EVOLUTION OF SHEAR MODULUS AND FABRIC DURING SHEAR DEFORMATION

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ABSTRACT

Numerical studies of shear modulus and fabric for sphere assemblies were performed by a discrete element code, TRUBAL. Results of numerical experiments were compared with physical experiments. In general, numerical experiments simulate the stress-strain behavior and shear modulus of physical results quite well both qualitatively and quantitatively. Shear modulus has a close relationship with the coordination number. After a cycle of loading and unloading, there is an induced anisotropic fabric and are locked-in inter-particle shear forces which reflect the previous stress history. This is also reflected in a decrease in both the shear modulus and the coordination number. During re-shear locked-in inter-particle shear forces would influence the behavior of shear modulus. When re-sheared in the previous direction inter-particle shear forces decrease first which results in an increase of shear modulus in the initial portion of loading. While re-sheared opposite to the previous direction initially high inter-particle shear forces would decrease the shear modulus.

Key words: dynamic shear modulus, fabric, granular material, numerical simulation (IGC: D3/D6)

INTRODUCTION

Anisotropy and dilation are two important features for granular materials and the state of fabric has been found to have a great influence on the behavior of granular materials. Previous studies of fabric have given better understanding of those behavior.

Direct measurements of fabric of granular materials showed that during shear deformation, contact normals tended to concentrate near the maximum principal stress axis, and fabric became more and more anisotropic during deformation (Oda and Konishi, 1974; Konishi, 1978; Oda et al., 1980; Konishi et al., 1982; Oda et al., 1985).

Chen et al. (1988), Ishibashi and Chen (1988), and Ishibashi, et al. (1988) correlated the state of fabric with dynamic shear modulus and found that shear modulus could reflect the minor changes of fabric along different stress paths.

The development of discrete element method makes it possible to look inside the micro–me-

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mechanics of granular materials. A discrete element code TRUBAL developed by Cundall and Strack (1979, 1983) was used by Chen and Ishibashi (1990) to study the dynamic shear modulus and fabric for sphere assemblies. Due to unrealistic linear contact stiffness used in the code, the units for TRUBAL have no physical meaning and are arbitrary. Also, when an assembly is loaded to a high stress level and unloaded, it has too much recovery in the strain during the initial portion of unloading than real experimental results. However, Chen and Ishibashi (1990) found that TRUBAL could still simulate the behavior of granular materials qualitatively. They found that shear modulus has a close relationship with coordination number during shear deformation.

To better simulate the behavior of granular materials, the linear contact stiffness has been changed to nonlinear contact formulation by Cundall (1988) based on the theory of Mindlin and Deresiewics. The new version is referred to as TRUBAL with Hertz contact formulation. As pointed out by Thornton and Randall (1988), a significant consequence of implementing such laws is the ability to quantify particle velocities, inter-particle forces, contact stresses and the macroscopic stress state in terms of real physical units, and it is possible for certain types of problem to perform the simulated experiments in real time. Numerical experiments performed by TRUBAL with Hertz contact showed much better results than linear version (Cundall, 1988; Chen, 1990). The discrepancy of the high recovery in the strain during the initial portion of unloading, as observed in the simulated results of the linear version, was corrected by using TRUBAL with Hertz contact formulation.

The purpose of this investigation is to study the fabric and shear modulus by TRUBAL with Hertz contact formulation. The results of numerical simulations were compared with those of physical tests (Chen, 1986) and the change of fabric and shear modulus during shear deformation was investigated.

**REPRESENTATION OF FABRIC**

Geometric fabric of granular material is defined as the spatial arrangement of particles. It usually includes orientation fabric (a measure of the orientation of the individual particles), coordination number (average contact number for each particle), and contact normal distribution (Oda, 1978).

Kanatani (1984) proposed a fabric tensor to represent the distribution of contact normals. It is found that a rank 2 fabric tensor is generally adequate to represent the contact normal distribution.

