SEISMIC BEARING CAPACITY OF A MOUNDED FOUNDATION NEAR A DOWN-HILL SLOPE BY PSEUDO-STATIC ANALYSIS

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ABSTRACT

The pseudo-static analysis has been used in engineering practice to simplify the computation of the upper limit of a slope under seismic load. In this paper, the upper bound technique of limit analysis of perfect plasticity is used to determine the bearing capacity of a foundation near a down-hill slope. In this analysis (Chen et al., 1969; Chen, 1975), a logarithmic spiral rupture (Abdul-Baki et al., 1970; Sawada et al., 1984) is assumed to start at an edge of the loaded area far from the slope. A landslide is assumed to be of rigid body type with the inertia force acted at its center of gravity. The collapse state is reached when the rate of kinematic energy of the landslide together with the load is equal to the rate of dissipation of internal energy along the sliding surface. This energy balance leads to the equation of ultimate load with several parameters. The optimized numerical results with respect to these parameters are compared with those by Kötter's method (Yamaguchi, 1959) and those obtained by the slice method.

Key words: bearing capacity, computer application, limit analysis, logarithmic spiral sliding surface, seismic yield acceleration coefficient, slope stability (IGC: E3/E6/E13)

INTRODUCTION

To estimate the load-carrying capacity of foundations on slopes, the finite element and boundary element analysis (Telles, 1983) with elastoplastic elements (Chen, 1975; Zienkiewicz, 1967) have been widely used.

Although these methods provide design guidelines with good accuracy, they involve generally complicated numerical calculations. At present, the bearing capacity of foundations is based on specification formulas for road bridge design. The bearing capacity so calculated is then multiplied by a correction factor (α) to account for the foundation on slopes.

In this paper, the upper bound technique of limit analysis is applied to obtain the seismic bearing capacity of a mounded foundation near a down-hill slope using the pseudo-static analysis (Chen et al., 1973; Drucker et al., 1952). The bearing capacity factor of foundations on slopes are determined by assuming the sliding surface to be a logarithmic-spiral function.

The bearing capacity factors so obtained are compared with those calculated by the conventional limit equilibrium method (Kubo et al., 1984).

THE UPPER BOUND SOLUTION

The power of the self-weight of the sliding region ABC shown in Fig. 1 and the power due to an earthquake inertia force about the center of rotation O are denoted by $\dot{W}$ and $\dot{W}'$, respectively. They can be obtained by simply subtracting the power for the region OBC and that for the region OAB from the power of the whole region OAC about the point O. All of these are calculated by assuming the whole region OAC is composed of soils. This leads to

$$\dot{W} = \dot{W}_{OAC} - \dot{W}_{OBC} - \dot{W}_{OAB} = \gamma r_0^2 \omega (f_1 - f_2 - f_3) \quad (1)$$

$$\dot{W}' = \dot{W}'_{OAC} - \dot{W}'_{OBC} - \dot{W}'_{OAB} = \gamma r_0^3 \omega (f_4 - f_5 - f_6). \quad (2)$$

Where $\dot{W} = \partial W / \partial t$ and $\omega (= V_0 / r_0)$ means angular velocity relative to the slip soil mass about the center of rotation O (see Fig. 1). Similarly, denoting the power due to the applied load $p (= q' B_t, q')$; applied uniform load/unit width) on the foundation and that due to its inertia force by $\dot{W}_p$ and $\dot{W}'_p$ respectively where K is the horizontal seismic coefficient for the effective foundation mass, and also x is the ratio, which contribute to the inertia

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By equating the total sum of the power of the sliding soil mass due to the external forces to the internal rate of energy dissipation along the sliding surface, we obtain

$$\dot{E}_i = \dot{W} + \dot{W}_p + \dot{W}' + \dot{W}_p'. \quad (6)$$

Solution of Function in Static Case

As shown in Fig. 2, the margin width b is equal to the difference of the collapsing length L at the levee crown of a slope and the length of the distributed load, B_ℓ . The horizontal seismic acceleration coefficient (K) is included in the equation of limit energy balance, Eq. (6). For a applied load p ($K=0$), the solution of the nonlinear optimization function is:

$$p(\theta_0, \theta_h) = \frac{cf_c - \gamma r_0(f_1 - f_2 - f_3)}{f_p}. \quad (7)$$

