THEORETICAL ASPECTS OF CONSTITUTIVE MODELLING FOR UNSATURATED SOILS

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ABSTRACT

In this paper, several theoretical aspects for preparation of constitutive equations of unsaturated soils are discussed. First, possible pore water states are described. Three states, insular air, fuzzy and pendular saturation are taken into account to examine the mechanical behavior of unsaturated soils. Suction effects are clarified for each state and these are used to interpret the overall mechanical behavior of unsaturated soils. The suction effects can be classified into two categories. One is that an increase in suction induces an increase in effective stresses and the other is that an increase in suction induces an increase in both yield stress and the stiffness of the soil skeleton against plastic deformations. The first suction effect can be estimated by formulating the relationship between suction and shear strength at the wet side of critical state. The second suction effect can be estimated from the state surface concept. The modified Cam clay model with these suction effects is proposed. The performance of this model will be examined in the paper (Kohgo et al., 1993).

Key words: capillary phenomena, constitutive equation of soil, effective stress, plasticity, unsaturated soil (IGC: D5/D6/E13)

INTRODUCTION

Many soils near the ground surface and compacted soils are unsaturated. Unsaturated soils consist of three phases, namely, solid (soil particles), liquid (pore water) and gas (pore air). The effects taken account of in the surface chemistry field act between each phase and make it difficult to understand clearly the mechanical behavior of unsaturated soils. From the mechanical view point, surface tension effects are the most significant in those

effects. The surface tension between water and air contributes to forming a meniscus and induces a pressure difference (suction) between pore water and pore air. On consideration of the mechanical behavior of unsaturated soils, it is very important to appreciate the suction effects.

In early work on unsaturated soils, suction effects were introduced into the framework of the principle of effective stress and attempts were made to explain the mechanical behavior of unsaturated soils in terms of effective

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stresses (Bishop, 1959; Aitchison, 1960; Jennings, 1960). At the Conference on Pore Pressure and Suction in Soils held in London in 1960, it was concluded that the expression defining the effective stress proposed by Bishop was the most generally applicable of those available. Bishop (1959) proposed the following expression defining effective stress for unsaturated soils.

$$\sigma' = (\sigma - u_a) + \chi s \quad (1)$$

$$s = u_a - u_w \quad (2)$$

where σ' is effective stress, σ total stress, s suction, u_a pore air pressure, u_w pore water pressure and χ a material parameter.

After the conference, the majority of investigations were concentrated on examining whether Eq. (1) was valid or not. Jennings and Burland (1962) showed that there were limitations in the use of Eq. (1) as the effective stress expression, since the volume change behavior due to wetting (the collapse phenomenon) could not be explained by Eq. (1). Moreover, Burland (1965) insisted that it was not essentially correct to compose $(\sigma - u_a)$ and s into a single effective stress as in Eq. (1) from the microscopic view point. However, the inconsistency is not reasonable, since effective stress must be essentially defined from the continuum mechanical view point (Atkinson and Bransby, 1978) and not from the microscopic view point. Thus, it is one main reason for limitations on the use of Eq. (1) that the collapse phenomenon cannot be explained solely by Eq. (1).

Several workers have attempted to express the stress-strain relationships for unsaturated soils using the two stress components $(\sigma - u_a)$ and s regarding them as independent variables in the way Coleman (1962) proposed (stress variable method). Matyas and Radhakrishna (1968) performed isotropic and K_0 compression tests on mixtures of kaolin and flint powder under controlling $(\sigma - u_a)$ and s . They used the above stress variable method to estimate the volume change behavior. The same approach was lately adopted by Lloret and Alonso (1980) to analyze the consolidation behavior of un-

saturated soils. Lloret et al. (1987) also proposed the nonlinear hyperbolic elastic model. Alonso et al. (1990) proposed the elastoplastic model which had two yield curves, LC and SI. The LC and SI denote respectively loading collapse and suction increase yield surfaces. As described in the later section, the suction increase, namely desaturated path is elastic when the suction is greater than air entry suction. The definition of SI regards the desaturated path as a plastic path. It is thus inconsistency with the experimental facts. Karube and Kato (1989) proposed an elastoplastic model based on effective vertical stress $f(s)$ due to suction. However, it is very complex to express the swelling and desaturated paths using the stress variable method. Moreover, it is very difficult to apply many constitutive models, which have been developed for saturated soils (for instance, subloading, two surface and multi surface models formulated on the base of the generalized elastoplastic theory), to unsaturated soils.

Kohgo (1987) interpreted the mechanical behavior of unsaturated soils using the elastoplastic theory and proposed a generalized critical state type elastoplastic model, which was formulated in terms of the effective stress defined as a function of suction instead of the above stress variables, with plastic volumetric strain and suction as the hardening parameters. Kohgo et al. (1991) furthermore discussed the suction effects in more detail and clarified to classify the suction effects into two categories. They proposed a more reasonable elastoplastic model.

The purpose of this paper is to interpret the mechanical behavior of unsaturated soils from micro and macro structure aspects and propose a simple elastoplastic model for unsaturated soils. First, possible pore water states should be described, since suction effects are influenced greatly by retentive situations of pore water. Suction effects are clarified for each state and used to interpret the mechanical behavior of unsaturated soils. Finally an elastoplastic model is made up through the above examinations. A more general version model, will be proposed and

verified in the paper (Kohgo et al., 1993).

