A GRANULAR MATERIAL MODEL BASED ON ASSOCIATED FLOW RULE APPLIED TO MONOTONIC LOADING BEHAVIOR

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ABSTRACT

The stress-strain behavior of a granular material is highly complicated relative to that of a cohesive soil. Due to the frictional nature of the material, a plasticity framework based on non-associated flow rule is generally considered to be most suitable for modeling its constitutive behavior. By employing two surfaces, it is shown in this paper that a plasticity model based on an associated flow rule provides a satisfactory means of simulating, under a monotonic loading, the stress-strain behavior of dense and loose sands under triaxial undrained and drained loading conditions.

Key words: constitutive equation, granular material, overconsolidation, plasticity, sand, triaxial test (IGC: E0/E13)

INTRODUCTION

Upon shearing, such as that involved during a triaxial loading, a granular material exhibits several, rather complex features. For instance, the volumetric behavior during a triaxial drained loading is found to change from compactive to dilative at a particular stress ratio. The line separating this behavior in the triaxial stress space is referred to as the phase transformation line (Ishihara et al., 1975; Habib and Luong, 1978). This line is found to be different in general from the failure line.

When a soil compacts during shear, it generally exhibits a hardening behavior. A granular material may, however, exhibit a hardening behavior while dilating, and in some cases (e.g., a very loose sand), a softening behavior while compacting. As a result, a combined density/deviatoric hardening rule is found to be necessary in simulating the behavior realistically (Nova, 1977; Wilde, 1977).

The effects of initial density and confining pressure are intermixed in a complex manner. The shape of the effective stress path during, say, an undrained triaxial shear loading can also be highly nonlinear. Furthermore, it has been shown by examining closely the behavior of some sands during stress probe experiments that they exhibit a non-associative behavior. These features render the mathematical modeling of the constitutive behavior of granular materials a difficult task.

Several plasticity models have been proposed in the past (e.g., Lade, 1977; Hashiguchi, 1979; Prevost, 1985; Pastor et al., 1985; Zienkiewicz et al., 1985; Bardet,

1986; Ghabhoussi and Momen, 1982; Poorooshasb and Pietruszczak, 1986), each with varying capabilities. Described in this paper is a new sand model, which is developed within the framework of the bounding surface concept (Dafalias, 1986). The unique feature of this model is that it employs an associated flow rule, thus overcoming some of the theoretical objections raised on the use of non-associated flow rules (Hashiguchi, 1989) and leading to a symmetric stiffness matrix, rendering finite element analyses very efficient. The key is the use of a second surface that is embedded within a bounding surface and is allowed to change continuously in size relative to that of the bounding surface.

The presentation in this paper is limited to application of the model in describing the behavior of sands during monotonic loading conditions. It is shown through extensive analysis of the model and comparison with experimental data that the model has the capability to describe most of the salient features of granular materials, including the nature of compaction and dilation, influence of such factors as the initial confining pressure, initial density, etc., and static liquefaction behavior of very loose sands.

Incorporating a special procedure to simulate more realistically the pore pressure buildup during stress reversals, the present model is shown in Anandarajah (1993, 1994) to be capable of describing the liquefaction behavior of sands during cyclic loading conditions. Finite element applications of a 9-parameter version of the liquefaction model for predicting liquefaction behavior of sand deposits subjected to earthquake loading may be

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found in Anandarajah (1993). While these models have been developed with the assumption of material isotropy, they may easily be extended for simulating anisotropic effects as in Anandarajah and Dafalias (1986).

THEORY

The theory is first presented for a three-dimensional state of stresses, and subsequently specialized for triaxial state for discussion and calibration purposes. In accordance with the soil mechanics convention, compressive stresses and strains are taken to be positive. Strains (ϵ_{ij}) are assumed to be small enough for decomposition of them into elastic (ϵ_{ij}^e) and plastic (ϵ_{ij}^p) parts to be valid. The effective stress is denoted by σ_{ij} . The material is assumed to be isotropic with respect to both elastic and plastic behavior. This requires that the constitutive functionals be expressed in terms of invariants, which are defined herein for the stress and plastic strain tensors as

$$I = \sigma_{kl} \delta_{kl} \quad (1)$$

$$J = \left(\frac{1}{2} s_{kl} s_{kl} \right)^{1/2} \quad (2)$$

$$S = \left(\frac{1}{3} s_{kl} s_{lm} s_{mk} \right)^{1/3} \quad (3)$$

$$-\frac{\pi}{6} \leq \alpha = \frac{1}{3} \sin^{-1} \left[\frac{3\sqrt{3}}{2} \left(\frac{S}{J} \right)^3 \right] \leq \frac{\pi}{6} \quad (4)$$