Solution of Function Considering the Inertia Force at the Time of an Earthquake

K_c is defined here as the seismic yield acceleration coefficient. That is, the smallest acceleration with which a slope begins to move due to the horizontal seismic inertia force divided by the acceleration of gravity (Sawada et al., 1984). The objective function $p(\theta_0, \theta_h)$ as given below considers an inertia term as shown in Fig. 3.

$$p(\theta_0, \theta_h) = \frac{cf_c - \gamma r_0(f_1 - f_2 - f_3) - K_c \gamma r_0(f_4 - f_5 - f_6)}{f_p + x K_c f_q} \quad (8)$$

Therefore, when the following conditions are satisfied, $p(\theta_0, \theta_h)$ takes the minimum value.

$$\frac{\partial p}{\partial \theta_0} = 0, \quad \frac{\partial p}{\partial \theta_h} = 0. \quad (9)$$

Accordingly, the bearing capacity q , can be expressed as

$$q = \min p(\theta_0, \theta_h). \quad (10)$$

The parameters L/r_0 and H/r_0 defining a logarithmic-spiral sliding surface can be expressed in terms of the angles θ_0 and θ_h as

Fig. 2. Static case

Fig. 3. Earthquake inertia force case

$$\frac{L}{r_0} = \frac{\sin(\theta_h - \theta_0)}{\sin \theta_h} - \frac{\sin(\theta_h + \beta)}{\sin \theta_h \sin \beta} \{ -\sin \theta_0 + \sin \theta_h \times \exp[(\theta_h - \theta_0) \tan \phi] \} \quad (11)$$

$$\frac{H}{r_0} = \sin \theta_h \exp[(\theta_h - \theta_0) \tan \phi] - \sin \theta_0. \quad (12)$$

RESULTS AND DISCUSSION

The computer program for the numerical calculation consists of three main parts: (1) the main program, (2) the subprogram for determining an objective function and calculating the smallest load, and (3) the subprogram describing restraint conditions. The main program has two objectives: (a) the determination of the initial values of parameters and (b) the analysis by a quasi-Newtonian method (Al-baali et al., 1985; Gallagher et al., 1977). For a safety factor equal to 1.0, the solution set for a load at the limit stationary state of a slope is determined. A uniaxial search is then carried out on a secondary plane, and the limit load q is thus determined by a quadratic approximation. This is the upper bound method of limit analysis. The smallest value of all these values satisfying the conditions is then adopted.

Comparison of the Statical Analysis Results with Those Conventional Method

To compare the results of the present analysis with those obtained by the conventional method recommended by the Honshi Connecting Bridge Investigation and Research Subcommittee (Kubo et al., 1984), the limit load q was calculated by the slice method, assuming the slope converged with $\gamma=0.0$ as a submerged slope. The analytical results are compared in Table 1 for the case of

Table 1. Comparison of bearing capacity by different methods

Methods	Units	b/B_t				
		Half Space (∞)	0.0	1.0	2.0	5.0
Simplified Method	q , tf/m ³ ($\times$ kPa)	17.9 (175.5)	12.6 (123.6)	13.7 (134.4)	14.7 (144.2)	17.7 (173.6)
	Correction Factor (α)	1.00	0.70	0.77	0.82	0.99
Bishop	q , tf/m ³ ($\times$ kPa)	30.3 (297.1)	21.8 (213.8)	24.5 (240.3)	26.7 (259.9)	30.3 (297.1)
	Correction Factor (α)	1.00	0.72	0.81	0.88	1.00
Kötter	q , tf/m ³ ($\times$ kPa)	30.3 (297.1)	19.6 (192.2)	20.9 (205.0)	23.0 (225.6)	30.3 (297.1)
	Correction Factor (α)	1.00	0.65	0.69	0.76	1.00
Limit Analysis	q , tf/m ³ ($\times$ kPa)	37.6 (368.7)	23.0 (225.6)	25.9 (254.0)	28.5 (279.5)	36.9 (361.9)
	Correction Factor (α)	1.00	0.61	0.69	0.76	0.96

$\beta=20^\circ$, $\phi=30^\circ$, $c=1.0$ tf/m² (9.8 kPa), $x=1.0$ and $B_t=10$ m. In the simplified Honshi method, it is assumed that all the statically indeterminate forces in the divided blocks are mutually balanced. As shown in Fig. 4, the absolute values of the correction factor α differ from the results obtained by the Bishop's method (Bishop, 1966) that considers the horizontal statically indeterminate forces. An approximate value of α is used for the actual design calculation. The values of α obtained as the ratio, for the results of analysis divided by the value of half space each other, are found quite close to those obtained