SUCTION EFFECTS AND SOIL PARTICLE BEHAVIOR

In this chapter, discussions are mainly concentrated on granular soils. Similar discussions may be acceptable in clayey soils except some kinds of expansive clayey soils, since clayey soils tend to form structures composed of “packets” of clay particles as they dry (Jennings and Burland, 1962).

Possible Pore Water States

The mechanical behavior of unsaturated soils is influenced greatly by saturation conditions in soils. Thus, first, a mention of the possible states of pore water is necessary. Here, we restrict our consideration to free water. Adsorbed water is regarded as the soil phase.

Bear (1979) shows possible water states in a water wet granular soil as in Fig. 1. At a very low saturation (Fig. 1(a)), water is retained in menisci formed around the grain contact points. This saturation condition is called “pendular saturation”. As water saturation increases, a continuous water phase is formed. This saturation condition is called “funicular saturation” (Fig. 1(b)). Both water and air phases are continuous. In both pendular and funicular saturation conditions, the mechanical behavior of soils is affected by

Fig. 1. Possible water saturation states (from Bear, 1979)

capillary forces, which are closely related to the magnitude of suction as described in the following section. If water saturation increases further, the pore air will lose its continuity and some parts of the air will remain as air bubbles surrounded by water. This condition is called “insular air (or occluded) saturation” (Fig. 1(c)). In this condition, the mechanical soil behavior can be explained in terms of effective stresses, the same as those estimated by Terzaghi’s equation for completely saturated soils. This fact will be discussed later.

In general, real soil has various pore sizes. Therefore, if the suction exceeds the air entry suction s_e and air enters the pores, the large pores will empty at low suction, while the small ones will be saturated until the higher suction. Thus, in real soil, retentive situations of pore water are not uniform and the three saturation conditions described above must coexist according to pore size except for suction lower than s_e and at extremely high suction.

In the following, it is convenient to take account of the three saturation conditions as shown in Fig. 2 to examine the mechanical behavior of unsaturated soils. Figures 2(a) and 2(c) correspond to Figs. 1(c) and 1(a), respectively. The condition as shown in Fig. 2(b) is transient from that in Fig. 2(a) to that in 2(c), and in this condition, three saturation condi-

Fig. 2. Possible saturation conditions in real soils

tions, that is, pendular, funicular and insular air saturation, coexist. Herein, we call this saturation condition "fuzzy saturation".

Suction and Capillary Forces

In real soils, it is very difficult to understand the relationship between suction and capillary forces. Here, it is assumed that the soil used is ideal and consists of uniform spheres. Accordingly, we must be able to examine the above relationship, comparatively easily.

At low saturation, water is retained in the menisci around grain contact points, as illustrated in Fig. 3. Each of the menisci tends to draw the particles together. This attractive force, called the capillary force, always acts perpendicular to the grain contact surface (Haines, 1925).

Now, supposing N_c is the capillary force, it can be obtained as follows considering the equilibrium on the C-C section in Fig. 3 (Fisher, 1926).

$$N_c = \frac{2\pi r T_s}{(1 + \theta_t)} \quad (3)$$

$$\theta_t = \tan(\theta_s/2) \quad (4)$$

where T_s denotes the surface tension between water and air, r the radius of a sphere and θ_s is defined in Fig. 3. It is assumed that the contact angle between the spheres and water is zero.

If r_1 and r_2 are the two principal radii of curvature of the meniscus, then the suction s is

$$s = T_s(1/r_1 - 1/r_2) = \frac{T_s(1 + \theta_t)(1 - 2\theta_t)}{2r\theta_t^2} \quad (5)$$

Fig. 3. Capillary force applying between two uniform spheres

Fig. 4. Normalized relationship between N_c and s

The relationship between N_c and s can be made clear by using Eqs. (3) and (5) regarding θ_t as a parameter. Figure 4 shows the normalized relationship between N_c and s . This figure indicates obviously that N_c increases with an increase of s . When s is a relatively small value, the rate of increase in N_c is comparatively high. However, it becomes lower as s increases. It may be seen from Eqs. (3) and (5) that the value of N_c becomes $2\pi r T_s$ as s approaches infinity. However, this limit cannot be reached, since Eq. (3) breaks down as r_1 and r_2 approach molecular dimensions (Baver, 1956).

The relationship between suction and capillary forces at each contact point, as illustrated in Fig. 4 could be acceptable in real soils.

Capillary Forces and Soil Particle Behavior

Here, we examine the suction effects on soil particle behavior at low saturation where water exists mainly within menisci around the grain contact points.

Application of external forces, as if they are isotropic, causes both normal and shear forces at each grain contact point, as shown in Fig. 5(a). Capillary forces are also applied at the same points in this condition. Thus, the normal force N_t and the shear force T_t caused by both external and capillary forces at any typical contact point are obtained as follows.

$$N_t = N + N_c \quad (6)$$

$$T_t = T \quad (7)$$

Fig. 5. Intergranular forces due to external and capillary forces

where N and T are respectively the normal and shear forces caused by external forces at any typical contact point.

Plastic deformations of the soil mass as a whole will be mainly induced by relative sliding between the particles (Lambe and Whitman, 1979). Hence, the resistance of soil to plastic deformations is strongly influenced by the shear resistance at grain contact points. The shear resistance between particles arises from the attractive forces acting among the surface atoms of the particles. According to the adhesion theory of friction (Bowden and Tabor, 1954), the mechanism of the shear resistance can be explained as follows. Since all surfaces are rough on a microscopic scale, the real contact between surfaces is at the asperities and the real contact area is normally extremely smaller than the nominal area. Therefore, actual contact stresses at the asperities are much higher than the nominal

stresses and the plastic flow will occur at the contacts. Thus, the real contact area A_c will be

$$A_c = N_i / h_s. \quad (8)$$

Where h_s is the normal stress required to cause yielding.