Cundall and Strack (1983) proposed a partitioned stress tensor which is composed of four stress partitions. They are shear, fabric, normal variation, and isotropic partitions, respectively. The first separation in the stress tensor is between stresses that derive from normal forces and stresses that derive from shear forces at contacts. The shear forces contribute only to the deviator components of the stress tensor and the isotropic part derives solely from normal forces at contacts. The isotropic stress is related to the average normal contact force for each particle, and is independent of the distribution of forces around the particle. The second split consists of separating, for each particle, the average normal force from the remaining normal forces. The latter forces constitute a partition of the stress tensor that is related to the angular variation of the normal forces. The average normal force gives rise to two other partitions, one being associated with the isotropic stress and the other with the fabric. Therefore, the stress tensor \( \sigma \) can be decomposed into shear \( \sigma_{ij}^{\alpha} \), fabric \( \sigma_{ij}^{\beta} \), normal variation \( \sigma_{ij}^{\omega} \), and isotropic partitions \( \sigma_\alpha \),

\[
\sigma_{ij} = \sigma_{ij}^{\alpha} + \sigma_{ij}^{\beta} + \sigma_{ij}^{\omega} + \sigma_\alpha \delta_{ij}
\]

in which, \( \delta_{ij} \) is Kronecker's delta.

The isotropic partition corresponds to the mean confining stress for assemblies. The shear partition corresponds to mobilized shear forces at contacts and is related to the tendency of contacts to slide. The fabric partition cor-
NUMERICAL SIMULATION

Description of Program

Numerical experiments are performed with a computer program TRUBAL which is a distinct element model developed by Cundall (1988). TRUBAL can generate random assembly of spheres with different radii as specified by users. The normal force at a contact between two particles is calculated from the normal displacement at the contact according to a nonlinear relation derived from the Hertz theory of contact. In order to perform quasi-static experiments, the assembly is loaded at a strain rate that is slow enough for inertial effects to be negligible. A small amount of mass-proportional viscous damping is also used.

The space occupied by the assembly is a periodic cell. Particle leaving one face of the cell will re-enter the cell at a corresponding location on the opposite face. By this arrangement boundary effects could be eliminated and it allows using a small assembly of spheres to simulate real experiments. The assembly is loaded by applying a uniform strain rate tensor to the periodic space to perform strain-controlled compression or extension tests. Stress components (σ₁₁, σ₂₂, and σ₃₃) and mean confining stress, (σ₁₁+σ₂₂+σ₃₃)/3, could be kept constant during shear by applying servo commands. The coordinate and the stress notation used in this paper are shown in Fig. 1.

Description of Simulation

The adequacy of the program was first examined by comparisons of the numerical results and the experimental ones by Chen (1986). The material properties used in the experiments and the numerical simulations are shown in Table 1. In experiments, hollow cylindrical samples were prepared by deposition through air and tamped to an initial void ratio of 0.58. The samples prepared by this method had initial anisotropic fabric in the vertical direction. After consolidation at 138 kPa, the samples were sheared to failure with a constant mean confining stress. During shear deformation, dynamic shear moduli at strain amplitude less
than $10^{-6}$ were measured at each stress increment by a quasi-static torsional simple shear/resonant column apparatus.

In numerical tests, after particles were generated randomly, the periodic cell was contracted isotropically until a void ratio of 0.58 was recorded at an isotropic stress of 138 kPa. During the compaction phase, contact friction was either increased or decreased in order to modify the final prorosity. The numerical servo-control was activated and the assembly was brought to equilibrium under the desired isotropic stress. The particle distribution on one section of the assembly for the initial state is shown in Fig. 2. Shearing of the assembly was performed by applying compression in $x_2$ direction and extension in $x_3$ direction with a strain rate equal to $10^{-7}$. The mean normal stress and the normal stress in $x_1$ direction were both kept constant throughout the shearing process. The stress condition was the same as the simple shear tests in the experiments except that the experiments were stress-controlled and the numerical tests were strain-controlled.

Numerical measurement of shear modulus of the assembly was performed as follows. Due to the strain-controlled testing condition in numerical experiments, particles are sliding at anytime during shear deformation. However, for shear modulus measured by a resonant column test at very low strain amplitude there should have no particle sliding. To simulate this condition, the inter-particle friction is increased to a higher value to stop particle sliding. In this study the friction is increased from 0.3 to 0.33. From this state the assembly is sheared in both along and opposite to the previous shearing direction to a certain strain amplitude (about $2\times10^{-5}$ in this study). The shear modulus is calculated by the shear stress increments and the strain amplitude as shown in Fig. 3. The ten percent increase of inter-particle friction from 0.3 to 0.33 was found to be more appropriate than other values, such as 0.3, 0.6, 1, 10, and 100.
Two-dimensional rose diagrams were constructed to show the distribution of contact normals by projecting all contact normals on some specific plane. The rose diagrams and fabric tensor for the initial state are shown in Fig. 4. The fabric tensor is plotted in dotted lines to show its intersection with some specific plane and is enlarged twice for better comparisons with rose diagrams. The component of the fabric tensor at a certain axis represents the relative concentration of contact normal distribution at that direction. As can be seen, the state of the initial fabric was quite isotropic. The fabric tensor component in $x_1$ direction was 1.024 which was slightly higher than the other two directions. The tensor components were about the same in $x_2$ and $x_3$ directions (0.978 and 0.995, respectively) which indicated that the fabric was more isotropic in the $x_2$–$x_3$ plane.