Fig. 4. Comparison of the correction factor α ; present method versus conventional methods

Figure 5 shows the comparison of the sliding surfaces. As a whole, these two sliding surfaces seem close to each other. The deepest distance in the sliding surface, 19.74

ϕ	5°	10°	20°	30°	40°
q , tf/m ³ ($\times$ kPa)	7.53 (73.84)	9.42 (92.38)	15.59 (152.89)	28.53 (279.78)	61.53 (603.40)
Depth of slip surface m	9.86	10.46	13.90	19.74	30.81

b/B_t	0.0	1.0	2.0	3.0	4.0	5.0
$K_c=0.0$						
q , tf/m ²	23.00	25.85	28.53	31.10	33.77	36.88
($\times$ kPa)	(225.55)	(253.50)	(279.78)	(304.99)	(331.17)	(361.67)
α	0.61	0.69	0.76	0.83	0.90	0.98
$K_c=0.1$						
q , tf/m ²	18.29	20.79	23.14	25.42	27.67	30.22
($\times$ kPa)	(179.36)	(203.88)	(226.93)	(249.29)	(271.35)	(296.36)
α	0.49	0.55	0.62	0.68	0.74	0.80
$K_c=0.3$						
q , tf/m ²	11.20	13.14	15.00	16.74	18.51	20.33
($\times$ kPa)	(109.84)	(128.86)	(147.10)	(164.16)	(181.52)	(199.37)
α	0.30	0.35	0.40	0.45	0.49	0.54

Table 4. Variation of bearing capacity with earthquake acceleration factor, K_c for the case of submerged slope ($\beta=20^\circ$, $b/B_t=2.0$, $B_t=10$ m, $\phi=30^\circ$, $c=1.0$ tf/m², $x=1.0$)

Bearing Capacity	K_c						
	0.1	0.15	0.20	0.25	0.35	0.40	0.45
q , tf/m ³ ($\times$ kPa)	23.14 (226.93)	20.77 (203.68)	18.62 (182.60)	16.69 (163.67)	13.42 (131.61)	12.06 (118.27)	10.87 (106.60)

m, was obtained by assuming a logarithmic spiral sliding. This value almost coincides with the 20.65 m obtained by the Honshi method by assuming a sliding composed of a circular arc and a straight line.

In Table 2, the results of the present analysis with several values of the internal friction angle ϕ are tabulated. It is apparent that the volume of sliding mass increases as ϕ increases. As ϕ becomes larger, the volume of sliding earth mass increases. Accompanying the increment of ϕ , the stability of a slope increase (see Fig. 6).

Results Considering the Inertia Force due to Earthquake

Table 3 compares the static case with the seismic cases for several values of K_c and b/B_t ratio. In Table 4, the variation of bearing capacity with seismic inertia

force at $b/B_t=2.0$ is given. As can be seen clearly in Tables 3 and 4, the influences of the bearing capacity (q), that is limit load, and of the inertia forces due to sliding earth mass are quite large; and The inertia forces divided by g ; (K_c) reduce q significantly.

In Tables 5 and 6, the bearing capacities are shown for several seismic yield accelerations K_c , and b/B_t ratios. In both cases, the larger the K_c or the smaller b/B_t value,

Table 6. Variation of bearing capacity with b/B_t ratio and γ (gf/cm³) ($K_c=0.5$, $\beta=20^\circ$, $\phi=30^\circ$, $c=1.0$ tf/m², $B_t=10$ m, $x=1.0$)

b/B_t	q , tf/m ² ($\times$ kPa)					
	γ ; 0.0	0.5	1.0	1.5	2.0	2.5
0.0	7.61 (74.63)	28.64 (280.86)	49.67 (487.10)	70.71 (693.44)	91.74 (899.67)	112.78 (1106.01)
	9.52 (93.36)	35.83 (351.37)	62.14 (609.38)	88.46 (867.50)	114.77 (1125.51)	141.09 (1383.63)
1.0	11.30 (110.82)	42.53 (417.09)	73.76 (723.37)	105.00 (1029.74)	136.23 (1336.01)	167.47 (1642.39)
	12.96 (127.09)	48.78 (478.35)	84.60 (829.61)	120.42 (1180.87)	156.25 (1532.23)	192.07 (1883.50)
2.0	14.53 (142.49)	54.69 (536.32)	94.85 (930.15)	135.01 (1323.99)	175.17 (1717.82)	215.34 (2111.75)
	16.21 (158.97)	61.01 (598.31)	105.82 (1037.76)	150.62 (1477.11)	195.43 (1916.56)	240.23 (2355.91)