Supposing τ_s is the shear strength of the adhered junctions, the maximum possible shear force $T_{\max}$ will be

$$T_{\max} = \tau_s A_c. \quad (9)$$

Substituting Eqs. (6) and (8) into Eq. (9), Equation (9) will be

$$T_{\max} = N \tan \phi_m + c_s \quad (10)$$

$$c_s = N_c \tan \phi_m \quad (11)$$

$$\tan \phi_m = \tau_s / h_s. \quad (12)$$

Where ϕ_m is the soil particle to particle friction angle and c_s denotes the contribution of shear resistance caused by the capillary force N_c .

In this case, "junction growth" caused by shear contact forces may not be considered, since this phenomenon only affects the values of ϕ_m and our purpose in this section is to gain knowledge of suction effects to soil particle behavior. If two surfaces in contact with each other have a very large number of asperities, Equations (10) and (11) will also be valid even though the individual asperities are deforming elastically (Archard, 1957).

The following information about suction effects to soil particle behavior can be obtained from processing the above formulation. Figure 4 has indicated obviously that the increase in suction s increased the capillary force N_c . This increase of N_c induces two effects. One is an increase of deformations occurring from the increase of A_c . Their quantity would usually be very small. The other is an increase of shear resistance between the soil particles. This inhibits the relative sliding between the particles. Therefore, overall plastic deformations are inhibited and the magnitude of shear resistance of soil increases. In most of the problems usually encountered in the field, c_s is independent of the external forces, since pore air is always connected with the atmosphere, both pressures are usually almost

the same and pore water pressure in such a low saturation condition is not affected by the external forces. Thus, c_s may be regarded as nominal cohesion.

SUCTION EFFECTS AND MECHANICAL BEHAVIOR OF UNSATURATED SOILS

Here, the overall mechanical behavior of unsaturated soils is examined taking account of the above discussions and aspects of the elastoplastic theory. First, we examine the volume change behavior of unsaturated soils and next, the shear resistance characteristics.

Before the discussion, we must mention about Terzaghi's effective stress concept. Terzaghi (1936) described the effective stress concept for saturated soils as follows. "The stresses in any point of a section through a mass of earth can be computed from the total principal stresses σ_1 , σ_2 and σ_3 which act in this point. If the voids of the earth are filled with water under a stress u_w , the total principal stresses consist of two parts. One parts, u_w , acts in the water and in the solid in every direction with equal intensity. It is called the neutral stress. The balance, $\sigma'_1 = \sigma_1 - u_w$, $\sigma'_2 = \sigma_2 - u_w$ and $\sigma'_3 = \sigma_3 - u_w$, represents an excess over the neutral stress u_w and it has its seat exclusively in the solid phase of the earth. This fraction of the total principal stresses will be called the effective principal stresses." Thus, the effective stress is defined as follows.

$$\sigma' = \sigma - u_w \quad (13)$$

or

$$p' = p + s. \quad (14)$$

Here, p' is the mean effective stress, and supposing σ_m is the mean total stress, p will be

$$p = \sigma_m - u_a. \quad (15)$$

In the following arguments, the term of effective stress means the stress σ' defined in Eq. (13), until the effective stress is newly defined.

Volume Change Behavior

The volume change behavior of unsaturated soils is affected by applying external stress and suction. First, we examine the volume change behavior due to an increase in suction and next that due to an increase in external stress. The former is called the suction consolidation (or shrinkage) and the latter is called the conventional consolidation here.

Figure 6 shows the typical test results of a suction consolidation in which a silt was initially in a slurry condition (Jennings and Burland, 1962). The stress path is desaturated. At suction smaller than $\bar{s}_e$ denoted in Fig. 6(a), the soil is seen to be fairly compressible. The suction value $\bar{s}_e$ is nearly equal to the air entry suction s_e value obtained from the soil water retention curve shown in Fig. 6(b). As p is constant during this test, if Terzaghi's effective stress concept is available, an increase in s is equal to an increase in p' from Eq. (14). The relationship between $e \sim \log s$ must be namely equivalent to that between $e \sim \log p'$. The in-

Fig. 6. Compression test results under suction pressures on silt (after Jennings and Burland, 1962)

Fig. 7. Consolidation and shrinkage curves on Scott clay (after Blight, 1966)

initial slope λ of the $e \sim \log s$ curve shown in Fig. 6(a) is almost the same as that of the $e \sim \log p'$ curve obtained from an isotropic compression test for this identical saturated soil under normal consolidation. The relationship between $e \sim \log s$ is namely equivalent to that between $e \sim \log p'$. This fact can also be seen in Fig. 7. Figure 7 shows triaxial consolidation and shrinkage data for Scott clay which was initially in a slurry condition (Blight, 1966). The results of the consolidation tests agree fairly well with those obtained from shrinkage tests. The relationship between $e \sim \log s$ is namely equivalent to that between $e \sim \log p'$. It is thus concluded that the volume change behavior can be explained by using Terzaghi's effective stress equation in the insular air saturation condition ($s \leq s_e$).