Two runs of the simulation were performed. In the first run (RUN-1), the assembly was sheared by compression in $x_2$ direction and extension in $x_3$ direction to its ultimate strength and unloaded from two different strain levels (about 0.18% and 0.45%) to the isotropic stress condition. Finally, the assembly was loaded again in the same direction up to a high strain condition. The second run (RUN-2) consisted of a complete cycle of loading, unloading, reverse loading, and reloading process. After the assemblies of RUN-1 returned to isotropic stress condition, the shearing direction was reversed, i.e., the assemblies were sheared by compression in $x_3$ direction and extension in $x_2$ direction. The assemblies were sheared in this reverse direction to certain strain levels before they were unloaded to isotropic stress and reloaded to high stress conditions. Therefore, the initial loading and unloading portions were the same for both RUN-1 and RUN-2.

RESULTS OF SIMULATION

Results of RUN-1

a. Comparisons with experiments

The stress–strain curves and volumetric behavior for RUN-1 are shown in Fig. 5 and Fig. 6, respectively. One corresponding set of experimental results for the simple shear test (Chen, 1986) was included in the figures for comparison. As shown in Fig. 5, similar behavior was observed except that the numerical test has a stiffer stress–strain response and lower strength than the experimental result. In Fig. 6 negative volumetric strain represents dilation. As can be seen, the numerical test dilated from the beginning of shear while the experiment contracted in the initial portion of shear, however, they had about the same dilation rate at large strain level. The difference in stress–strain curve and volume change between numerical and experimental results probably comes from different initial fabric and different particle size distribution. Experimental samples had anisotropic initial fabric which was 45° away from the major principal stress.
axis during shear and numerical assemblies had isotropic initial fabric. Numerical assemblies consisted of two sizes of particles while experimental samples consisted of two ranges of particles.

Fig. 7 and Fig. 8 show the evolution of the shear modulus during shear deformation for numerical and experimental results. Similar trend of the change of shear modulus for numerical and physical tests was observed. The shear modulus decreased with increasing maximum shear stress and returned to a lower value on unloading. It increased a little in the beginning portion of reloading and decreased again at high stress level. The shear modulus decreased with increasing shear strain and approached to a limiting value at high strain level (Fig. 8).

From the above comparisons, it is clear that the computer program TRUBAL simulates the results of the experiments quite well in the loading, unloading and reloading process.

b. The evolution of fabric

The coordination number is plotted against the maximum shear stress in Fig. 9. As shown in the figure, the coordination number decreased with increasing maximum shear stress, and returned to a lower value than its original value after a complete unloading. On reloading it did not change much until the stress level reached its previous maximum, then it decreased again at high stress condition. During shear, the contacts increased in the major principal stress direction and decreased in the minor principal stress direction. However, the total number of contacts decreased with increasing maximum shear stress. The decrease in contact number would reduce the rigidity of the assembly, since each particle experiences less restraint from its neighbors.

Good similarity was observed between Fig. 7 and 9. When the shear modulus was plotted against the coordination number in Fig. 10, it can be seen that the shear modulus is more or less proportional to the coordination number during shear deformation.

Stress partitions for RUN-1 during loading and unloading are shown in Fig. 11. The maximum shear stress is equal to the components of
the three stress partitions and could be calculated as follows.

\[ (\sigma_{22} - \sigma_{55})/2 = (\sigma_{22}^{(\alpha)} - \sigma_{55}^{(\alpha)})/2 + (\sigma_{22}^{(\beta)} - \sigma_{55}^{(\beta)})/2 + (\sigma_{22}^{(\gamma)} - \sigma_{55}^{(\gamma)})/2 \]

(2)

The three stress partitions all increased with increasing maximum shear stress during loading. This implies that fabric is becoming more and more anisotropic and inter-particle shear forces increase during loading. On unloading from two different strain levels to isotropic stress condition (denoted as A and B in Fig. 11), normal variation partition returned to zero, however, fabric partition retained a positive value and shear partition passed zero to a negative value. The positive fabric partition means that an anisotropic fabric had been induced by the loading and unloading process and the major axis of the induced fabric was parallel to the previous major principal stress axis. To compensate for this residual fabric partition the shear partition became negative which means that some locked-in inter-particle shear forces had been created and their directions were opposite to the previous shearing direction. This is the same behavior as has been found by Cundall and Strack (1983).