$$\epsilon_v^p = \epsilon_{kl}^p \delta_{kl} \quad (5)$$

$$\xi^p = \left(\frac{1}{2} e_{kl}^p e_{kl}^p \right)^{1/2} \quad (6)$$

where,

$$s_{ij} = \sigma_{ij} - \frac{1}{3} I \delta_{ij} \quad (7)$$

$$e_{ij}^p = \epsilon_{ij}^p - \frac{1}{3} \epsilon_v^p \delta_{ij} \quad (8)$$

and δ_{ij} is the Kronecker delta ($=0$ for $i \neq j$ and 1 for $i = j$). The surfaces involved in this model, as well as other functions, are expressed in terms of the stress invariants I , J and α (Lode angle). The plastic strain invariants ϵ_v^p and ξ^p are employed as the plastic internal variables defining the hardening behavior of the model.

Constitutive Equations

The rate of elastic strain is related to the rate of effective stress as

$$\sigma_{ij} = E_{ijkl} \dot{\epsilon}_{kl} \quad (9)$$

where

$$E_{ijkl} = \left(K - \frac{2}{3} G \right) \delta_{ij} \delta_{kl} + G (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (10)$$

and G and K are the bulk and shear moduli. The shear modulus is assumed as (Richart et al., 1970)

$$G = G_0 p_a \frac{(2.97 - e)^2}{1 + e} \left(\frac{p}{p_a} \right)^{n_e} \quad (11)$$

where G_0 and n_e are dimensionless material parameters, p_a is the atmospheric pressure, p ($=I/3$) is the mean normal pressure and e is the void ratio. While n_e is found to be approximately 0.5 for most sands (Richart et al., 1970), it is treated herein as a material parameter for the sake of generality.

The rate of plastic strain is expressed as

$$\dot{\epsilon}_{ij}^p = \langle L \rangle n_{ij} \quad (12)$$

where

$$L = \frac{1}{K_p} \dot{\sigma}_{kl} n_{kl} \quad (13)$$

$$(n_{kl} n_{kl})^{1/2} = 1 \quad (14)$$

and $\langle L \rangle = 0$ for $L \leq 0$ and $\langle L \rangle = L$ for $L > 0$, and K_p is the plastic modulus. L in the above equation, referred to as the loading index, defines the loading/unloading events. Note that K_p is included within the Heaviside function $\langle . \rangle$ as was done by Dafalias and Herrmann (1986), which allows strain softening to be simulated as discussed subsequently. The plastic constitutive law thus requires the definition of K_p and n_{ij} .

Bounding and Loading Surfaces

As most readers in soil mechanics are familiar with shapes of yield surfaces in the triaxial, p - q stress space, we present a sketch of the surfaces employed in the present theory in this space. A triaxial loading takes place in a principal plane with $\sigma_1 \neq \sigma_2 = \sigma_3$, where σ_1 , σ_2 and σ_3 are the principal stresses. With the mean normal pressure p and the deviatoric stress q defined as (Schofield and Wroth, 1968)

$$p = \frac{\sigma_1 + 2\sigma_3}{3} \quad (15)$$

$$q = \sigma_1 - \sigma_3 \quad (16)$$

it is easy to verify that $p = I/3$ and $|q| = \sqrt{3} J$, and thus p and q are invariants as well. A loading where $\sigma_1 > \sigma_3$ is referred to as the triaxial compression, and that where $\sigma_1 < \sigma_3$ is referred to as the triaxial extension. It then follows that $q > 0$ and $\alpha = \frac{\pi}{6}$ for triaxial compression loading and $q < 0$ and $\alpha = -\frac{\pi}{6}$ for triaxial extension loading. α will assume any value between $\frac{\pi}{6}$ and $-\frac{\pi}{6}$ for a general three-dimensional loading (i.e., $\sigma_1 \neq \sigma_2 \neq \sigma_3$).

