 + 1.939 \chi$
For $0.13 < \chi < 0.50$: $\eta_e = 0.982 + 0.936 \chi$

(ii) Uniform-flow type
For $0.0 < \chi < 0.14$: $\eta_e = 0.881 + 1.859 \chi$
For $0.14 < \chi < 0.50$: $\eta_e = 1 + \chi$

CONCLUSION

An apparatus was designed for one-dimensional seepage-failure tests. The apparatus consists of the three parts: (1) seepage cylinder, (2) constant-head device, and (3) open piezometers. The details of the apparatus, the preparation of specimens and the test procedure were then described.

A series of seepage-failure experiments was conducted on the one-layered sand column ($p = 0$), the one-layered sand column ($p > 0$) and the two-layered sand column ($k_{r'} > 1$, $p = 0$) which are typical examples of a multi-layered sand column. The effects of the flow conditions, a loaded filter, the residual effective stress and the surface friction on the stability for seepage failure of the sand column were investigated. It is found from the experiments that the flow condition: (i) $\frac{k_0}{k} > 15$ (exerted-pressure type) or (ii) $1 < \frac{k_0}{k} < 5$ (uniform-flow type) is one of the most important factors in the seepage-failure problem of the sand columns. The following results were then obtained:

(1) The mode of failure of a multi-layered sand column is similar to the mode of failure predicted theoretically from the residual effective stress diagram in its critical state.

(2) The one-layered sand column ($p = 0$) collapses in quite different ways according to the flow conditions: the exerted-pressure type and the uniform-flow type.

(3) In the case of the flow condition of the exerted-pressure type a multi-layered sand column collapses at the smaller difference in total hydraulic head than that in the uniform-flow type.

(4) The relative density $D_r$ is considered to be one of the parameters which represent the resistance to quicksand of the one-layered sand column ($p = 0$).

(5) For $D_{r0} < 50\%$ almost all the specimens of the one-layered sand columns ($p = 0$) collapse at the smaller hydraulic gradient than the theoretical critical hydraulic gradient in the case of the flow condition of the uniform-flow type, and for $D_{r0} < 80\%$ the same things are said in the case of the flow condition of the exerted-pressure type.

(6) For the case of $0.0 < \frac{p}{r/l} < 0.1$ the one-layered sand column ($p > 0$) collapses at the smaller difference in total hydraulic head between the topmost level and the bottom level of the sand column than the theoretical critical hydraulic head difference $h_{r, t}$ (Eq. (2b)) neglecting the effect of surface friction, for $0.1 < \frac{p}{r/l} < 0.3$ at the same hydraulic head difference as $h_{r, t}$, and for $0.3 < \frac{p}{r/l}$ at the larger hydraulic head difference than $h_{r, t}$. It is considered that for $0.0 < \frac{p}{r/l} < 0.1$ the residual effective stress is small and has no effect on the stability of the sand column, and for $0.3 < \frac{p}{r/l}$ the residual effective stress is large and the friction between the vertical cylindrical surface and sand particles is large and the
resistance to failure of the sand column increases.

(7) The two-layered sand column \((k\gamma_c') > 1, p = 0\) collapses for \(0.0 < l < 0.1\) at the smaller difference in total hydraulic head than the theoretical critical hydraulic head difference \(h_{e, t}\) (Eq. (3b)) neglecting the effect of the surface friction, for \(0.1 < l < 0.3\) at the same hydraulic head difference as \(h_{e, t}\), and for \(0.3 < l\) at the larger hydraulic head difference than \(h_{e, t}\). It is considered that for \(0.0 < l < 0.1\) the residual effective stress is small and has no effect on the stability of the sand column, and for \(0.3 < l\) the residual effective stress is large and the friction between the vertical cylindrical surface and sand particles is large and the resistance to failure of the sand column increases.

(8) The following three effects of a loaded filter are clarified from the experimental results: As \(p\) increasing, 

(i) \(h_e\) increases,

(ii) stability of the sand column in its critical state increases, and

(iii) the critical location goes down.

(9) In the case of the uniform-flow type, a multi-layered sand column collapses for \(0.0 < \chi < 0.06\) at the smaller hydraulic head difference than the theoretical critical hydraulic head difference neglecting the effect of surface friction \((\gamma_c = 1)\), for \(0.06 < \chi < 0.14\) at the hydraulic head difference between the theoretical critical hydraulic head differences neglecting the effect of surface friction \((\gamma_c = 1)\) and considering the effect of surface friction \((\gamma_c = 1 + \chi)\), and for \(0.14 < \chi < 0.50\) at the same hydraulic head difference as the theoretical critical hydraulic head difference considering the effect of surface friction \((\gamma_c = 1 + \chi)\). It is considered that for \(0.0 < \chi < 0.06\) the residual effective stress is small and the surface friction due to it cannot function as the resistance to seepage failure of the sand column, and for \(0.14 < \chi < 0.5\) the residual effective stress is large and the surface friction due to it can function as the resistance to seepage failure of the sand column.

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