Rose diagrams of the contact normal distribution for state A and B were also constructed in Fig. 12 to show the effect of loading and unloading on fabric. As can be seen state A and B both have a preferred contact normal distribution in \( x_2 \) direction which is the pre-
Fig. 13. Stress-strain curves for RUN-2 and experimental results at lower strain level.

Fig. 14. Volumetric behavior for RUN-2 and experimental results at lower strain level.

Fig. 15. Shear modulus versus maximum shear stress for RUN-2 and experimental results at lower strain level.

Fig. 16. Shear modulus versus maximum shear strain for RUN-2 and experimental results at lower strain level.

vious major principal stress direction. The components of fabric tensor at \( x_2 \) and \( x_3 \) directions are 1.024 and 0.894 for state A, and 1.051 and 0.832 for state B. Therefore, the level of anisotropy on the \( x_2-x_3 \) plane is higher for state B than for state A which indicates that preshearing with higher stress or strain would generate more anisotropic fabric.

Results of RUN-2

a. Comparisons with experiments

The stress-strain curve and volumetric behavior for RUN-2 at lower strain level (about 0.18%) with one corresponding set of experimental results are shown in Fig. 13 and Fig. 14, respectively. As shown in Fig. 13, the stress strain curve for RUN-2 is similar to the experimental results. The numerical test still had more dilation than experiments (Fig. 14). The experimental and numerical shear modulus are plotted against the maximum shear stress in Fig. 15, and against the maximum shear strain in Fig. 16. Similar behavior was observed between numerical and experimental results. The shear modulus decreased with increasing maximum shear stress during loading and returned to a lower value upon unloading and did not change much during reverse loading and reloading process. The experimental test had less decrease in the shear modulus than numerical test during loading and had better recovery on unloading. It has been found that when unloading from lower stress level shear
Fig. 17. Stress-strain curves for RUN-2 and experimental results at higher strain level

Fig. 18. Volumetric behavior for RUN-2 and experimental results at higher strain level

Fig. 19. Shear modulus versus maximum shear stress for RUN-2 and experimental results at higher strain level

Fig. 20. Shear modulus versus maximum shear strain for RUN-2 and experimental results at higher strain level

Fig. 21. Coordination number versus maximum shear stress for RUN-2

Fig. 22. Shear modulus versus coordination number (RUN-2)
modulus would show better recovery (Ishibashi et al., 1990). The numerical test was unloaded after its ultimate strength while the experimental test was unloaded before its ultimate strength.

The stress–strain curves and volumetric behavior for RUN–2 at higher strain level (about 0.45%) with one corresponding set of experimental results are shown in Fig. 17 and Fig. 18, respectively. As shown in Fig. 17, the stress–strain curve for RUN–2 is similar to the experimental results. As shown in Fig. 18, the numerical test still had more dilation than experiments in the beginning of shear but they had about the same dilation rate at a higher strain level. The shear modulus is plotted against the maximum shear stress in Fig. 19, and against the maximum shear strain in Fig. 20. These two figures show that the numerical test simulates the experimental results quite well during a complete cycle of loading, unloading, reverse loading and reloading process.

It could be concluded from results of RUN–1 and RUN–2 that the numerical tests compare well with experimental tests. The numerical tests have higher initial stiffness and dilate from the beginning of shear. The shear modulus measured by simulation is qualitatively and quantitatively consistent with the shear modulus measured by the resonant column test.

b. The evolution of fabric

The coordination number is plotted against the maximum shear stress for RUN–2 in Fig. 21. The coordination number decreased with increasing maximum shear stress during shear, increased a little on unloading, kept on increasing when the assembly was sheared in the reverse direction, and then decreased at a high stress level. It can be seen again that the coordination number plot is very similar to the plot of experimental as well as numerical shear modulus. In Fig. 22, the shear modulus is plotted against the coordination number and it can be seen that the shear modulus increases with coordination number more or less linearly. Together with the results of RUN–1 shown in Fig. 10, this indicates that a close relationship exists between the shear modulus and the coordination number.

