TWO-DIMENSIONAL ORIENTATION OF FINE QUARTZ GRAIN SUBJECTED TO DIRECT SHEAR

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ABSTRACT

The degree of grain orientation given by the magnitude of vector strength by visual measurements is compared with that of the birefringence ratio by an indirect one. To link both expressions, a concentration degree of distribution is introduced on the assumption that the grain arrangement obeys the circular normal distribution. The longitudinal section illustrating the shear-induced fabrics is put under a polarizing microscope, and quartz grains in it are measured by the trigonometric treatment. The grain orientation brought forth during shearing is evaluated as a random orientation, because grains rotate as the horizontal displacement increases. However, it is also shown that the parallel grain arrangement exists in a certain direction such as the slip line and its conjugate at the top of crack occurring during shearing.

Key words: birefringence ratio, direct shear, grain orientation, quartz grain, shear-induced fabric, vector strength (IGC: D3)

INTRODUCTION

In studying soil structure, much interest has been focused on grain or particle orientation, which reflects the geological and geochemical conditions. It is well known that such conditions relate strongly to anisotropic behavior of strength, compressibility and permeability of soil.

Recently, digital techniques of image processing and image analysis have been developed for the evaluation of the preferred particle orientation within micrographs (Luo et al., 1992). This new two-dimensional technique is basically the same procedure as the

trigonometric treatment performed in the field of geology (Pincus, 1956; Matalucci et al., 1969), because the image processing is to find individual boundaries of particles and the result of its analysis is expressed automatically on an angular histogram of the direction of long axis of particle in the micrograph. On the other hand, the optical method based on the birefringence phenomenon of clay aggregate was proposed by Morgenstern and Tchalenko (1967a). The birefringence ratio defined by the ratio of minimum to maximum transmitted light intensities evaluates from zero for the perfect parallel orientation to unity for the perfect random one. By applying the circular

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Manuscript was received for review on February 10, 1992.

Written discussions on this paper should be submitted before October 1, 1993 to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4 F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

normal distribution function (Gumbel et al., 1953), Fujikawa and Miwa (1974) presented the relationship between the birefringence ratio and vector strength obtained by the trigonometric treatment. The vector strength gives from zero for the perfect random orientation to unity for the perfect parallel one, as mentioned later.

For researches on the micromechanics of granular materials, numerical simulations are usually concerned with idealized systems. Although those simulations are used to verify the so-called stress-force-fabric relation (e.g. Rothenburg et al., 1989), the microscopic mechanism is not easily seen in experiments. And also it may be important that the degree of orientation is determined by a common parameter, because various indexes are used to express the preferred orientation.

This paper presents the microfabric showing the grain rearrangement with progressive failure, and evaluates the degree of grain orientation in the shear-induced fabric by using the vector strength, which is a common index for both trigonometric and optical procedures.

METHOD OF DATA ANALYSIS

Numerous fabric studies have been based on measurements of the direction of apparent long axis of grain seen in a thin section, which have become a more common tool in the study of directional properties of deposits. This treatment involves the measurements of the direction of long axis from a reference line. The well known procedure is applied for such a directional data analysis.

For grouped data, the vector strength $\bar{a}$ may be written in the following form,

$$\bar{a} = (1/M) \sqrt{\left(\sum_{i=1}^n f_i \sin \alpha_i \right)^2 + \left(\sum_{i=1}^n f_i \cos \alpha_i \right)^2} \quad (1)$$

where f_i is the number of grains counted at each interval. α_i is the mean direction of each interval, and M is the total number of grains ($\sum f_i$). We employ here the density function $f(\alpha)$ of the circular normal distribution given in the following form (Gumbel et al., 1953).

$$f(\alpha) = \{1/2\pi I_0(k)\} \exp [k \cos (\alpha_i - \alpha_0)] \quad (2)$$

where k is the concentration degree of distribution and $I_0(k)$ is the modified Bessel function of the first kind. α_0 is the mean direction of grain distribution. In order to obtain the relation of $\bar{a}$ and k , the likelihood function (L) is also given in the following form.