Once air enters the pores ($s > s_e$), the rate of volume change decreases very rapidly, as illustrated in Fig. 6(a). This situation is the fuzzy saturation condition shown in Fig. 2(b). As described before, in the large pores, water is retained in the menisci formed at the grain contact points and soil particles are confined by the application of capillary force, while the small pores are still saturated and the soil particles may move comparatively easily. However, overall deformations of the soil must be inhibited because of the confinement of the soil particles forming the large pores. If suction becomes very high, the soil will be nearly incompressible. This situation is the

pendular saturation condition (Fig. 2(c)). In this condition, water only exists in the menisci and all soil particles are confined.

The inhibition of overall deformations of the soil caused by the confinement of the soil particles forming the large pores as described above can be regarded as an internal confinement. This internal confinement expands as suction increases because of the expansion of unsaturated area. A similar confinement of soils is also induced by application of preconsolidation pressure. Though the latter refers to the interlocking effects, the former seems to be almost equivalent to the latter from the aspect of overall mechanical behavior of soils under the application of external forces. Both have the effect of inhibiting the plastic deformations of soils, that is, the interparticle sliding. The effect can be estimated as the increase of the yield stress in the elastoplastic theory. Further discussions about the internal confinement will be presented in the next section. Thus, the volume change behavior on a desaturated path in fuzzy saturation is one where the soil particles are confined and the plastic deformations of soil are inhibited. Therefore, the path is an elastic one.

Next, let us see the volume change behavior of the conventional consolidation. Figure 8 presents the consolidation test paths with con-

Fig. 8. The $e \sim \log p$ curves under constant suction (after Iwasaki, 1978)

stant suction plotted in $e \sim \log p$ space (Iwasaki, 1978). This figure indicates obviously the following three points. (a) The yield stress increases as the suction increases. (b) In greater suction, the smaller slope λ^* of the $e \sim \log p$ curves in the plastic range (normally consolidation region). In other words, the stiffness of the soil skeleton in the plastic range increases as suction increases. (c) The slope κ of the $e \sim \log p$ curves at unloading will not be affected by suction. These characteristics refer to the effect that suction inhibits the slippage of soil particles.

A typical example of the test results, which unitedly represent the volume change behavior of the conventional and suction consolidation tests as described above, can be seen in Fig. 9. Figure 9 shows the volume change behavior of an unsaturated soil caused by increasing applied external stress or decreasing suction (Matyas and Radhakrishna, 1968). In this series of test, the initial density of the specimens is not so high because of statical compaction. The yield stress due to compaction is namely not so high. As these stress paths may immediately enter the plastic range after loading because of the low yield stress, they are similar to the $e \sim \log p$ paths in the normally consolidated range in Fig. 8. The

decreasing suction (soaking) paths of the specimens, which were in the normally consolidation range before soaking, traversed these $e \sim \log p$ lines in the normally consolidated range. Matyas and Radhakrishna (1968) called these stress paths, which traced a unique surface in the space with e , p and s axes, the "state surface". Thus, the state surface defines the plastic range. Matyas and Radhakrishna (1968) also presented that the increasing suction (desaturated) paths at constant applied stress or the decreasing applied stress (unloading) ones at constant suction fell below this surface. These paths are elastic as described before.

Shear Resistance Characteristics

The relationships between suction and shear strength obtained from direct shear tests under controlled suction (Escario and Sáez, 1986; Gan et al., 1988) are shown in Fig. 10. Here, τ_0 is the shear strength without applying suction at each confining stress. The $(\tau - \tau_0)$ stress therefore expresses the increase in the shear strength with increasing suction. The rate of $(\tau - \tau_0)$ at low suction is greater than that at high suction. The gradient of $(\tau - \tau_0)$ to s at low suction is nearly equal to ϕ' . ϕ' is the angle of internal friction with respect to effective stress. Thus, an increase in suction may be regarded as an increase in effective stress, when the suction is smaller than a certain value which is approximately equal to the air entry suction s_e (Fredlund et al., 1987). Therefore, Terzaghi's effective stress equation is also accepted at shear in the insular air saturation condition ($s \leq s_e$) accounting Eq. (14).

If suction exceeds s_e , the rate of $(\tau - \tau_0)$ will become small. The saturation condition is fuzzy saturation until suction becomes very high. Small pores are still saturated and pore water in large pores is retained in menisci formed around grain contact points as illustrated in Fig. 2(b). Soil particles forming large pores are confined by an increase in capillary force as described in the previous section. While in small pores, effective stress may only increase with an increase in suction as effective stress increases in insular air saturation. The

Fig. 9. State surface for isotropic compression (after Matyas and Radhakrishna, 1968)

Fig. 10. Relationships between shear strength and suction ((a) after Gan et al., 1988, (b) after Escario and Sáez, 1986)

internal confinement as described in the previous section may also be induced. The magnitude of the internal confinement is affected not only by magnitude of suction but also by magnitude of applied external stress. It becomes smaller with the greater applied external stress. The fact concerned with the internal confinement can be seen in the following experimental results.