The stress space shown in Fig. 1 is a triaxial compression loading (meridional) plane. The surface mathematically represented by $g(\sigma_{ij}^q, p_0) = 0$ is the bounding surface (Dafalias, 1986) and the region outside of it is inaccessible. The surface $f(\sigma_{ij}^f, p_0, M_p) = 0$ is used to define n_{ij} and L (Eqs. (12)–(14)), and this will be referred to herein as the loading surface. n_{ij} and r_{ij} are normal to $f=0$ and $g=0$ respectively. The lines OG and OQ are the failure and phase transformation lines, each having slopes of M_f and M_p respectively. The size and shape of $g=0$ depend on p_0 , R and M_f , of which p_0 is allowed to vary depending

with $r=1$ for $\eta^g > 1$ and $r=R-1$ for $\eta^g < 1$, and n_1 and n_2 are parameters defining the shape of the surface. p^g and q^g are mean normal and deviatoric stresses derived from σ_{ij}^g . The surface is assumed to be an ellipse for $\eta^g < 1$, and thus $n_1=n_2=2$ in this region. In the region for $\eta^g > 1$, the surface is assumed to be a distorted ellipse, i.e., $n_1 \neq n_2 \neq 2$. The ellipse and distorted ellipse are joined smoothly at $\eta^g=1$ with the normal being parallel to q -axis (i.e., at Point G). Equation of $f=0$ is similarly defined using p^f, q^f (derived from σ_{ij}^f) and M_p in Eq. (18).

In order to show the shapes represented by Eq. (18), the surfaces shown in Fig. 1 are drawn to scale with $M_f=1.5, M_p=0.5, R=2, p_0=1$, and $n_1=2.0$ and $n_2=1.2$ for $\eta^g, \eta^f > 1$. Based on our analyses to date, it was decided to fix the values of n_1 and n_2 at 2.0 and 1.2 respectively for all soils.

Hardening Rule

The evolution of p_0 is assumed as

$$\dot{p}_0 = C_1 \dot{\xi}^p + C_2 \dot{\xi}^p \quad (19)$$

$$C_1 = \frac{1+e_0}{\lambda^*} p_0 \quad (20)$$

$$C_2 = \beta(\xi^p, p_0) p_0 \quad (21)$$

where $\lambda^* = \lambda - \kappa$, and λ and κ are the slopes of consolidation and swelling lines in the $e - \ln(p)$ space obtained from the results of an isotropic consolidation test. e is the void ratio and e_0 is its initial value. It may be pointed out that the first term in Eq. (19) gives rise to the usual density hardening employed in most of the clay models (e.g., Schofield and Wroth, 1968) and some of the sand models (e.g., Bardet, 1986). The second term in Eq. (19) allows the material to harden or soften as a function of the plastic deviatoric strain rate. The above, therefore, is a combined density/deviatoric hardening rule, which was first used by Nova (1977) and Wilde (1977). The key to success of deviatoric hardening in describing material behavior over a wide range of conditions is proper choice of β . It is assumed herein that

$$\beta = \left(\gamma_0 - m \frac{p_0}{p_a} \right) e^{-\gamma_1 \xi^p} \quad \text{for } \beta > \beta_{\min} \\ = \beta_{\min} \quad \text{for } \beta \leq \beta_{\min} \quad (22)$$

where γ_0, m and γ_1 are material parameters, which may in general depend on the initial void ratio e_0 of the soil. $\beta_{\min}$ is the minimum value of β .

γ_1 and m are always defined to be positive. For a range of initial densities that are encountered normally in practice (i.e., for sands that are not too loose), γ_0 is assigned a positive value such that the initial value of β is positive, and $\beta_{\min}$ is chosen to be zero. For example, say, $\gamma_0=120, m=2$ and $p_0=50$ kPa (initially). This leads to an initial value (Note $\xi^p=0$ initially.) for β of 119 ($p_a=100$ kPa). During loading, β decreases from its positive initial value to zero as a result of increasing ξ^p and/or p_0 , and thus gradually erasing the deviatoric hardening effect. Note that the soil can still possess the density hardening effect,

which is independent of the deviatoric hardening effect.

On the other hand, when γ_0 is chosen so that β is initially negative, and $\beta_{\min}$ is set to this initial negative value, β will increase from $\beta_{\min}$ during loading to zero as ξ^p and/or p_0 increase. For example, say, $\gamma_0=-120, m=2$ and $p_0=50$ kPa (initially). This leads to an initial value for β of -121 . $\beta_{\min}$ will be set to -121 . Such a choice for β allows peculiar behavior such as the static liquefaction experimentally observed for very loose sands (Castro, 1969) to be simulated by the model. The technique, therefore, is very powerful in simulating the real behavior, and will further be discussed subsequently.