$$L = (2\pi)^{-n} I_0^{-n}(k) \exp \left[k \sum_{i=1}^n \cos (\alpha_i - \alpha_0) \right] \quad (3)$$

Equating $\partial/\partial \alpha_0 \log L$ and $\partial/\partial k \log L$ to zero, we obtain Eqs. (4) and (5).

$$\sum_{i=1}^n \sin (\alpha_i - \alpha_0) = 0 \quad (4)$$

$$I'_0(k)/I_0(k) = (1/M) \sum_{i=1}^n \cos (\alpha_i - \alpha_0) \quad (5)$$

From Eqs. (4) and (5), the parameter k is the solution of Eq. (6).

$$I_1(k) - \bar{a} I_0(k) = 0, \text{ where } I'_0(k) = I_1(k) \quad (6)$$

Eq. (6) was solved for k by interpolating in a table of Bessel functions.

From the measurements obtained at intervals of 10-degree in the range of 360 degrees, the vector strength can be calculated from Eq. (1) and the parameter k corresponding to it is given by using Eq. (6). For plotting both the observed data in frequency and its distribution curve in a linear scale on the same polar paper, the density function $g(\alpha)$ which is adequate for tracing equiareal frequency ovals on the polar paper is introduced in the following form

$$g(\alpha) = \sqrt{\exp (k \cos \alpha_i) / I_0(k)} \\ = \sqrt{1 + [2/I_0(k)] \sum_{n=1}^{\infty} I_n(k) \cos n \alpha_i} \quad (7)$$

in which $I_n(k)$ is the modified Bessel function of n -th kind. The radii of the distribution curve in an equiareal polar paper scale are obtained by multiplying the values given in Gumble's table (Gumble et al., 1953) or our table (Fujikawa et al., 1974) by $\sqrt{F/N}$, where F and N denote the total frequency and the number of wedges - in our case $F=100\%$ and $N=36$.

MATERIAL AND PREPARATION OF THIN SECTION

For observing visually the behavior of grain subjected to the direct shear, quartz sand powder was used and its physical properties are given in Table 1. The specimen compressed in a shear box (60 mm in diameter, 20 mm in height) at a normal stress of 196 kN/m^2 for thirty minutes was sheared under the conditions of a displacement rate of 1 mm/min and the space of upper and lower boxes of 0.2 mm . The shear stress-displacement curve is shown in Fig. 1 for illustrating the specimen achieved

Table 1. Physical properties and results of direct shear test

specific gravity	2.635
liquid limit (%)	29.0
plastic limit (%)	non.
sand fraction (%)	6.0
silt fraction (%)	52.5
clay fraction (%)	41.5
water content (%)	21.19
dry density (Mg/m^3)	1.548
void ratio	0.702
internal friction angle (deg.)	44.0
cohesion (kN/m^2)	1.0

Fig. 1. Shear stress-horizontal displacement curves (normal stress; 196 kN/m^2)

at the peak strength, and the result of the direct shear test is also given in Table 1.

The epoxy resin (U-Loid #120) was utilized to solidify the specimen achieved at the peak strength; that is, the epoxy resin with adjusted the gelling-time was impregnated in the specimen in the shear box. The specimen taken out was hard enough for thin-sectioning in the same manner as a rock sample, and was cut for taking the longitudinal section shown in Fig. 1.

SHEAR-INDUCED FABRICS

As shown in Photo. 1, details of the shear-induced fabric may be observed and photographed through the polarizing microscope under crossed nicols. The crack brought forth at the edge was about 25 degrees to the forced shear plane as illustrated diagrammatically in Photo. 1. Grains arranged at the directions of about 15 and 80 degrees to the forced shear plane were observed at the top zone of this crack. Such a fabric would reflect the mechanics of shear-box test itself, since the slip line and its conjugate may be considered to be inclined at $\phi/2=22$ and $(\pi/2-\phi/2)=68$ degrees to the horizontal from Table 1. Photo. 2 shows the grain arrangement of those two directions. Although those two parts of oriented grains move towards the center of specimen as the displacement increases, the preferred grain orientation is extinguished around the crack.

