LINKING BETWEEN JOINT GEOMETRY MODELS AND A DISTINCT ELEMENT METHOD IN THREE DIMENSIONS TO PERFORM STRESS ANALYSES IN ROCK MASSES CONTAINING FINITE SIZE JOINTS

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ABSTRACT

To use the distinct element method to perform stress analysis in three dimensions (3D), the domain considered has to be discretized into distinct polyhedral blocks by the joints present in rock masses. Unfortunately, most of the joint networks present in rock masses do not produce distinct polyhedral blocks. Therefore, to use the distinct element method to perform stress analysis in actual jointed rock masses, it is necessary to find a procedure to discretize the rock mass into polyhedral blocks starting from a 3D joint network which does not discretize a rock mass into polyhedral blocks. Also, this procedure should not alter the mass deformation and strength properties of the jointed rock. This paper provides needed procedures to link 3D stochastic or deterministic joint geometry modeling schemes to a 3D distinct element method to perform stress analysis in jointed rock masses which contain finite size joints. The procedure is illustrated through an example.

Key words: deformability, distinct element method, fictitious joints, joint geometry, rock mass, strength (IGC: G1/G2/G13)

INTRODUCTION

To analyze, design and predict the performance of structures built in and on rock masses, it is necessary to have a good understanding of the mechanical properties (strength and deformability) of jointed rock masses. Since discontinuities control the behaviour of jointed rock, it is important to know the effect of both persistent (through-go-

ing) as well as non-persistent (finite size) discontinuities on the mechanical behaviour of rock masses. This paper provides a procedure to study the effect of non-persistent joints on the mechanical behaviour of rock masses at the 3D level.

Results of laboratory model studies (Brown, 1970a, b; Ladanyi and Archambault, 1970; John, 1970; Einstein and Hirschfeld, 1973; Chappel, 1974) show that many different

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failure modes are possible in jointed rock and that the internal distribution of stresses within a jointed rock mass can be highly complex. Currently, there are two ways to estimate the mechanical properties of jointed rock masses: direct and indirect methods. Direct methods include laboratory and in-situ tests. To obtain realistic results for jointed rock mass mechanical properties, many large volumes of rock having a number of different known joint configurations should be tested under different stress paths at significant stress levels. Such an experimental program is almost impossible to carry out in the laboratory. With in-situ tests, such an experimental program would be very difficult, time-consuming and very expensive. A few in-situ tests have been carried out to study the effect of size on rock mass compressive strength (Bieniawski, 1968; Pratt et al., 1972; Bieniawski and Van Heerden, 1975) and on rock mass modulus (Bieniawski, 1978). Heuze (1980) has reviewed the previous work on scale effects on mass strength and deformability. The results of these investigations clearly show the reduction of mass strength and modulus with size up to a certain size at which change becomes insignificant. It is important to note that these relations are highly site-dependent, since the scale effect is primarily governed by the fracture network. In these investigations, no attempt had been made to map the fracture network.

An indirect approach to obtain estimates of the strength and deformability of a jointed rock mass is by empirical correlation. Empirical correlations attempt to statistically relate deformability and strength to index properties such as RQD, or to rock mass classifications (Deere et al., 1967; Bieniawski, 1978; Hoek and Brown, 1980). Such correlations are limited to characteristics of the rocks at the sites.

Some researchers have treated jointed rocks (or rock masses) as two separate portions-intact rock and joints-and have developed expressions for overall moduli or compliances analytically. This procedure assumes that the average stresses are distributed throughout jointed rock and the overall average strains are contributed by both the intact rock and the

joints. This approach may be termed as the analytical decomposition technique. Salamon (1968), Singh (1973), Morland (1976), Amadei and Goodman (1981), Gerrard (1982a, b, c), and Fossum (1985) have derived the global deformation moduli (or compliances) for rock masses with continuous persistent joints by considering the load-deformation relationship for each comprising material (joints and intact rock) and assuming that the global behaviour of the jointed rock mass is the summation of each component response. In order to apply these expressions, it is necessary for rock masses to have continuous persistent joint sets which are spaced regularly with deterministic orientations. In reality, most of the joints are non-persistent with finite size, and all the joint geometry parameters are inherently statistical. Also, the interaction between joints is not considered in this method. Therefore, the assumptions made in these models are quite difficult to satisfy for most in-situ rock masses. These models may be applicable to a limited number of rock masses.

