PROGRESSIVE FAILURE AND SCALE EFFECT OF TRAP-DOOR PROBLEMS WITH GRANULAR MATERIALS

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ABSTRACT

The progressive failure and scale effect of trap-door problems were evaluated, by comparing experimental results with finite element analyses employing the elasto-plastic model including the shear band effect. The experiments were performed in passive and active mode by measuring load-displacement relationships and observing shear band developments using two sizes of apparatus (the trap-door width of the smaller one was 5 cm and of the larger one 10 cm.) The load was measured with soil stress measurement cells embedded in the face of the trap-door. The shear band developments were observed using thin horizontal colored sand layers placed in the sand mass.

The distribution of earth pressure by finite element analysis achieved good agreement with experimental results. The localized narrow zone of shear strain by the analysis was approximately identical with the observed shear band in both passive and active modes. The scale effect was recognized by the analysis and the experiment only in passive mode.

Key words: finite element method, model test, progressive failure, sandy soil, slip surface (IGC: E14)

INTRODUCTION

De Beer (de Beer, 1965) discussed the effect of the scale of footings on the bearing capacity of dense sand in the light of progressive failure. Although the scale effects of footing problems have been investigated by the centrifugal technique (Ovesen, 1979; Kimura et al., 1985), those of anchor problems have not been adequately investigated. Ovesen(1981) discussed the scale effects of uplift capacity of circular and square anchors using the centrifugal testing technique.

The trap-door problem provides a useful analogue for soil structure interaction problems such as vertical anchors (Meyerhof, 1968), tunnel design (Terzaghi, 1936) and embedded pipes (Takagi et al., 1983). Vardoulakis (1981) discussed solutions for the trap-door force in active and passive modes. Vermeer (1985) derived solutions for the trap-door force in passive mode using empirical data from various researchers. Rowe (1982) presented an elasto-plastic finite element analysis and compared the results with experimental data. Walters (1982) presented a numerical model for the analysis of shear zone development which utilizes an incremental elasto-plastic...
finite element approach with strain softening, but did not evaluate limiting loads. Previous investigators have not yet considered the progressive failure and scale effects of the trap-door problem.

In this study, we attempt to explain the progressive failure and the scale effects of trap-door problems, by comparing the experimental results with finite element analysis. The experiments were performed in passive and active modes by measuring the load-displacement relationships and observing shear band developments using two types of trap-door apparatus. The trap-door width of the smaller apparatus was 5 cm and that of the larger was 10 cm. Shear band development was observed by placing thin horizontal layers of colored sand in the sand mass. The earth pressure was measured by a soil stress measurement cell embedded in the face of the trap-door. Many separate rectangular stress measurement cells were used in the larger apparatus in order to evaluate the progressive failure. The finite element analysis employed two constitutive models for non-associated strain softening of elasto-plastic material. One was the simple strain softening constitutive model and the other was the refined constitutive model. The shear band thickness was introduced as a characteristic length into the constitutive equation.

**TESTING APPARATUS**

The testing apparatus consisted of a glass (or transparent)-walled box, trap-door and hopper. The smaller box was 100 cm long, 50 cm high, and 20 cm wide (Fig. 1). The out-of-plane sides of the box consisted of 1 cm-thick plate glass. The width of the trap-door was 5 cm. Two soil stress measurement cells were used, the locations of which are shown in Fig. 1. The larger box was 182.9 cm long, 100 cm high, and 40 cm wide (Fig. 2). The out-of-plane sides of the box consisted of 3 cm-thick transparent walls. The width of the trap-door was 10 cm. 11 separate rectangular stress measurement cells were used to measure the earth pressure (Tani, 1986). The load was measured at the middle third of the trap-door.

Fig. 1. Trap-door testing apparatus (smaller box)

Fig. 2. Trap-door testing apparatus (larger box)
(Fig. 2). The trap-door was moved up (in passive mode) or moved down (in active mode) by screw jacks until the primary failure surface had completely passed into the sand mass.

The sand mass was prepared by pouring the air-dried sand into the testing apparatus through the hopper. The drop height of the sand was kept approximately constant in each test. In order to observe shear band development in sand mass, thin horizontal colored sand layers were placed at intervals of approximately 1.5 cm in the sand mass adjacent to the glass wall on the side of the box. The sand used for the tests was Toyoura Sand (\(D_{50} = 0.16\) mm, \(U_0 = 1.46\), \(G_t = 2.64\), \(e_{\max} = 0.98\), \(e_{\min} = 0.61\), fines content = 0%). The tests were performed so that the ratio of sand mass height (\(h\)) and width of the trap-door (\(B\)) was 2.0. The dry density was 1.59 g/cm\(^3\). All the tests were conducted under normal gravity conditions.