Figure 11 shows the triaxial test results on the specimens with and without wetting after they are compacted in the same initial conditions (Nobari and Duncan, 1972). The range of the diameter of soil particles was 1.19~2.38 mm (coarse sand). The specimens were compacted by vibration in a 3.56 cm diameter mold to a relative density of 90%. The “dry” specimens were tested at an air-dry water content of about 2%, while the “wet” specimens were saturated after consolidation. Suction in the “dry” specimens is almost constant during the tests because water content is very low and almost constant during the tests. Comparing the “dry” specimens with the confining stresses $\sigma_3 = 98, 294$ and 588 kN/m^2 , the amount of dilation becomes greater and softening behavior becomes more remarkable as σ_3 becomes lower. Comparing between the “dry” and “wet” specimens with the same $\sigma_3 = 98 \text{ kN/m}^2$, the amount of dilation in the

Fig. 11. Drained triaxial test results on wet and dry samples with same initial compacted conditions (after Nobari and Duncan, 1972)

“dry” specimen is greater than that in the “wet” one. As dilatancy can be usually regarded as the phenomena induced by the internal confinement (for example, interlocking), the fact that the amount of dilation in the “dry” specimen is greater than that in the “wet” one expresses that the intensity of the internal confinement in the “dry” specimen is stronger than that in the “wet” one. The difference of the intensity of the internal confinement between the “dry” and “wet” specimens refers to the internal confinement due to suction or N_c described above.

Summarizing the above discussion, the shear strength in fuzzy saturation is affected by the following 4 points.

- (1) Effective stress increases with an increase in suction in saturated zone.
- (2) The saturation zone decreases with an increase in suction.
- (3) Capillary force N_c increases with an increase in suction in unsaturated zone.
- (4) Internal confinement, that is, the inhibition of overall deformations of the soil caused by the Point (3) becomes greater as suction increases.

As slippage between soil particles occurs anywhere in soil on the critical state line at the wet side of critical state, the behavior is uniform. Here, uniform behavior means the behavior where no internal confinement can be found. The wet and dry sides of critical state respectively mean the normally consolidation and overconsolidation sides of critical state. The definition is the same as that in the Critical State Soil Mechanics (Atkinson and Bransby, 1978). The phenomenon of the Point (4) induces the dilatancy as shown in Fig. 11, which is the same as that induced by interlocking, and makes the soil behavior non-uniform. The Point (4) is the overall confinement of soil induced by the Point (3) and cannot be estimated until the elastoplastic theory is introduced. It is namely treated as an expansion of yield surface (an increase in yield stress) due to increasing suction which is the same manner as that employed in the case of consideration of interlocking effects due to application of preconsolidation pressure. If the Point (3) is not superior to the applied external forces, the

uniform soil behavior, which you can see in the shear behavior with $\sigma_3 = 588 \text{ kN/m}^2$ shown in Fig. 11, may occur. If the soil behavior due to the applied external forces is uniform, the behavior may be expressed by using stresses defined in the continuum mechanics. Comparing shear strength between saturated and unsaturated soils in the uniform shear behavior which you can see in the shear behavior of “dry” and “wet” specimens with $\sigma_3 = 588 \text{ kN/m}^2$ shown in Fig. 11, we may consider that the extra confining stresses applies in the unsaturated soil. These extra confining stresses are the effective confining stresses and may be regarded as the effective stresses.

Effective Stress

Here, effective stress in unsaturated soils is defined as follows. When the effects concerned with Points (1)~(3) as described in the previous section can be regarded as the effect of an increase in effective confined stress as described above the effective confined stress will be defined as effective stress.

In unsaturated soils, an increase of suction induces an increase of effective stresses, yield stress and stiffness of soil skeleton. Therefore, the effective stress concept in unsaturated soils may not be the same as that in saturated ones. The original definition of effective stress in saturated soils requires that changes in volume and shearing strength of a soil are due exclusively to changes in effective stress. However, in unsaturated soils, effective stresses do not only play significant roles. The softening or hardening of soil skeleton sometimes plays a more significant role. Collapse is a typical example of the latter. Swelling, shrinkage of the samples with elastic initial conditions and shearing strength of a soil at the wet side of the critical state may be controlled by the effective stresses. As swelling and shrinkage of the samples with elastic initial conditions are elastic behavior and the capillary force N_c does not affect the elastic behavior as described in the previous section, the effect due to suction may only be regarded as the effect of effective stresses. The effective stress equation in unsaturated soils can namely be defined using these relationships. Accord-

ing to the reason described in the previous section, the effective stress can also be estimated formulating the relationship between suction and shear strength at the wet side of critical state. Such a formulation of effective stress will be presented in the following section.

ELASTOPLASTIC CONSTITUTIVE MODELLING FOR UNSATURATED SOILS

The suction effects described above can be summarized as following two suction effects.

(1) An increase in suction induces an increase in effective stresses.

(2) An increase in suction induces an increase in both yield stress and the stiffness of the soil skeleton against plastic deformations.

The effects (1) and (2) can be respectively estimated formulating the effective stress and the state surface. These processes will be described as follows in this chapter. An elastoplastic model for unsaturated soils will be proposed taking account of these suction effects. The elastoplastic model proposed here is a simple version of those models.

Formulation of Effective Stress Equations

Here, effective stress equations are formulated using the relationship between shear strength of soil and suction on the wet side of the critical state. According to the experimental result as illustrated in Fig. 12, it seems to be reasonable that the relationship is postulated

Fig. 12. Increase of shear strength due to suction

as a hyperbolic function. The following effective stress equations may be empirically defined.

$$\sigma' = \sigma - u_{eq} \quad (16)$$

$$u_{eq} = u_a - s \quad (s \leq s_e) \quad (17)$$

$$u_{eq} = u_a - \left(s_e + \frac{s_c - s_e}{s^* + a_e} s^* \right) \quad (s > s_e) \quad (18)$$

$$s^* = 0 \quad (s \leq s_e) \quad (19)$$

$$s^* = s - s_e \quad (s > s_e) \quad (20)$$

where u_{eq} is the so called equivalent pore pressure (Nakase, 1969), a_e a material parameter, s_c the critical suction and s^* the effective suction which represents an excess over the air entry suction.