Effects of Preshear

The initial portions of loading and unloading were the same for RUN–1 and RUN–2. Assembly at state A was unloaded from 54 kPa (about 0.18%) and assembly at state B was unloaded from 56 kPa (about 0.45%). After that the assemblies were reshared either in the same direction as the previous direction for RUN–1 or they were reshared reversely for RUN–2. The effect of preshear could be investigated by comparing their results of reshair after state A and B.

One set of experimental results (Chen, 1986) was selected here for comparisons. Experimental samples were first presheared up to 62 (or 69) kPa and unloaded before they were reshared along different directions. The stress–strain curves and volume changes for numerical and experimental tests are shown in Fig. 23 and Fig. 24, respectively. In the figures, R is the angle which the major principal stress axis changes during reshare and preshear. The number of stress shown in the parentheses stands for the maximum stress level in preshear. As can be seen in Fig. 23, when samples were sheared in the preshear direction (R = 0°), they had higher stiffness than samples which were sheared opposite to the preshear direction (R = 90°). Samples dilated from the beginning of shear for R = 0° test and samples contracted first then dilated later for R = 90° tests (Fig. 24).
The change of normalized shear modulus versus maximum shear stress is shown in Fig. 25. To eliminate experimental scatter and to provide easy comparisons, the values of shear modulus were divided by their values before shearing. For R=0° test, the shear modulus increased first in the initial portion of loading then decreased again at high stress level. The shear modulus for R=90° test had more reduction than R=0° test.

It could be seen from the above results that the numerical tests simulate the experimental ones very well in terms of stress-strain curves, volume change, and shear modulus evolution. The difference between R=0° and R=90° tests was more pronounced for state B than state A. This is due to the effect of the induced fabric. The fabric of state B is more aniso-

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**Fig. 24. Volumetric behavior for numerical and experimental results with preshear**

**Fig. 25. Normalized shear modulus versus maximum shear stress for numerical and experimental results with preshear**

**Fig. 26. Coordination number versus maximum shear stress for numerical results with preshear**

**Fig. 27. Evolution of stress partitions for re-shearing from initial state A (R=0° test)**

**Fig. 28. Evolution of stress partitions for re-shearing from initial state A (R=90° test)**
tropic than that of state A.

The coordination number is plotted versus
maximum shear stress in Fig. 26. For \( R=0^\circ \)
test the coordination number decreased with
increasing maximum shear stress, while for
\( R=90^\circ \) test the coordination number increased
first then decreased later at a high stress level.
This is different from shear modulus which in-
dicates that other factors would influence the
shear modulus besides coordination number
during reshear.

To explain the behavior of shear modulus
during reshear the evolution of stress partitions
after state A are plotted in Fig. 27 for \( R=0^\circ \)
test and in Fig. 28 for \( R=90^\circ \) test. As shown
in Fig. 27, the shear partition for \( R=0^\circ \) test
was negative initially (inter-particle shear forces
opposite to reshear direction) and it decreased
to zero in the initial portion of loading then
increased to positive values at a high stress
level. This means that the inter-particle shear
forces at contacts are decreasing and particles
are more difficult to slide in the initial portion
of loading which results in stiffer stress-strain
behavior, more dilation, and an increase of the
shear modulus. As shown in Fig. 28, the
shear partition for \( R=90^\circ \) test is pretty high ini-
tially which means that the particles are very
easy to slide, so assemblies are softer and con-
tractive and the shear modulus decreases with
shearing. The evolution of stress partitions
after state B was the same as state A.

**CONCLUSIONS**

1. The computer program TRUBAL with
Hertz contact law could simulate the behavior
of granular material both qualitatively and
quantitatively.

2. The numerical tests could simulate the
evolution of shear modulus during loading, un-
loading, reverse loading, and reloading process.

3. The shear modulus is sensitive to the
coordination number, the distribution of con-
tact normals, and the inter-particle forces. Dur-
ing loading stage the decrease of coordination
number and increase of contact shear forces
would reduce the rigidity of samples and thus
result in a decrease of shear modulus.

4. After a cycle of loading and unloading,
there was a net loss in coordination number,
some remaining anisotropic fabric, and some
locked-in inter-particle shear forces. This was
also reflected in a decrease in the shear modulus.

5. The induced fabric and the locked-in in-
ter-particle shear forces would influence the
behavior of shear modulus during reshear.
When resheared along the previous shearing
direction, the initial decrease of inter-particle
shear forces results in an increase of shear
modulus. When resheared opposite to the pre-
vious shearing direction, the high inter-particle
shear forces would decrease the shear modulus.

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