**SOLUTION METHODS**

It is assumed that the yield function \(f\) is defined by the stress \(\sigma\) and the parameter \(\kappa\).

\[
f(\sigma, \kappa) = 0
\]

\[
\kappa = \int d\varepsilon^p,
\]

\[
d\varepsilon^p = 2(\varepsilon_x)\dot{\varepsilon} + 2(\varepsilon_y)\dot{\varepsilon} + 2(\varepsilon_z)\dot{\varepsilon} + (\dot{\gamma}_{xy})^2
\]

where \(\varepsilon_x, \varepsilon_y, \varepsilon_z\) are deviatoric components of the plastic strain rate. The ratio \(S\) is built into the elasto-plastic constitutive model as a characteristic length scale

\[
S = \frac{F_b}{F_e}
\]

where \(F_b\) is the area of the shear band in one element and \(F_e\) is the total area of the element. The ratio is approximately given by the equation

\[
S = \frac{W}{\sqrt{F_e}}
\]

where \(W\) is the thickness of the shear band and intrinsic to the soil.

We consider additive decomposition

\[
d\varepsilon = d\varepsilon^e + S \cdot d\varepsilon^p
\]

where \(d\varepsilon^e\) and \(d\varepsilon^p\) are elastic and plastic components of the strain rate. The plastic strain rate can be given as

\[
d\varepsilon^p = \frac{\partial \Phi}{\partial \sigma} = \lambda b
\]

where \(\Phi\) is the plastic potential and \(\lambda\) is the proportionality factor. When the plastic flow occurs, \(df = 0\) has to be satisfied and the following equation is obtained

\[
a^T d\sigma - A\lambda = 0, \ A = -\frac{1}{\lambda} \frac{df}{d\kappa}
\]

where \(a = (df/d\sigma)\) and \(A\) is the hardening-softening modulus.

The elastic stress-strain relation is given by

\[
d\sigma = [D]^e d\varepsilon^e
\]

where \([D]^e\) is an elastic matrix. From Eqs. (5)–(8) the factor is given by

\[
\lambda = \frac{a^T [D]^e d\varepsilon}{A + S a^T [D]^e b}
\]

The elasto-plastic constitutive relationship is obtained as

\[
d\sigma = \left( [D] - \frac{S [D]^e a^T [D]^e}{A + S a^T [D]^e b} \right) d\varepsilon
\]

In order to avoid numerical instability due to singularity of the non-associated Mohr-Coulomb model, a constitutive model based on the yield function of Mohr-Coulomb type and the plastic potential function of Drucker-Prager type is employed.

For predicting deformations in a post-peak regime, the elastic strain-softening plastic model is developed. The yield function is given by the following expression

\[
f = \alpha I_1 + \frac{1}{g(\theta)} \sqrt{J_2} - K = 0
\]

where \(I_1\) is the first invariant of stresses, and \(J_2\) is the second invariant of deviatoric stresses. In the case of Mohr-Coulomb material, \(g(\theta)\) in Eq. (11) takes the form
\[ g(\theta) = \frac{3 - \sin \phi}{2\sqrt{3} \cos \theta - 2 \sin \theta \sin \phi} \quad (12) \]

where \( \phi \) is the mobilized friction angle and \( \theta \) is the Lode angle.

The simple strain-softening functions are specified: i.e. \( \alpha \) in Eq. (11) are expressed as

\[ \alpha = \frac{\alpha_p + \alpha_t \kappa}{B + \kappa}, \quad \alpha_t = -(\alpha_p - \alpha_R) \quad (13) \]

where \( B \) is a material constant. \( \alpha_p \) and \( \alpha_R \) in Eq. (13) are defined as

\[ \alpha_p = \frac{2 \sin \phi_p}{\sqrt{3}(3 - \sin \phi_p)} \quad (14) \]

\[ \alpha_R = \frac{2 \sin \phi_R}{\sqrt{3}(3 - \sin \phi_R)} \quad (15) \]

where \( \phi_p \) is the peak friction angle and \( \phi_R \) is the residual friction angle.

The mobilized friction angle is given by

\[ \phi = \sin^{-1} \left( \frac{3 \sqrt{3} \alpha}{3 + \sqrt{3} \alpha} \right) \quad (16) \]

The plastic potential function is given by the following equation of Drucker-Prager type

\[ \Phi = \alpha' I_1 + \sqrt{J_2} - K' = 0 \quad (17) \]

Under plane strain conditions, coefficient \( \alpha' \) is expressed as

\[ \alpha' = \alpha_0 \left( \frac{1 - \kappa}{F + \kappa} \right) \quad (18) \]

\[ \alpha_0 = \frac{\tan \psi_0'}{\sqrt{9 + 12 \tan^2 \psi_0'}} \quad (19) \]

where \( \psi_0' \) is the initial dilatancy angle and \( F \) is the material constant.