M_p is assumed to vary from $\alpha_g M_f$ to M_f ($0 \leq \alpha_g \leq 1$) as ζ^g varies from 0 to 1 as

$$M_p = M_f [\alpha_g + (1 - \alpha_g)(\zeta^g)^{n_g}] \quad (23)$$

where α_g and n_g are material parameters. While the above equation provides satisfactory results for the experimental data considered herein, other forms may be found more suitable for other sands. Note that a state of stress is not allowed to lie outside of the surface $g=0$, and thus the maximum value of M_p is M_f .

Consistency Condition and Bounding Plastic Modulus

Once a stress state reaches the bounding surface $g=0$, it continues to remain on $g=0$ for $\dot{\sigma}_{ij}$ directed outward from $g=0$. This is enforced by applying the consistency condition to $g=0$ as

$$\dot{g}(\sigma_{ij}, p_0) = 0 \quad (24)$$

and it is easy to verify that Eq. (24) yields the following expression for the plastic modulus on $g=0$ (at a point such as C)

$$K_b = K_{b1} + K_{b2} \quad (25)$$

$$K_{b1} = -\frac{1+e_0}{l\lambda^*} \frac{\partial g}{\partial p_0} p_0 r_{kk} \quad (26)$$

$$K_{b2} = -\frac{1}{l} \beta \frac{\partial g}{\partial p_0} p_0 r^d \quad (27)$$

where $l = ((\partial g / \partial \sigma_{ij})(\partial g / \partial \sigma_{ij}))^{1/2}$, r_{kk} is the trace of r_{ij} and r^d is the second invariant defined from the deviatoric part of r_{ij} as

$$r_{ij}^s = r_{ij} - \frac{1}{3} r_{kk} \delta_{ij} \quad (28)$$

$$r^d = \left(\frac{1}{2} r_{ij}^s r_{ij}^s \right)^{1/2} \quad (29)$$

The sign of K_b , therefore, depends on the sign of $\partial g / \partial p_0, r_{kk}, \beta$ and r^d . $g=0$ will be defined such that $\partial g / \partial p_0$ will always be negative. r^d is always positive. For a positive β , K_{b2} is always positive. For $\eta^g < 1$ (i.e., on the portion of the bounding surface P_0 -C-Q-F-G), $K_{b1} > 0$ since $r_{kk} > 0$. $K_{b1} < 0$ for $\eta^g > 1$ since $r_{kk} < 0$. $K_{b1} = 0$ when $\eta^g = 1$ since $r_{kk} = 0$.

Shape Hardening Plastic Modulus

Let us consider a stress state that lies within the bound-

ing surface. $g \neq f$ for $\alpha_g < 1$ in this case (Eq. (23)). The plastic modulus consists of K_b and H as given by Eq. (17). The following form for H is found to be satisfactory for stress-strain behavior under monotonic loading considered thus far.

$$H = h_0 K_0 H_1(p) H_2(\zeta^g) H_3(\eta^g) \quad (30)$$

$$H_1(p) = \left(\frac{p}{p_a} \right)^{n_e} \quad (31)$$

$$H_2(\zeta^g) = \langle 1 - \zeta^g \rangle^{m_1} \quad (32)$$

$$H_3(\eta^g) = 1 - (\eta^g)^{h_1} \quad \text{for } \eta^g \leq 1 \quad (33)$$

$$H_3(\eta^g) = 0 \quad \text{for } \eta^g > 1 \quad (34)$$

where h_0 , h_1 , m_1 and n_e are material parameters, all of which are defined to be positive. K_0 is the value of the bounding modulus (Eq. (25)) evaluated at the point where the surfaces intersect the p -axis; i.e., at $q=0$ and $p=p_0$. n_e is the same parameter as the one appearing in Eq. (11), thus allowing H to decrease with p , just as the elastic moduli G and K decrease with p . h_0 controls the behavior for $\eta^g=0$, i.e., on the p -axis. h_1 controls variation of H as η^g increases. m_1 governs the variation of H as the stress state moves between the bounding surface ($\zeta^g=1$) and the origin ($\zeta^g=0$) along a radial line ($\eta^g=\text{constant}$). The influence of η^g was considered previously in Bardet (1986), where a different form based on a trigonometric function was employed.

Generalization to Three-Dimensional Loading Conditions

The three-dimensional behavior may approximately be simulated by allowing certain material parameters to vary as a function of Lode angle α . For example, M_f at any α may be expressed in terms of M_{fc} (the value of M_f in compression) and n ($=M_{fe}/M_{fc}$ with M_{fe} denoting the value of M_f in extension) as

$$M_f = b(n, \alpha) M_{fc} \quad (35)$$

where b is a suitable function. The following function is chosen here.

$$b(n, \alpha) = \frac{2n}{1 + n - (1 - n) \sin 3\alpha} \quad (36)$$

The value of a parameter in triaxial compression and extension may be determined by calibrating the model independently using data obtained from compression and extension tests. All of the model parameters involved in the present model may be allowed to vary as a function of α , with the exception of λ^* and h_0 , which are defined based on the behavior during an isotropic loading.