The third indirect approach available is the numerical decomposition method. This method calculates strength and deformability of rock masses using a numerical method, incorporating the strength and deformation properties of the joints and of the intact rock. This methodology allows incorporation of any joint network to the jointed rock mass and also includes interaction between joints and intact rock. The finite element method has been used with Goodman's joint element (Goodman et al., 1968) to study the effect of joint geometry parameters on rock mass strength and deformability properties (Kulatilake, 1985). However, since the finite element method is based on continuum mechanics, simulation of large displacements and large rotations is difficult with the method, even though they may occur in jointed rock masses.

The distinct element method introduced by Cundall (1971) and further developed by Cundall and co-workers (Cundall, 1988; Hart et al., 1988; Lemos et al., 1985; Lorig et al., 1986) is a powerful technique to perform stress analysis in blocky rock masses formed by persistent discontinuities. In this method,

the rock mass is modeled as an assemblage of rigid or deformable blocks. Discontinuities are regarded as distinct boundary interactions between these blocks; joint behaviour is prescribed for these interactions. The distinct element algorithm includes not only continuum theory representation for the blocks, but also force displacement laws which specify forces between blocks and a motion law which specifies the motion of each block due to unbalanced forces acting on the block. By taking into account the interaction of intact blocks and joints, the distinct element method can effectively calculate the mechanical behaviour of block systems under different stress and displacement boundary conditions. The method employs an explicit solution procedure. An advantage of the explicit method is that, because matrices are never formed, large displacements, rotations and complex constitutive behaviour for both intact material and joints are possible with no additional computing effort.

To use the distinct element method to perform stress analysis in three dimensions (3D), the domain considered has to be discretized into distinct polyhedral blocks by the joints present in rock masses. Unfortunately, most of the joint networks present in rock masses do not produce distinct polyhedral blocks. Therefore, in order to use the distinct element method to perform stress analysis in actual rock masses, it is necessary to find a way to discretize the rock mass into polyhedral blocks starting from a 3D joint network which does not discretize a rock mass into polyhedral blocks.

This paper shows how 3D stochastic joint geometry modeling schemes were linked to a 3D distinct element method to perform stress analysis in a rock mass which contains finite size joints.

THREE-DIMENSIONAL STOCHASTIC JOINT GEOMETRY MODELING SCHEMES

In this paper, the word "joints" means minor discontinuities such as joints, bedding planes and fissures. Due to the statistical

nature, the geometry of joints should be characterized statistically.

Since joint geometry patterns can vary from one statistically homogeneous region to another, each statistically homogeneous region should be represented by a separate joint geometry model. Therefore, the first step in the procedure of joint geometry modeling in a rock mass should be the identification of statistically homogeneous regions. To model joint geometry in 3D space, for a statistically homogeneous region, it is necessary to know the number of joint sets, and for each joint set, the intensity, spacing, location, orientation, shape and dimension distributions. These joint parameters are inherently statistical. Sample values of joint parameters provided by the field data usually subject to errors due to sampling biases and represent only one or two dimensional properties. Therefore, before inferring these parameters from sampling values, sampling biases should be corrected on field data. In addition, principles of stereology need to be used in order to infer 3D parameters of the joint sets from one-dimensional or two-dimensional values.