The pre-peak regime also influences the limiting load in the case of softening material. Therefore, for accurate prediction of the pullout resistance of anchors, we have to take into account this and the pressure level effect which is due to the effect of mean stress on both internal friction and deformation properties of sand.

For the refined constitutive model, the frictional hardening and softening functions are expressed as

\[ \alpha(\kappa) = \left( \frac{2\sqrt{\kappa e_f}}{\kappa + e_f} \right)^m \alpha_p \]

\[ \quad (\kappa \leq e_f: \text{hardening-regime}) \quad (20) \]

\[ \alpha(\kappa) = \alpha_R + (\alpha_p - \alpha_R) \exp \left\{ -\left( \frac{\kappa - e_f}{e_r} \right)^2 \right\} \]

\[ \quad (\kappa > e_f: \text{softening-regime}) \quad (21) \]

where \( m \), \( e_f \) and \( e_r \) are constants. \( m \) is a constant which takes into account the plastic modulus, \( e_f \) is the plastic strain at peak and \( e_r \) is a constant which takes into account the stress-strain relation in the softening-regime. Similar expressions are used by de Borst (1986). \( \alpha_p \) and \( \alpha_R \) are the same as expressed in Eqs. (14) and (15).

The peak friction angle \( \phi_p \) can be estimated from the following empirical relations proposed by Bolton (1986).

\[ I_R = D_r \left\{ 5 - \ln \left( \frac{\sigma_m}{150} \right) \right\} - 1 \quad (\sigma_m \geq 150 \text{ kPa}) \]

\[ (22) \]

\[ I_R = 5D_r - 1 \quad (\sigma_m < 150 \text{ kPa}) \]

\[ (23) \]

\[ \phi_p - \phi_R = 5I_R \]

\[ (24) \]

where \( D_r \) is the relative density and \( \sigma_m \) is the mean stress.

The dilatancy angle \( \psi' \) can be estimated from the modified Rowe's stress-dilatancy relation.

\[ \sin \psi' = \frac{\sin \phi - \sin \phi_0'}{1 - \sin \phi \sin \phi_0'} \quad (25) \]

\[ \phi_0' = \phi_R \left[ 1 - \beta \exp \left\{ -\left( \frac{\kappa}{e_d} \right)^2 \right\} \right] \quad (26) \]

where \( \beta \) and \( e_d \) are material constants.

The increment for \( \kappa \) is obtained as

\[ d\kappa = d\delta_p = \lambda \]

\[ (27) \]

The strain hardening softening modulus is expressed as

\[ A = -\frac{1}{\lambda} \frac{\partial^2 f}{\partial \kappa} \quad (28) \]

Elastic moduli are estimated using the following equations.
\[ G = G_0 \frac{(2.17 - e)^2}{1 + e} \sigma_m^{1/2} \]  
\[ k = \frac{2(1 + v)}{3(1 - 2v)} G \]

where \( e \) is the void ratio.

The element employed for the analysis was a pseudo-equilibrium model using a special reduced integration of the 4-noded Lagrange type element. The modified Newton-Raphson iteration method was applied in the nonlinear analysis solution.

The dry density which corresponded with the model test was 1.60 g/cm\(^3\). Peak and residual friction angles (\( \phi_p \), \( \phi_R \)) initial dilatancy angle (\( \psi_0 \)), Poisson's ratio (\( v \)) and elastic modulus (\( k \)) were chosen based on data from plane strain tests of air-pluviated dense Toyoura sand (Tatsuoka et al., 1986).

\[ \phi_p = 50^\circ, \phi_R = 35^\circ, \psi_0 = 20^\circ, \]
\[ v = 0.3, k = 50 \text{ kPa}. \]

Material constants \( B, C, m, \varepsilon_f, \varepsilon_r, \varepsilon_d \) and \( \beta \) used the following values.

\[ B = C = 0.5, m = 0.3, \varepsilon_f = 0.1, \varepsilon_r = 0.6 \]
\[ \varepsilon_d = 0.3, \beta = 0.2 \]

These constants were fitted by the trial and error method based on the data set of plane strain tests (Tatsuoka, et al., 1986; Tatsuoka, et al., 1990). A parametric study of model parameters was carried out on the footing problems (Siddique et al., 1991).

For the post-peak regime, deformations are localized to the shear band and material constants should be determined by local strain fields measured, for example, by means of the laser speckling method.

Vardoulakis et al. (1981) reported the shear band thickness was about 20 times the mean grain diameter (\( D_{50} \)) and the thickness of shear band (s.b.) was 0.3 cm. Tanaka (1987) discussed the influence of shear band thickness for collapse loading of strip footings and showed that as the thickness of shear band increases, the strain softening becomes less marked.

The back-prediction of plane strain compression tests by the finite element method using one element was carried out employing the above mentioned material properties. The calculated stress-strain-volume change relationship under \( \sigma_3 = 100 \text{ kPa} \) condition is shown in Fig. 3.