The increment of σ' can be obtained from Eqs. (16), (17) and (18).

$$d\sigma' = d\sigma - du_a + \chi ds \quad (21)$$

$$\chi = 1 \quad (s \leq s_e) \quad (22)$$

$$\chi = \frac{a_e(s_c - s_e)}{(s^* + a_e)^2} \quad (s > s_e). \quad (23)$$

Formulation of State Surface

An increase in both the stiffness of the soil skeleton at the plastic range and yield stress refers to the inhibition of the relative sliding between the soil particles due to applied suction in the unsaturated parts of a soil. In other words, such an increase of the soil stiffness arises from the inhibition of plastic deformations. Thus, the effect can be estimated from formulation of the state surface which represents the plastic range and defines the state boundary of volume change behavior in unsaturated soils. It is similar to the state boundary concept in saturated soils defined in Critical State Soil Mechanics (Atkinson and Bransby, 1978).

If the state surface as illustrated in Fig. 9 is replotted in the space with the axes, $\log p'$, s^* and e , it will be represented as Fig. 13(a). Figures 13(b) and 13(c) show projections of the test paths on the state surface in Fig. 13(a) to s^* constant plane and p' constant plane, respectively. If the relationships between e and $\log p'$ are linear, this family of test results will be expressed as follows.

Fig. 13. The generalized state surface

$$e = -\lambda^* \log p' + \Gamma^* \quad (24)$$

In the case of saturation, $\lambda^* = \lambda$ and $\Gamma^* = \Gamma$. Therefore, Equation (24) is

$$e = -\lambda \log p' + \Gamma \quad (25)$$

If the $s^* \sim e$ curves shown in Fig. 13(c) fit into the family of hyperbolic equations, they will be

$$y = \frac{(e^n - e_0^n)}{(e_0^n - e^n)} = \left(\frac{s^*}{a_s} \right)^{n_s} \quad (26)$$

where a_s and n_s are the material parameters, e_0^n and e^n are defined in Fig. 13(c), and superscript n denotes the n th $s^* \sim e$ curve. $e_0^n, \dots, e_0^n$ can be expressed by Eq. (25), since they are void ratios at the saturation condition. The void ratio e at any point on the state surface can therefore be obtained from Eq. (24) in which λ^* and Γ^* are defined as follows.

$$\lambda^* = \frac{\lambda}{1+y} \quad (27)$$

$$\Gamma^* = \frac{\Gamma + e_0^n y}{1+y} \quad (28)$$

To assess the applicability of the equations formulated here, test data on kaolin and flint powder mixture compacted soil (Matyas and Radhakrishna, 1968) were compared with the values estimated by using the above equations. Material parameters for definitions of effective

stress and the state surface are as follows, $s_e = 20 \text{ kN/m}^2$, $s_c = 118 \text{ kN/m}^2$, $a_e = 98 \text{ kN/m}^2$, $\lambda = 0.176$, $\Gamma = 1.156$, $e_0^0 = 0.928$, $a_s = 217 \text{ kN/m}^2$, $n_s = 0.87$. The parameters concerned with effective stress were obtained from the result of suction consolidation test on the specimen which was initially in the elastic range. If the specimen is in the elastic range, we can estimate effective stress from the relationship between the amount of volume change and suction because the compression index κ is constant. Other parameters concerned with the state surface can be obtained plotting the test data in the space of e and $\log p'$. The more details can be seen in the paper (Kohgo et al., 1993). Figure 14 shows the agreement between the measured void ratios and those predicted. Both agree well. Thus, the equations formulated here can indeed express the state surface.

Elastoplastic Model for Unsaturated Soils

The yield stress σ_y is assumed to be a function of the plastic volumetric strain ϵ_v^p and s^* according to the argument presented before. σ_y values namely reduce to a family of curves for all possible s^* values. Therefore, σ_y is

$$\sigma_y = \sigma_y(\epsilon_v^p, s^*). \quad (29)$$

Fig. 14. Comparison of predicted and measured void ratios on Kaolin + Flint powder

The simplest version of proposed elastoplastic models will be made up on the basis of the modified Cam-clay model (Roscoe and Burland, 1968). The modified Cam-clay model is

$$f_1 = M^2(p' - P_0)^2 + q^2 - (MP_0)^2 = 0 \quad (30)$$

$$P_0 = \frac{P_c}{2} \quad (31)$$

$$p' = (\sigma'_1 + \sigma'_2 + \sigma'_3)/3 \quad (32)$$

$$q = \sigma'_1 - \sigma'_3 \quad (33)$$

$$M = \frac{6 \sin \phi'_{cs}}{3 - \sin \phi'_{cs}} \quad (34)$$

where ϕ'_{cs} is the angle of internal friction of critical state line, σ'_1 , σ'_2 , and σ'_3 the three principal effective stresses which are estimated using Eq. (16), and P_0 and P_c as defined in Fig. 15.

P_c is the yield stress of this model. Hence, P_c is defined as follows from Eq. (29).

$$P_c = \sigma_y(\varepsilon_v^p, s^*). \quad (35)$$

P_c may be estimated by means of the state surface concept (see Appendix 1).

$$P_c = \exp\left(\frac{B^* + \varepsilon_v^p}{A^*}\right) \quad (36)$$

$$A^* = \frac{(\lambda^* - \kappa)}{2.3(1 + e_0)} \quad (37)$$

$$B^* = \frac{(\lambda_0^* - \kappa)}{2.3(1 + e_0)} \ln(p'_0) - \frac{(\Gamma_0^* - \Gamma^*)}{(1 + e_0)} \quad (38)$$

where κ is the slope of $e \sim \log p'$ curve at unloading, e_0 the initial void ratio, p'_0 the initial mean effective stress, Γ_0^* the initial value of Γ^* and λ_0^* is the initial value of λ^* .