Figure 1 shows the flow chart used to develop 3D stochastic joint geometry modeling schemes. The present status on investigation of statistical homogeneity of rock masses is reported in (Kulatilake et al., 1990). Procedures for delineation of joint sets, and corrections for orientation bias and modeling of orientation distribution for each joint set are available in (Wathugala et al., 1990). Joint trace length modeling, including corrections for sampling biases and joint size modeling in 3D are explained in (Kulatilake et al., 1993). Joints are considered as thin circular discs in modeling joint size in 3D. This is not an unreasonable assumption for a granitic rock mass. Joint spacing modeling including corrections for sampling biases, joint intensity modeling in 1D, 2D and 3D, and joint location modeling are explained in detail in (Kulatilake et al., 1993). Building of joint geometry network schemes in 3D by combining the statistical models obtained for the aforesaid joint geometry parameters is given in (Kulatilake et al., 1993). The same paper

- Step 1: Obtain a statistically homogeneous region
- Step 2: Delineate joint sets
- Step 3: Apply correction for orientation bias for each joint set
- Step 4: Model the true orientation distribution for each joint set
- Step 5: Estimate the trace length distribution for each joint set taking into account the sampling biases
- Step 6: Infer joint size distribution in 3D for each joint set taking into account the sampling biases (joints are considered as circular discs)
- Step 7: Determine joint spacing distributions along scanlines taking into account the sampling bias on spacing
- Step 8: Infer joint Spacing distribution along the mean pole direction (true spacings) for each joint set
- Step 9: Estimate the mean joint center density in 3D (number/volume) for each joint set
- Step 10: Obtain a distribution, for the random variable, "number of joint centers per chosen volume".
- Step 11: Suggest a 3D stochastic joint geometry model by describing the joint geometry parameters: (a) number of joint sets (b) orientation distribution for each joint set (c) Spacing distribution for each joint set (d) distribution for density in 3D for each joint set (e) size (diameter when joint shape is considered as circular) distribution for each joint set

Fig. 1. Flow chart used to develop 3D stochastic joint geometry modelling schemes

also shows how joint networks can be generated in 3D, based on the built-up models and how these built-up schemes can be validated. Figure 2 shows a typical configuration of generated joints in 3D according to a modeling scheme. It is very clear that the domain in Fig. 2 is not divided into polyhedral blocks by the joints generated in the particular

Note: Joints generated in a volume $4.69 \text{ m} \times 2.69 \text{ m} \times 2.69 \text{ m}$ are shown

Fig. 2. Typical configuration of generated joints in 3D according to a modelling scheme

volume. Therefore, it is not possible to perform distinct element stress analysis on a rock mass such as that shown in Fig. 2. The next section shows how to discretize a domain such as the one in Fig. 2 into polyhedral blocks, starting from the generated joints.

DISCRETIZATION OF A 3D DOMAIN INTO POLYHEDRAL BLOCKS STARTING FROM THE GENERATED JOINTS

First the disc-shaped real joints are converted to square-shaped joints having the same area. Then fictitious joints are introduced so that when they are combined with real joints, the domain is discretized into polyhedral blocks.

Conversion of Disc-Shaped Joints into Square Joints

First consider the X - Y - Z coordinate system with origin 0 shown in Fig. 3. For each disc-shaped generated joint, the coordinates of the joint center (x_c, y_c, z_c) , the orientation (dip angle and dip direction), and the joint radius, r , are known. Consider the second coordinate system, x' - y' - z' , with O' as the origin, in which O' is the joint center location, and the z' axis coincides with the normal direction, n , of the disc. The x' axis is selected on the joint plane in such a way as to be parallel to the horizontal plane (x - y) (Fig. 3). Finally, y' is selected on the joint plane to be perpendicular to both x' and z' .