Material Parameters

The model requires in its most general form the following parameters: (1) Elastic parameters G_0 , ν and n_e , (2) geometrical parameters M_f , α_g , n_g and R , (3) density hardening parameter λ^* , (4) deviatoric hardening parameters γ_0 , γ_1 and m , and (5) shape hardening parameters h_0 , h_1 and m_1 . Except for λ^* and h_0 , the parameters may be deter-

mined independently for triaxial compression and extension cases. Additionally, the initial values of σ_{ij} , p_0 and e are required.

In the event of limited amount of data, some of the above parameters may be assigned fixed values. For example, $R=2.7$ in the Cam-Clay theory (Schofield and Wroth, 1968) and 2.0 in the modified Cam-Clay theory (Roscoe and Burland, 1968). One of these values, say 2.0, may be assigned to R . Normally, a suitable value may be assigned to ν (say, 0.3). For most sands, $n_e=0.5$ is found to be satisfactory (Richart et al., 1970). Depending on the range of strain of interest, for some soils, either a density hardening including shape hardening or a deviatoric hardening excluding shape hardening may be used, thus eliminating the need for parameters associated with one of them. This will further be discussed subsequently. Parameters may also be assumed to be equal in compression and extension, except M_f , whose value in extension may be determined from the Mohr-Coulomb criterion. Examples of the use of such simplified models may be found in Anandarajah (1993) and Anandarajah and Agarwal (1991).

QUALITATIVE TRIAXIAL MODEL BEHAVIOR

Triaxial Stress-Strain Relationships

In order to analyse the behavior of the model, the constitutive equations are specialized herein for triaxial loading conditions. In this case, the generalized equations given by Eqs. (9), (12) and (13) can be reduced to volumetric and deviatoric scalar equations as

$$\dot{\epsilon}_v^e = \frac{\dot{p}}{K} \quad (37)$$

$$\dot{\epsilon}_d^e = \frac{\dot{q}}{3G} \quad (38)$$

$$\dot{\epsilon}_v^p = \frac{1}{K_p^*} (n_p \dot{p} + n_q \dot{q}) n_p \quad (39)$$

$$\dot{\epsilon}_d^p = \frac{1}{K_p^*} (n_p \dot{p} + n_q \dot{q}) n_q \quad (40)$$

where $\dot{\epsilon}_v = \dot{\epsilon}_1 + 2\dot{\epsilon}_3$ is the volumetric strain rate, $\dot{\epsilon}_d = \frac{2}{3}(\dot{\epsilon}_1 - \dot{\epsilon}_3)$ is the deviatoric strain rate, and $n_p = (1/l^*)(\partial f / \partial p)$ and $n_q = (1/l^*)(\partial f / \partial q)$ are component of a unit normal to $f=0$ in the p - and q -directions respectively. The strain variables (ϵ_v , ϵ_d) as defined here are energy conjugates of the stress variables (p , q). Note that $n_p^2 + n_q^2 = 1$ and $l^* = \sqrt{(\partial f / \partial p)^2 + (\partial f / \partial q)^2}$. Also note that a superscript $*$ is used with the plastic modulus to emphasize that it is defined with respect to triaxial loading condition as in Eqs. (39)–(40).

Triaxial Undrained Behavior with an Initial $\beta \geq 0$

Let us now consider a triaxial undrained loading. In this case

$$\dot{\epsilon}_v = 0 \quad (41)$$

$$\dot{\epsilon}_v^e + \dot{\epsilon}_v^p = 0 \quad (42)$$

$$\frac{\dot{p}}{K} + \dot{\varepsilon}_v^p = 0 \quad (43)$$

$$\dot{p} = -K\dot{\varepsilon}_v^p \quad (44)$$

By substituting Eq. (44) into Eqs. (37)–(40), the differential equations for undrained effective stress path (i.e., p vs q) and the deviatoric stress-strain relation (i.e., ε_d vs q) are obtained as

$$\frac{\dot{q}}{\dot{p}} = -\frac{n_p}{n_q}(1+y) \quad (45)$$

$$\frac{\dot{\varepsilon}_d}{\dot{q}} = -\frac{n_q^2}{K_p^*} \left(\frac{y}{1+y} \right) + \frac{1}{3G} \quad (46)$$

where,

$$y = \frac{K_p^*}{Kn_p^2} \quad (47)$$

A number of observations can be made on the basis of Eqs. (45)–(47) concerning the model behavior. For clarity, the magnitude and sign of relevant quantities are summarized in Table 1. A typical effective stress path along with the phase transformation line (PTL), the critical state line (CSL) and certain key points are shown in Fig. 2. Behavior of a sample loaded from an isotropically-consolidated initial state (Point A) is considered. On the p -