Fig. 15. Yield surface of Cam clay model

Associated flow rules are adopted here. Therefore, the plastic potential function is

$$\psi_1 = f_1. \quad (39)$$

CONCLUSIONS

We obtained the following conclusions.

(1) In real soils, it is practical to take account of the three saturation conditions, insular air saturation, fuzzy saturation and pendular saturation.

(2) In insular air saturation ($s \leq s_e$), an increase in suction only contributes to an increase in effective stresses. In fuzzy saturation ($s > s_e$), it not only inhibits the relative sliding between the soil particles but also increases the effective stresses. In pendular saturation ($s > s_e$), it only induces inhibition of the relative sliding between the soil particles.

(3) The inhibition of the relative sliding between the soil particles caused by suction increases both yield stress and the stiffness of soil skeleton against plastic deformations. Such an suction effect may be formulated defining the state surface in the space e , s^* and p' as three axes.

(4) Effective stress equations may be formulated using shear strength of soil on the wet side of the critical state line. Effective stress equations can be defined as a function of s^* in Eqs. (16) ~ (20).

(5) An elastoplastic model was proposed taking account of these two suction effects. The model is formulated using the newly defined effective stress for unsaturated soils. In the model with the plastic volumetric strain as a plastic parameter, the amount of the plastic volumetric strain can be estimated formulating the state surface in the space e , s^* and p' as three axes. The verification of the model and the more general version model will be performed in the paper (Kohgo et al., 1993).

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REFERENCES

- 1) Aitchison, G. D. (1960): "Relationships of moisture stress and effective stress functions in unsaturated soils," Conf. Pore Pressure and Suction in Soils, London, Butterworths, pp. 47-52.
- 2) Alonso, E. E., Gens, A. and Josa, A. (1990): "A constitutive model for partially saturated soils," *Géotechnique* Vol. 40, No. 3, pp. 405-430.
- 3) Archard, J. F. (1957): "Elastic deformations and the laws of friction," *Proc. Royal Soc., A* 243, pp. 190-205.
- 4) Atkinson, J. H. and Bransby, P. L. (1978): *The Mechanics of Soils—An introduction to Critical State Soil Mechanics*, McGraw-Hill, pp. 18-144.
- 5) Baver, L. D. (1956): *Soil Physics*, New York; Wiley, pp. 81-122.
- 6) Bear, J. (1979): *Hydraulics of Groundwater*, New York; McGraw-Hill, pp. 190-224.
- 7) Bishop, A. W. (1959): "The principle of effective stress," *Teknisk Ukeblad* 39, pp. 859-863.
- 8) Blight, G. E. (1966): "Strength characteristics of desiccated clays," *Proc. ASCE* 92, SM6, pp. 19-37.
- 9) Bowden, F. P. and Tabor, D. (1954): *The Friction and Lubrication of Solids*. Oxford; Clarendon Press., pp. 1-332.
- 10) Burland, J. B. (1965): "Some aspects of the mechanical behavior of partly saturated soils," *Moisture Equilibria and Moisture Changes in Soils beneath Covered Areas*, Sydney, Australia, Butterworths, pp. 270-278.
- 11) Coleman, J. D. (1962): "Stress/strain relations for partly saturated soil," *Correspondence, Géotechnique* Vol. 12, No. 4, pp. 348-350.
- 12) Escario, V. and Sáez, J. (1986): "The shear strength of partly saturated soils," *Géotechnique* Vol. 36, No. 3, pp. 453-456.
- 13) Fisher, R. A. (1926): "On the capillary forces in an ideal soil; correction of formulas by W. B. Haines," *J. Agr. Sci.* 16, pp. 492-505.
- 14) Fredlund, D. G., Rahardjo, H. and Gan, J. K. M. (1987): "Non-linearity of strength envelope for unsaturated soils," *Proc. 6th Int. Conf. Expansive Soils*, New Delhi, India, pp. 49-54.
- 15) Gan, J. K. M., Fredlund, D. G. and Rahardjo, H. (1988): "Determination of the shear strength parameters of an unsaturated soil using the direct shear test," *Can. Géotech. J.* Vol. 25, pp. 500-510.
- 16) Haines, W. B. (1925): "Studies in the physical properties of soils. A note on the cohesion developed by capillary forces in an ideal soil," *J. Agr. Sci.* 15, pp. 529-535.
- 17) Iwasaki, Y. (1978): "Reassessment of the effective stress coefficient χ in unsaturated soils," *Master of Science Thesis*, Kobe Univ., pp. 1-55 (in Japanese).
- 18) Jennings, J. E. (1960): "A revised effective stress law for use in the prediction of the behavior of unsaturated soils," *Conf. of Pore Pressure and Suction in Soils*, London, Butterworths, pp. 26-30.
- 19) Jennings, J. E. and Burland, J. B. (1962): "Limitations to the use of effective stresses in partly saturated soils," *Géotechnique* Vol. 12, No. 2, pp. 125-144.
- 20) Karube, D. and Kato, S. (1989): "Yield functions of unsaturated soil," *Proc. 12th ICSMFE*, Rio de Janeiro, Vol. 1, pp. 615-618.
- 21) Kohgo, Y. (1987): "The interpretations and analyses of mechanical behavior of partly saturated soils using elastoplastic models," *Proc. Symp. on Unsaturated Soils*, Osaka, Japan, JSSMFE, pp. 59-68 (in Japanese).
- 22) Kohgo, Y., Nakano, M. and Miyazaki T. (1991): "Elastoplastic constitutive modelling for unsaturated soils," *Proc. 7th Int. Conf. Computer Meth. Adv. Geomech.*, Cairns, Balkema, pp. 631-636.
- 23) Kohgo, Y., Nakano, M. and Miyazaki, T. (1993): "Verification of the generalized elastoplastic model for unsaturated soils," *Soils and Foundations*, Vol. 33, No. 4, pp. 64-75.
- 24) Lambe, T. W. and Whitman, R. V. (1979): *Soil Mechanics*. SI Version, New York; Wiley, pp. 61-70.
- 25) Lloret, A. and Alonso, E. E. (1980): "Consolidation of unsaturated soils including swelling and collapse behavior," *Géotechnique* Vol. 30, No. 4, pp. 449-477.
- 26) Lloret, A., Gens, A., Batlle, F. and Alonso, E. E. (1987): "Flow and deformation analysis of partially saturated soils," *Groundwater effects in geotechnical engineering. Proc. 9th ECSMFE*, Dublin, Vol. 2, pp. 565-568.
- 27) Maswoswe, J. (1985): "Stress paths for a compacted soil during collapse due to wetting," *Ph. D. Thesis*, University of London, pp. 1-484.
- 28) Matyas, E. L. and Radhakrishna, H. S. (1968): "Volume change characteristics of partially saturated soils," *Géotechnique* Vol. 18, No. 4, pp. 432-448.
- 29) Nakase, A. (1969): *Soil Mechanics*, Chapter 5 *Shear Characteristics of Soil*, edited by Mogami, T., Gihodoh Pub., pp. 479-622 (in Japanese).
- 30) Nobari, E. S. and Duncan, J. M. (1972): "Effect of reservoir filling on stresses and movements in earth and rockfill dams," *Géotechnical Engineering Report No. TE-72-1*, University of California, Berkeley, pp. 1-198.
- 31) Roscoe, K. H. and Burland, J. B. (1968): "On the generalized stress-strain behavior of 'wet' clay," *Engineering plasticity*, J. Heyman and F. A. Leckie eds., Cambridge Univ. Press, Cambridge, pp. 535-609.