Fig. 3. Conversion of discs to equivalent square joints

Let the directional cosines for x' , y' and z' axes be (l_1, m_1, n_1) , (l_2, m_2, n_2) and (l_3, m_3, n_3) , respectively. The relationship between the two coordinate systems can be given by:

$$x = l_1 x' + l_2 y' + l_3 z' + xc \quad (1a)$$

$$y = m_1 x' + m_2 y' + m_3 z' + yc \quad (1b)$$

$$z = n_1 x' + n_2 y' + n_3 z' + zc \quad (1c)$$

in which directional cosines for the x' , y' and z' axes are given below by Eqs. (2), (3) and (4), respectively:

	First Set	Second Set	
	$l_1 = +\sin \theta$	$-\sin \theta$	(2a)

x' axis	$m_1 = -\cos \theta$	$+\cos \theta$	(2b)
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	$n_1 = 0$	0	(2c)
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	$l_2 = +\cos \theta \cos \phi$	$-\cos \theta \cos \phi$	(3a)
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y' axis	$m_2 = +\sin \theta \cos \phi$	$-\sin \theta \cos \phi$	(3b)
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	$n_2 = -\sin \phi$	$+\sin \phi$	(3c)
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and

	$l_3 = \sin \phi \cos \theta$	(4a)
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z' axis	$m_3 = \sin \phi \sin \theta$	(4b)
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	$n_3 = \cos \phi$	(4c)
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For directional cosines of the x' and y' axes, two sets of equations are possible (first set and second set in Eqs. (2) and (3)). Both these sets satisfy Eq. (1). We have selected the first set for our formulations.

The area of a disc-shaped joint is πr^2 . Therefore, the size of the square joint having the same area is $\sqrt{\pi r}$. With respect to the coordinate system $x'-y'-z'$, the coordinates of the vertices A, B, C and D of the square can be given by:

Vertex A:	$x'a = -\sqrt{\pi}(r/2)$	(5a)
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	$y'a = -\sqrt{\pi}(r/2)$	(5b)
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	$z'a = 0$	(5c)
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Vertex B:	$x'b = \sqrt{\pi}(r/2)$	(6a)
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	$y'b = -\sqrt{\pi}(r/2)$	(6b)
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	$z'b = 0$	(6c)
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Vertex C:	$x'c = \sqrt{\pi}(r/2)$	(7a)
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	$y'c = \sqrt{\pi}(r/2)$	(7b)
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$$z'c = 0 \quad (7c)$$

Vertex D:	$x'd = -\sqrt{\pi}(r/2)$	(8a)
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	$y'd = \sqrt{\pi}(r/2)$	(8b)
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	$z'd = 0$	(8c)
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These values now can be used in Eq. (1) to obtain the coordinates of the vertices A, B, C and D with respect to the coordinate system $X-Y-Z$. Note that the sides AB and CD of the square joint are always parallel to the horizontal plane $X-Y$. To introduce fictitious joints it is necessary to know the coordinate values of the vertices of real joints.

Geometric Determination of the Introduced Fictitious Joints

Different fictitious joint systems are introduced for different real joint configurations. First, the simplest case is considered, where only one real joint ABCD exists inside the block (Fig. 4). If the dip angle of the real joint is greater than or equal to 45° , two horizontal fictitious joint planes (planes EHIL and MPQT) are created to contain the two horizontal sides AB and CD, as shown in Fig. 4. These two fictitious joint planes divide the total block into 3 sub-blocks (blocks EHILZ₂YZZ₁, MPQTLEHI and UVWX-TMPQ). Between the two horizontal planes EHIL and MPQT, two more fictitious joint planes (planes GORJ and FNSK) are created, including real joint sides AD and BC, respectively, and to be parallel to either X axis or Y

Fig. 4. Formation of polyhedral blocks with one real joint of dip angle greater than or equal to 45° and dip direction between 45 and 135° or between 225 and 315° and the introduced fictitious joints

axis, depending on the dip direction of the real joint. If the dip direction is between $45\text{--}135^\circ$ or $225\text{--}315^\circ$, the planes GORJ and FNSK are created parallel to the y axis. Otherwise, they are created parallel to the x axis. In Fig. 4, a real joint with dip angle 60° and dip direction 60° is shown along with the related fictitious joint planes. A real joint with dip angle 60° and dip direction 20° is shown along with the related fictitious joint planes in Fig. 5. The whole block in each Figure (4 and 5) is divided into polyhedral blocks through the combined system of the real joint and the fictitious joint planes.