Fig. 2. Typical triaxial compression undrained effective stress path (refer to Table 1)

Table 1. Theoretical triaxial undrained behavior at different states with initial $\beta > 0$

Stress State (Points in Fig. 2)	n_p	n_q	K_{b1}^*	K_{b2}^*	H^*	K_p^*	y	$ y $	$\frac{\dot{q}}{\dot{p}}$	$\frac{\dot{\varepsilon}_d}{\dot{q}}$
$q=0$ (A)	>0	0	>0	0	>0	>0	>0		∞	$\frac{1}{3G}$
$0 < \frac{q}{p} < M_p$ (A-B)	>0	>0	>0	>0	>0	>0	>0		<0	>0
$\frac{q}{p} = M_p$ (B)	0	1	>0	>0	>0	>0	∞		∞	$\frac{3G+K_p^*}{3GK_p^*}$
$M_p < \frac{q}{p} < M_f$ (B-C)	<0	>0	>0	>0	>0	>0	>0		>0	>0
$\frac{q}{p} > M_f$ (C-D) $\frac{q}{p}$ increases	<0	>0	<0	>0	0	>0	>0		>0	>0
$\frac{q}{p} > M_f$ (D) $\left(\frac{q}{p}\right)_{\max}$	<0	>0	<0	>0	0	0	0		>0	>0
$\frac{q}{p} > M_f$ (D-E) $\frac{q}{p}$ decreases	<0	>0	<0	≥ 0	0	<0	<0	<1	>0	>0
$\frac{q}{p} = M_f$ (E) CS failure	0	1	0	0	0	0			0	∞

axis, $n_q=0$, which leads to $\dot{q}/\dot{p}=\infty$ and $\dot{\varepsilon}_d/\dot{q}=1/3G$. That is, the undrained effective stress path is normal to p -axis and the initial slope of ε_d vs q relationship depends only on shear modulus.

For stress states that lie between PTL and the p -axis (i.e., between Points A and B in Fig. 2), $n_p>0$ and $n_q>0$. In this case, if $K_p^*>0$, then $y>0$ and thus $\dot{q}/\dot{p}<0$ and $\dot{\varepsilon}_d/\dot{q}>0$. The effective stress path will lie on the left side of vertical and ε_d vs q curve will be a rising one (i.e., hardening type). In this region, K_{b1}^* and H^* are always positive, and when $\beta>0$, K_{b2}^* will also be positive, thus $K_p^*>0$.

On PTL (i.e., at Point B), $n_p=0$ and $n_q=1$. Thus with $K_p^*\neq 0$, $y=\infty$, giving rise to $\dot{q}/\dot{p}=\infty$ and $\dot{\varepsilon}_d/\dot{q}=(3G+K_p^*)/3GK_p^*$. The effective stress path crosses PTL parallel to the q -axis.

For stress states lying between PTL and CSL (i.e., from B to C), $n_p<0$ and $n_q>0$. H^* is always positive. r_{kk} (defined on $g=0$) is positive and thus $K_{b1}^*>0$. When $\beta>0$, $K_{b2}^*>0$, and thus $K_p^*>0$, leading to $y>0$. Therefore, $\dot{q}/\dot{p}>0$ and $\dot{\varepsilon}_d/\dot{q}>0$. That is, the effective stress path will lie on the right side of vertical and ε_d vs q curve will still be a rising one. The model thus simulates a hardening behavior even though the material dilates ($n_p<0$) in this region.

In the region above CSL, $n_p<0$, $n_q>0$ and $r_{kk}<0$. $H^*=0$, $K_{b1}^*\leq 0$ and $K_{b2}^*\geq 0$. $K_p^*(=K_{b1}^*+K_{b2}^*)$ may either be positive, zero or negative, and depending on this, the model may give rise to strain-hardening, peak or strain-softening behavior. An important subtle point to note here is that, after K_p becomes negative, although q/p will always decrease, q will increase for $|y|<1$ and decrease for $|y|>1$. The former case is what is shown in Fig. 2 (and described in Table 1).