TRIGONOMETRIC TREATMENT

As shown in Photo. 1, the final direction of the crack was almost parallel to the forced shear plane, so that we choose this plane for the reference line. The trigonometric treatment was carried out in a range of 360 degrees at intervals of 10-degree. 105 grains for the position containing the crack were chosen out of grains from 0.1 to 0.3 mm long. 96 grains and 107 grains were also chosen at the top portion of the crack for evaluating the grain arrangement along the conjugate lines.

Fig. 2 shows a half of the frequency on the equiareal polar paper. In this figure, the

Photo. 1. Shear-induced fabric (OO; forced shear plane)

Photo. 2. Grain arrangements at top portion of crack

reference line is vertical to the forced shear plane in Fig. 2(C) only. The distribution curve given by k corresponding to $\bar{a}$ is also shown in Fig. 2. From Eq. (6), the perfect random orientation is given by $\bar{a}=0$ and $k=0$. In the case of perfect parallel orientation, $\bar{a}$ is unity and k is infinite. As shown in Fig. 2(B) and (C), the

two concentration degrees of distribution given at the top portion of the crack were almost the same and the mean value was 0.46, while at the position around the crack the concentration degree was 0.16 as shown in Fig. 2(A).

Fig. 2. Distribution of apparent long axis of grains ((A), (B); reference line is parallel to forced shear plane. (C); reference line is vertical to it)

CORRELATION OF VECTOR STRENGTH ($\bar{a}$) AND BIREFRINGENCE RATIO (β)

The light intensity (I) transmitted through a crystal grain under the crossed nicols is expressed by

$$I = I_0 \sin^2 2\theta \sin^2 (\pi d \Delta n / \lambda) \quad (8)$$

where I_0 denotes the intensity of incident illumination, θ the angle between optic axis and polarizer, d the thickness, Δn the double refraction and λ the wave length of incident illumination. The light intensity transmitted through the clay aggregate shown in Fig. 3 may be expressed by

$$I = 2I_0 \int_0^\pi g(\alpha) \sin^2 2(\theta + \alpha) \sin^2 (\pi d \Delta n / \lambda) d\alpha \quad (9)$$

If the aggregate is at the extinction position when $\theta=0$, the illumination position is obtained when $\theta=\pi/4$. Accordingly, minimum and maximum intensities can be written as follows.

$$I_{\min} = 2I_0 \int_0^\pi g(\alpha) \sin^2 2\alpha \sin^2 (\pi d \Delta n / \lambda) d\alpha \quad (10)$$

$$I_{\max} = 2I_0 \int_0^\pi g(\alpha) \cos^2 2\alpha \sin^2 (\pi d \Delta n / \lambda) d\alpha \quad (11)$$

Therefore, the birefringence ratio (β) defined by the ratio of the minimum transmitted light intensity to the maximum intensity is given in the following form.

$$\beta = \frac{\int_0^\pi g(\alpha) \sin^2 2\alpha d\alpha}{\int_0^\pi g(\alpha) \cos^2 2\alpha d\alpha} \quad (12)$$

The relationship between β and k shown in Fig. 4 was calculated by using the following form at an interval angle of 10 degrees (Fujikawa et al., 1974).

$$\beta = \frac{\sum_{i=0}^{180} \left[\sqrt{\left(1 + (2/I_0(k)) \sum_{n=1}^{\infty} I_n(k) \cos n\alpha_i \right)} \sin^2 2\alpha_i \right]}{\sum_{i=0}^{180} \left[\sqrt{\left(1 + (2/I_0(k)) \sum_{n=1}^{\infty} I_n(k) \cos n\alpha_i \right)} \cos^2 2\alpha_i \right]} \quad (13)$$

Fig. 3. Circular normal distribution (POP; direction of polarizer, COC; reference line in thin section, $g(\alpha)$; radii vector at α degree, θ ; stage angle)

Fig. 4. Relations of both β versus k and β versus R (after Fujikawa et al. and Morgenstern et al.)