Now, consider a block with one real joint ABCD having a dip angle less than 45° (Fig. 6).

Fig. 5. Formation of polyhedral blocks with one real joint of dip angle greater than or equal to 45° and dip direction between 0 and 45 , or between 135 and 225 , or between 315 and 360° and the introduced fictitious joints

Fig. 6. Formation of polyhedral blocks with one real joint of dip angle less than 45° and the introduced fictitious joints

First, two vertical fictitious planes (planes LX-VJ and FRTH) are created to contain the two horizontal sides AB and CD as shown in Fig. 6. These two fictitious joint planes divide the total block into 3 sub-blocks (blocks LJKWXV, LEFHIJ VXQRTU and FGHTRS). Between the two vertical planes LXVJ and FRTH, two more vertical fictitious joint planes (planes MPCZ₂YB and NODZ₁ZA) are created to include real joint sides BC and AD, respectively. In Fig. 6, a real joint with dip angle 30° and dip direction 60° is shown along with the related fictitious joint planes. The real and all the aforementioned fictitious joints discretize the whole block into 6 polyhedral sub-blocks.

When the block contains a square-shaped real joint set, first the intersections between the real joints and the boundary of the block is checked. The parts of the joints which are outside the boundary are then removed. Then for each actual joint, the fictitious joint system is generated as explained in the previous paragraphs. All the real and fictitious joints then discretize the whole block into small polyhedral blocks. Obviously, the number of polyhedral blocks created depends on the number of real joints and their relative locations. Figure 7 shows a polyhedral block

Fig. 7. A polyhedral block system formed by 5 real finite size joints and introduced fictitious joints

system formed in a cube of side dimension 2 m by 5 real joints of dip angle 45° , dip direction 60° , and size 0.4 m square, with centers uniformly distributed and the created fictitious joints.

After rock domains are discretized into polyhedral blocks, such domains can then be subjected to distinct element stress analysis. To perform stress analysis, it is necessary to specify the mechanical property values for the selected constitutive models of intact rock, real joints and fictitious joints. Laboratory tests can be performed to evaluate necessary mechanical property values for intact rock and real joints. The next question which arises is what values should be assigned for the mechanical properties (strength and deformation) of the fictitious joints so that they behave as intact rock.

REPRESENTATIVE VALUES FOR MECHANICAL PROPERTIES OF FICTITIOUS JOINTS TO SIMULATE INTACT ROCK BEHAVIOUR IN DISTINCT ELEMENT STRESS ANALYSIS

This topic should be addressed according to the constitutive models chosen for both the intact rock and the fictitious joints. If one decides to use linear elastic, perfectly-plastic constitutive models with the Mohr-Coulomb failure criterion, including a tension cut-off for both the intact rock (Fig. 8) and the fictitious joint (Fig. 9), then the strength parameters of the fictitious joints should be assigned the same values as for the strength parameters of the intact rock. The deformation parameters for the isotropic linear elastic portion of the intact rock are specified by either the Young's modulus, E , and the Poisson's ratio, ν , combination, or by the shear modulus, G , and the bulk modulus, K , combination. On the other hand, linear elastic fictitious joint deformation needs to be characterized by a joint normal stiffness, JKN , and a joint shear stiffness, JKS . This leads to the problem of selecting proper values for JKN and JKS of fictitious joints so that they behave as intact rock. For real joints, usually both JKN and JKS show normal stress-depen-