- 32) Terzaghi, K. (1936): "The shearing resistance of saturated soils and the angle between the planes of shear," Proc. 1st ICSMFE, Harvard, Mass., Vol. 1, pp. 54-56.

APPENDIX 1: Determination of P_c

A change of void ratio Δe^t may be obtained as follows when a pair of stresses (p', s^*) increases from (p'_0, s^*_0) to (p', s^*) (See Fig. 16).

$$\Delta e^t = e_0 - e \quad (\text{A-1})$$

where e_0 and e are defined in Fig. 16. Δe^t can be divided into Δe^e and Δe^p where Δe^e and Δe^p are the elastic and plastic changes of void ratio, respectively.

$$\Delta e^t = \Delta e^e + \Delta e^p. \quad (\text{A-2})$$

Supposing the $e \sim \log p'$ curves at unloading

Fig. 16. Idealized $e \sim \log p'$ curves for unsaturated soils

are linear, Δe^e can be obtained as follows.

$$\Delta e^e = \kappa \log (p' / p'_0). \quad (\text{A-3})$$

As e_0 and e are the void ratios on the state surface, e_0 and e may be estimated by Eq. (24). Thus,

$$e_0 = -\lambda_0^* \log (p'_0) + \Gamma_0^* \quad (\text{A-4})$$

$$e = -\lambda^* \log (p') + \Gamma^*. \quad (\text{A-5})$$

Therefore, substituting Eqs. (A-4) and (A-5) into Eq. (A-1), Δe^t will be

$$\Delta e^t = -\lambda_0^* \log (p'_0) + \lambda^* \log (p') + (\Gamma_0^* - \Gamma). \quad (\text{A-6})$$

Hence, Δe^p can be determined from Eqs. (A-2), (A-3) and (A-6).

$$\begin{aligned} \Delta e^p = & -(\lambda_0^* - \kappa) \log (p'_0) \\ & + (\lambda^* - \kappa) \log (p') + (\Gamma_0^* - \Gamma^*). \end{aligned} \quad (\text{A-7})$$

ε_v^p is defined by the following equation.

$$\varepsilon_v^p = \frac{\Delta e^p}{1 + e_0}. \quad (\text{A-8})$$

Substituting Eq. (A-7) into (A-8), ε_v^p will be as

$$\varepsilon_v^p = A^* \ln (p') - B^* \quad (\text{A-9})$$

$$A^* = \frac{(\lambda^* - \kappa)}{2.3(1 + e_0)} \quad (\text{A-10})$$

$$B^* = \frac{(\lambda_0^* - \kappa)}{2.3(1 + e_0)} \ln (p'_0) - \frac{(\Gamma_0^* - \Gamma^*)}{(1 + e_0)}. \quad (\text{A-11})$$

As $P_c = p'$, P_c can be obtained from Eq. (A-9),

$$P_c = \exp \left(\frac{B^* + \varepsilon_v^p}{A^*} \right). \quad (\text{A-12})$$