If the values of K_{b1}^* and K_{b2}^* are such that K_p^* is positive, the behavior will be similar to that in the region between Points B and C, i.e., strain hardening. This will be the behavior from Points C to D.

During continued shearing, β will decrease reducing the value of K_{b2}^* . At one point, K_p^* can become zero and then negative. The stress ratio reaches the maximum at Point D, where $K_p^*=0$ but $n_p\neq 0$.

Since K_p^* is included within the equation for L (Eq. (13)), the plastic loading may take place with negative K_p^* for stress rate directions such that $\dot{\sigma}_{kl}n_{kl}<0$. y is negative and it follows from Eqs. (45)–(47) that when $|y|<1$, $\dot{q}/\dot{p}>0$ and $\dot{\varepsilon}_d/\dot{q}>0$, i.e., strain hardening (from Points D to E). The convex nature of the bounding surface ensures that q/p is maximum when K_p changes from positive to negative, i.e., at Point D. Eventually, K_p^* again becomes zero at Point E, where $n_p=0$, resulting in $\dot{q}/\dot{p}=0$ and $\dot{\varepsilon}_d/\dot{q}=\infty$. The material experiences a critical state failure at this point.

On the other hand, when $y<0$ and $|y|>1$ (case not shown in Fig. 2 or described in Table 1), $\dot{q}/\dot{p}<0$ and $\dot{\varepsilon}_d/\dot{q}<0$ (i.e., q decreases, thus exhibiting a strain softening), and when $|y|=1$, $\dot{q}/\dot{p}=0$ and $\dot{\varepsilon}_d/\dot{q}=\infty$ (peak). Even in this case, K_p^* eventually becomes zero with $n_p=0$, and $\dot{q}/\dot{p}=0$ and $\dot{\varepsilon}_d/\dot{q}=\infty$. The material experiences a critical state failure.

Remark:

While the behavior described above represents a general case, it must be noted that in the absence of deviatoric hardening (i.e., $\beta=0$ and thus $K_{b2}^*=0$) and shape hardening (say, the stress state is on the bounding surface and thus $H^*=0$), the stress path never goes above CSL, i.e., the sample fails as soon as the stress path reaches Point C since K_p is zero at this point. This is the typical behavior of a loose sand.

Triaxial Drained Behavior with Initial $\beta>0$

In this section, the model's behavior of a soil sample, which is subjected to a drained triaxial compression loading starting from an isotropically-consolidated initial state, is examined. Specifically, ε_d vs ε_v and ε_d vs q relationships are of interest. The essential details are summarized in Table 2. Figure 3 shows a typical stress path, along with CSL and PTL.

Without loss of generality, a triaxial loading such that $\dot{p}=0$ (i.e., a constant mean normal pressure test) is considered. It is easy to show from Eqs. (37)–(40) that

$$\frac{\dot{\varepsilon}_d}{\dot{\varepsilon}_v} = \frac{n_q}{n_p} \left(1 + \frac{K_p^*}{3Gn_q^2} \right) \quad (48)$$

$$\frac{\dot{\varepsilon}_d}{\dot{q}} = \frac{1}{3G} \left(1 + \frac{3Gn_q^2}{K_p^*} \right) \quad (49)$$

With the aid of Table 2, it is easy to verify that the soil compacts from Points A to B shown in Fig. 3, with the slope of ε_d vs ε_v relationship changing from zero at Point A to positive to zero again at Point B. ε_d vs q relationships is a rising (i.e., strain hardening) type, with its slope changing from $1/3G$ at Point A to $(3G+K_p^*)/3GK_p^*$ at Point B.

The key change as the stress path crosses PTL (i.e., at Point B) is that n_p changes from positive to negative. Consequently, the volumetric behavior changes from compactive to dilative at Point B, and remains dilative from this point on. However, r_{kk} (and therefore, K_{b1}^*) changes from positive to negative only as the stress path crosses CSL, i.e., at Point D. Therefore, q still increases with ε_d between Points B and D.

The behavior above CSL depends on the relative magnitudes of K_{b1}^* (≤ 0) and K_{b2}^* (≥ 0). (Note that $H^*=0$ above CSL.) Typically, as K_p^* changes from positive (say, from D to C) to negative (say, from C to D), the behavior changes from strain-hardening type to strain-softening in the corresponding regions. As K_p^* is included within the definition of the loading index (Eq. (13)), plastic behavior is permitted from Points C to D. Point C signifies the “peak” and Point D the ultimate, and the slope of ε_d vs q relationship is zero at these points. The slope of ε_d vs ε_v relationship remains negative until the material reaches the ultimate state, at which point, it becomes zero, thus resulting in a critical state failure.