Fig. 8. Assumed constitutive model for intact rock: (a) stress vs. strain, (b) Mohr-Coulomb failure criterion with a tension cutoff

dent behaviour. However, it is important to realize that here the interest is on the pre-failure behaviour of fictitious joints which should behave as intact rock. For most of the rocks, especially for hard rocks, the linear elastic assumption of pre-failure behaviour of intact rock is quite reasonable from a practical point of view. Therefore, the assumption of constant JKN and JKS values for fictitious joints is reasonable. Investigations performed to estimate representative JKS and JKN values for fictitious joints to simulate intact rock behaviour are reported by Kulatilake et al. (1992). Through these investigations, it was found that by using the parameter values given below for the constitutive models considered here, it is possible to simulate the intact material behaviour for fictitious joints:

(a) For both intact material and fictitious joints, the same strength parameter values should be used.

(b) A JKS value for fictitious joints should be chosen to satisfy a G/JKS ratio between 0.008 and 0.012 m.

(c) A JKN/JKS ratio between 2 and 3 seems satisfactory. If one requires a single

Fig. 9. Assumed constitutive model for fictitious joint: (a) normal stress vs. normal displacement, (b) shear stress vs. shear displacement (b) Mohr-Coulomb failure criterion with a tension cutoff

value, the E/G value for the particular rock is recommended for the JKN/JKS ratio.

DISTINCT ELEMENT STRESS ANALYSIS OF A ROCK BLOCK WHICH CONTAINS FINITE SIZE REAL JOINTS

A cubical rock block of side dimension 2 m was used for this example. Five real joint centers were generated according to a uniform distribution. Using these joint centers, five real joints of diameter 1.2 m, dip angle 45° and dip direction 60° were placed in the rock block. Fictitious joints were then introduced into this rock block to discretize the real block into polyhedra when they are connected with the real joints. Figure 10 shows the discretized rock block.

Linear elastic, perfectly plastic constitutive models with Mohr-Coulomb failure criterion, including a tension cut-off were used to repre-

Fig. 10. The discretized rock block used to perform distinct element stress analysis

sent the mechanical behaviour of the intact rock, the fictitious joints and the real joints in the rock block. The values used for the parameters in the constitutive models are

Table 1. Mechanical Property Values Used to perform Distinct Element Stress Analysis on the Rock Block Shown in Fig. 10.

For intact rock	Bulk modulus	$K=40$ GPa
	Shear modulus	$G=24$ GPa
	Density	$D=2500$ Kg/m ³
	Cohesion	$C=50$ MPa
	Friction coefficient	$f=0.839$
	Tensile strength	$t=10$ MPa
For real joints	Normal stiffness	$JKN=67.2 \times 10^3$ MPa/m
	Shear stiffness	$JKS=2.7 \times 10^3$ MPa/m
	Cohesion	$C_j=0.4$ MPa
	Friction coefficient	$f=0.654$
	Tensile strength	$jt=0.0$
For fictitious joints	Normal stiffness	$JKN=5 \times 10^6$ MPa/m
	Shear stiffness	$JKS=2 \times 10^6$ MPa/m
	Cohesion	$C_j=50$ MPa
	Friction coefficient	$f=0.839$
	Tensile strength	$jt=10$ MPa

given in Table 1. First, an isotropic compressive stress of 5 MPa was applied to the rock block. Then the stress in the z direction (Fig. 10) was increased in steps until the failure of the rock block was observed. This stress analysis was performed using the 3D distinct element method (Wang 1992). Under each stress increment it was necessary to monitor the velocity of the block to check whether the rock block has reached the stable equilibrium state. The next stress increment was applied after the block has reached the equilibrium status under the previous stress increment. Displacements of the rock block in the X , Y , and Z directions were monitored throughout this stress analysis to compute average block strains in the X , Y and Z directions, respectively. Figure 11 shows the stress-strain plot obtained for the rock block in the z direction. Local failure of real joints causes the stress-strain relation to be nonlinear.