The finite element mesh used for the trapdoor analyses is shown in Fig. 4. The calcula-
tion was carried out by displacement control. In order to avoid the trap-door corner singularities, the elements beside the trap-door were given a linear incremental displacement distribution.

NUMERICAL AND EXPERIMENTAL RESULTS

We carried out calculations for prediction of the larger model test in which the sand mass height was 20 cm, and trap-door width was 10 cm.

1. Passive Mode

The calculated and experimentally observed load-displacement curves are shown in Fig. 5. In this figure, the load is the averaged earth pressure with the value of the initial self-weight subtracted. The calculated curves are comparable to the experimental ones. However, the load using the analysis with the simple strain softening model is rather high at initial displacement compared with the experiment, but the analysis with strain hardening-softening model agrees with the experiment. Figs. 6, 7 and 8 show the load displacement curves obtained by soil stress measurement cells (from No. 6 to No. 11, shown in Fig. 2) and calculated at locations corresponding to the stress cells. We can see that peak loads and displacements at peak load are both different in the experiment and analysis and these are considered to be the effect of progressive failure. In Fig. 9 the distribution of earth pressures at 0.6 mm displacement is shown. The calculated earth pressures are approximately identical with the observed earth pressures.
pressure. Both earth pressures show rather low values at middle and high at the edges of the trap-door. The progressive failure was also recognized by this phenomenon. Fig. 10 show the comparison between strain-softening (with the simple strain softening constitutive model) and perfect plasticity models. It is seen that the load obtained employing the strain-softening model is between those of the perfect plasticity model with peak and residual friction angles. Although the effect of strain-softening is clearly seen, it is not so remarkable as in footing problems. This is due to the difference of failure mode. Figs. 11, 12

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Fig. 9. Distribution of earth pressures (at 0.6 mm displacement)

Fig. 10. Comparison between the strain softening and perfect plasticity models

Fig. 11. Propagation of the shear band (experimental)

Fig. 12. Shear strain distribution (simple strain softening model)
and 13 show the observed and calculated propagation of the shear band. The shear strain shown in the figure is apparent maximum shear strain averaged at element level. The concentrated zone of shear strain is progressing upward and the direction of the localized narrow zone is approximately identical with the direction of outermost shear band development observed experimentally.

2. Active Mode

Fig. 14 shows the calculated and experimentally observed averaged load-displacement curves. The calculated curves are identical to the experimental ones. Figs. 15, 16 and 17 show the curves obtained by 11 stress measurement cells (from No. 1 to No. 6, shown in Fig. 2) and calculated at locations corresponding to each stress cell. We can see that minimum
loads and displacements at minimum load are nearly the same in both experiments and analyses, and are not considered to be the effect of progressive failure. In Fig. 18 the distribution of earth pressures at 0.3 mm displacement is shown. The calculated pressures are identical to the observed earth pressure. As both earth pressures show uniform distribution, the progressive failure was less marked. Fig. 19 shows the comparison between the strain-softening (using the simple strain softening model) and perfect plasticity models. It is seen that the load obtained employing the strain-softening model was approximately equal to that of the perfect plasticity model at peak friction angle. Fig. 20 shows the propagation of the shear band observed experimentally and Figs. 21 and 22 show the calculated apparent maximum shear strain. The localized narrow zone by the
an analysis is approximately identical with the experimentally observed direction of shear band development.

**SCALE EFFECTS**

We investigated the so called 'scale effect' under conditions where the ratio of sand mass height ($h$) and width of the trap door ($B$) is 2.0. Fig. 23 shows the relationship between normalized earth pressure ($\sigma/\rho_d \cdot h$) and the trap-door width in passive mode. Both experiment and finite element analyses show the reduction of the normalized earth pressure with increasing trap-door width. This phenomenon is called the scale effect. The progressive failure was evaluated and recognized in passive mode.
Since this experiment was carried out at very low confining pressure, crushing of the sand particles may be negligible (Tatsuoka et al., 1986), i.e. peak internal friction may not depend on the confining pressure. Accordingly the scale effect is considered to be due to the shear banding.

Fig. 24 shows the relationships between normalized earth pressure and the trap-door width in active mode. In this case the scale effect is not recognized by both experiment and finite element analysis, because the progressive failure is less marked in active mode.

**CONCLUSION**

The progressive failure and scale effect in trap-door problems were evaluated by comparing experimental results with elasto-plastic finite element analysis including the shear band effect. The distribution of earth pressure by finite element analysis achieved good agreement with the experimental results. The propagation of shear bands was investigated by experimental and calculated results in passive and active mode. In passive mode, the progressive failure and scale effect were recognized and evaluated by the analysis in the same manner as the experiment. In active mode, the progressive failure and scale effect were not recognized in both experiments and analyses.

**REFERENCES**