For determining optically the preferred orientation of clay particles, Morgenstern et al. (1967a) used the model distribution, in which the half-length of the major axis of the ellipse is fixed at unity and the length of the minor axis (R) changes from 0 to 1. This distribution function $m(\alpha)$ may be written in the form

$$m(\alpha) = \frac{R}{\sqrt{1 - (1 - R^2) \cos^2 \alpha}} \quad (14)$$

where R is called the orientation degree. By using Eq. (14), the birefringence ratio is given as follows.

Fig. 5. Relations of both k versus R and $\bar{a}$ versus β

$$\beta = \frac{\int_0^{\pi/2} m(\alpha) \sin^2 2\alpha d\alpha}{\int_0^{\pi/2} m(\alpha) \sin^2 2\alpha d\alpha} \quad (15)$$

When k equals zero, the distribution expressed by Eq. (7) degenerates into the uniform circular distribution. This distribution corresponds to $R=1$ in Eq. (14). In this case the grain orientation is a perfect random with $\bar{a}=0$ and $\beta=1$. Otherwise, a perfect parallel orientation having $\bar{a}=1$ and $\beta=0$ is given when $k=\infty$ and $R=0$. The relation of β and R is also shown in Fig. 4. From the parameter k , the relations of $\bar{a}$ versus β and k versus R are shown in Fig. 5. Since the evaluation of the consolidated kaolin obtained by Morgenstern et al. (1967a) is the parallel orientation with $R=0.02$ on average, the preferred orientation can be expressed by $k > 10$. Therefore, by using k , the degree of orientation from coarse grain to fine can be evaluated, and the vector strength is related to the birefringence ratio, as shown in Fig. 5.

GRAIN ORIENTATION IN SHEARED SPECIMENS

The values of k and $\bar{a}$ given in Fig. 2 correspond to $R=0.99$ and $\beta=1$ for the portion containing the crack, and to $R=0.90$ and $\beta=1$ for the top portion of it. Accordingly, the random

orientation is evaluated by both methods, because grains rotate during shearing. However, as shown in Photo. 2, the parallel orientations along the slip line and its conjugate occur at the top zone of the crack during the shear, but it is difficult to evaluate their grain arrangements because of a limited number of grains. In general, 100 grains are required at least to identify the grain orientation by means of vector strength.

CONCLUSIONS

By using the concentration degree of distribution, the degree of orientation from coarse grains to fine particles can be evaluated. Therefore, the vector strength is related to the birefringence ratio.

The grain arrangement evaluated by both methods presents the random orientation in the sheared specimen, because of the rotation of grains with the progress of failure. On the other hand, it is also a fact that the parallel orientation exists in a certain direction such as the slip line and its conjugate, although it is temporary and in a very small region.

ACKNOWLEDGEMENTS

The research was carried out at Kyushu University, Fukuoka with the financial support by grant-in-aid for scientific research, Ministry of Education, Japan. The author is thankful to Dr. T. Fujikawa for providing an opportunity to work at his laboratory.

NOTATIONS

$\vec{a}$ =vector strength
 $f(\alpha)$ =density function of circular normal distribution
 $g(\alpha)$ =radii of probability density for equiareal tracing of $f(\alpha)$

I =light intensity transmitted through crystal grain under crossed nicols

I_0 =intensity of incident illumination

$I_{\min}, I_{\max}$ =minimum and maximum light intensity

$I_0(k), I_n(k)$ =modified Bessel function of first and n -th kinds

k =concentration degree of distribution

L =likelihood function

M =total number of grains

$m(\alpha)$ =model distribution function

R =orientation ratio

α, α_i =direction of apparent long axis of grain

α_0 =mean direction of grain distribution

β =birefringence ratio

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