The principal stress difference at failure of the jointed rock block (with finite size joints) in the z direction ($\sigma_z - \sigma_x$ or $\sigma_z - \sigma_y$) was found to be between 55 and 60 MPa. Unfortunately, when a rock mass contains finite size joints, no analytical solution is available to compute rock mass strength. However, when a rock mass contains only a single persistent joint, an analytical solution is available in two dimen-

Fig. 11. Calculated stress-strain plot for the rock block shown in Fig. 10 in the Z direction, obtained through three-dimensional distinct element stress analysis

sions to calculate rock mass strength under the boundary loadings considered (Brady and Brown, 1985, p. 114). According to this solution, when the rock mass has a 45 degree persistent joint, the principal stress difference at failure ($\sigma_z - \sigma_x$ or $\sigma_z - \sigma_y$) is 21.2 MPa. This strength value was also obtained through a distinct element analysis. In this case, the rock mass failure is governed by shear failure along the joint. On the other hand, for these boundary loadings, the principal stress difference at failure for intact rock according to the two-dimensional Mohr-Coulomb failure criterion is 673.2 MPa. Here, shear failure through intact rock governs the failure. The strength obtained using the distinct element method for a rock block with finite size joints seems reasonable compared to the strengths obtained for intact rock and the rock block with a single persistent joint.

Using the distinct element stress analysis, the tangent modulus of deformation of the jointed rock block (with finite size joints) in the z direction at the initial portion of the stress-strain curve was found to be 40.9 GPa. The ratio of this deformation modulus to the intact rock Young's modulus was found to be 0.68. Poisson's ratios for the jointed rock block from distinct element stress analysis were found to be $\nu_{zx}=0.237$ and $\nu_{zy}=0.286$.

The Poisson's ratio used for the intact rock was 0.25.

Unfortunately, when the rock mass contains finite size joints, no analytical solution is available to compute deformability. However, to estimate rock mass equivalent elastic modulus, an approximate analytical solution is available in two dimensions when the joints are of the persistent type. This procedure assumes that the average stresses are distributed throughout jointed rock and the overall average displacements are influenced by both the intact rock and the joints. To compute the normal and the shear displacements of a joint, it is necessary to obtain the normal and the shear stresses acting on the joint plane. These stresses can be computed from the boundary stresses using the stress transformation equations. The results obtained using the analytical solution for various persistent joint configurations are given in Table 2. For these persistent joint configurations, distinct element analyses also produced almost the same results. Also, at the bottom of Table 2, the distinct element solution obtained for the rock block with finite size joints is given for comparison purposes. The distinct element analysis result obtained for the rock block

with finite size joints seems reasonable compared to the result obtained for the intact rock and for the rock blocks with different configurations of persistent joints.

This example shows how one can use the distinct element method to perform a stress analysis in jointed rock masses which contain finite size real joints.

CONCLUSIONS

The suggested procedures in the paper to link 3D stochastic joint geometry modeling schemes to a 3D distinct element method to perform stress analysis in rock masses which contain finite size joints were found to work very well. This improves the capabilities of the distinct element method in performing stress analysis in discontinuous rock masses. At present, the authors are using the technique explained in the paper to study the effect of finite size joints on the strength and deformability of jointed rock masses.

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Table 2. Values of Deformability Modulus in the z-Direction for Intact Rock and Jointed Rock Blocks with Various Joint Configurations

Solution Type	Joint Configuration	Deformability Modulus in z-Direction (GPa)
Analytical solution based on elasticity	No Joints, intact rock	60
Both the analytical solution and the distinct element solution	One persistent joint with dip angle 45°	9.3
	Two persistent joints with dip angle 45°	6.6
	Three persistent joints with dip angle 45°	4.5
	Four persistent joints with dip angle 45°	3.5
	Five persistent joints with dip angle 45°	2.8
Only the distinct element solution	Five finite size joints of diameter 1.2 m with dip angle 45° in a cubical rock block of side dimension 2 m	40.9

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