Table 5. Variation of bearing capacity with K_c for the case of submerged slope ($\beta=20^\circ$, $\phi=30^\circ$, $c=1.0$ tf/m², $B_t=10$ m, $b/B_t=0.0$, $x=1.0$)

Seismic Yield Acceleration Factor	Bearing Capacity q , tf/m ³ ($\times$ kPa)
$K_c=0.1$	18.29 (179.36)
$K_c=0.3$	12.15 (119.15)
$K_c=0.4$	9.51 (93.26)
$K_c=0.5$	7.61 (74.63)

Table 7. Variation of bearing capacity with x and γ (gf/cm³) ($\beta=20^\circ$, $\phi=30^\circ$, $c=1.0$ tf/m², $B_t=10$ m, $b/B_t=2.0$)

K_c	x	q , tf/m ² ($\times$ kPa)					
		γ ; 0.0 (submerged)	0.5	1.0	1.5	2.0	2.5
0.1	0.1	28.09(275.47)	105.73(1036.89)	183.38(1798.32)	261.02(2359.74)	338.66(3321.17)	416.31(4082.59)
	0.5	26.34(258.31)	99.15 (972.30)	171.95(1686.29)	244.76(2400.29)	317.56(3114.28)	390.37(3828.27)
	1.0	23.71(232.31)	89.25 (875.19)	154.78(1517.87)	220.32(2160.55)	285.86(2803.23)	351.39(3445.90)
0.2	0.1	27.65(271.15)	104.08(1020.63)	180.50(1770.12)	256.93(2519.60)	333.36(3269.08)	409.79(4018.57)
	0.5	24.21(237.42)	91.13 (893.67)	158.03(1549.92)	224.97(2206.17)	291.88(3296.45)	358.80(3518.67)
	1.0	20.22(198.29)	76.11 (746.38)	145.06(1294.47)	187.88(1842.56)	243.78(2390.66)	299.67(2938.75)
0.5	0.1	26.34(258.31)	99.15 (964.77)	171.95(1686.29)	244.76(2400.29)	317.56(3114.28)	390.70(3828.27)
	0.5	18.40(180.44)	69.26 (679.19)	120.12(1177.95)	170.98(1676.67)	221.84(2175.45)	272.70(2674.20)
	1.0	11.30(110.82)	42.53 (417.14)	73.77 (723.45)	105.00(1029.77)	136.23(1336.01)	167.47(1642.40)

the smaller the bearing capacity q , that is to say, the slope tend to reduce stability.

Let x be the mass ratio of the surcharge to the ground. As shown in Table 7, as x increases the bearing capacity decreases and the slope becomes unstable. Accordingly, the influence of x becomes large within $x=1.0$.

Also, in Tables 6 and 7, Considering γ , the slope is more stable. Expressing in other words, as the γ value increases, the slope becomes much firmer.

SUMMARY

The results of the present study can be summarized as follows:

1. When the sliding surface is of logarithmic-spiral shape, the upper bound limit analysis results in a close approximation to the result obtained previously by the slice method using a compounds sliding surface.
2. The correction factor α determined by the present method is approximately equal to the results of the conventional slice methods.
3. The values of bearing capacity of foundations on slopes by the present method are found close to those by Bishop and Kötter. But, the values by Honshi appears about half of those by the above three methods, though the correction factor (α) are approximative each other.
4. The seismic inertia force has a significant influence on bearing capacity. Using the applied load p as the objective function, the evaluation of the bearing capacity of foundations on slopes can be carried out simply and conveniently. The effect of seismic inertia force on the bearing capacity of foundations can be determined simply by the present method.

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NOTATIONS

The following symbols are used in this paper:

- B_i = Location of surcharge load P
- b = Length from the crown to the end of surcharge load
- c = Cohesion of soil
- g = Acceleration of gravity
- K = Horizontal acceleration factor of earthquake (or horizontal seismic coefficient)
- L = Length of failure mechanism
- ℓ = Arm length of failure mechanism
- r = Radius of rotational failure mechanism
- V_0 = initial velocity of the sliding soil mass on sliding surface
- θ_0 = Angle of starting point of failure mechanism
- θ_h = Angle of ending point of failure mechanism
- ϕ = Internal friction angle
- γ = Gravity of soil per unit volume