Simulation of Undrained Static Liquefaction with Initial $\beta<0$

Let us now consider the possibility of simulating the be-

Table 2. Theoretical triaxial drained behavior at different states with initial $\beta > 0$

Stress State (Points in Fig. 3)	n_p	n_q	K_{b1}^*	K_{b2}^*	H^*	K_p^*	$\frac{\dot{\epsilon}_d}{\dot{\epsilon}_v}$	$\frac{\dot{\epsilon}_d}{\dot{q}}$
$q=0$ (A)	>0	0	>0	0	>0	>0	∞	$\frac{1}{3G}$
$0 < \frac{q}{p} < M_p$ (A-B)	>0	>0	>0	>0	>0	>0	>0	>0
$\frac{q}{p} = M_p$ (B)	0	1	>0	>0	>0	>0	∞	$\frac{3G+K_p^*}{3GK_p^*}$
$M_p < \frac{q}{p} < M_f$ (B-D)	<0	>0	>0	>0	>0	>0	<0	>0
$\frac{q}{p} > M_f$ (D-C)	<0	>0	<0	>0	0	>0	<0	>0
Strain Hardening								
$\frac{q}{p} > M_f$ (C)	<0	>0	<0	>0	0	0	<0	∞
Peak $\left(\left(\frac{q}{p} \right)_{\max}, q_{\max} \right)$								
$\frac{q}{p} > M_f$ (C-D)	<0	>0	<0	≥ 0	0	<0	<0	<0
Strain Softening								
$\frac{q}{p} = M_f$ (D)	0	1	0	0	0	0	∞	∞
Critical State

Fig. 3. Typical triaxial compression drained stress path (refer to Table 2)

havior of a very loose sand, whose typical behavior is shown in Fig. 4. The sample liquefies in the sense that p

becomes zero (Castro, 1969) when it is sheared under undrained condition from an isotropically-consolidated initial state. Say, we simulate the behavior using an initial, negative value for β and assuming $p_{init} = p_0$, where p_{init} is the initial confining pressure. The magnitude and sign of relevant quantities at various stages during the loading are summarized in Table 3, where undrained relations given by Eqs. (45)–(47) are utilized.

Since the state of stress is initially on $q=0$, it follows that $\zeta^q = 1$ and thus $H^* = 0$. n_p will remain positive provided the stress path never goes above CSL, and n_q is always positive. Assume that the initial value of β is chosen such that $K_{b1}^* > K_{b2}^*$ and thus $K_p^* > 0$. The initial behavior during shear will be the same as in the case with $\beta > 0$ discussed earlier, i.e., the effective stress path will emerge from the p -axis at 90° (Point A) and lean towards left from vertical, and q will increase with ϵ_d . K_{b1}^* will decrease since r_{kk} decreases. K_{b2}^* will increase (i.e., $|K_{b2}^*|$ will decrease) since ξ^p increases, the rate of which will, however, be initially small because ξ^p will be small.

At some stage of loading, K_p^* becomes zero. At this point, $\dot{q}/\dot{p} = -(n_p/n_q) < 0$ and $\dot{\epsilon}_d/\dot{q} = (n_q^2/K_p^2) + (1/3G) > 0$. A point such as Point B in Fig. 4 represents this stage. As the loading continues, K_p^* becomes more negative with $|K_p^*|$ increasing. Now the nature of the curves depend on the value of $|y|$ (Eq. (47)). $\dot{q}/\dot{p} < 0$ and $\dot{\epsilon}_d/\dot{q} > 0$ when $|y| < 1$ (Point C, hardening), $\dot{q}/\dot{p} > 0$ and $\dot{\epsilon}_d/\dot{q} < 0$ when $|y| > 1$ (Point E, softening) and $\dot{q}/\dot{p} = 0$ and

Fig. 4. Typical static liquefaction behavior of a very loose sand under triaxial undrained compression

Table 3. Theoretical static liquefaction behavior at different states with initial $\beta > 0$

Points	n_p	n_q	K_{b1}^*	K_{b2}^*	H^*	K_p^*	γ	$ y $	$\frac{\dot{q}}{\dot{p}}$	$\frac{\dot{\epsilon}_d}{\dot{q}}$
A	>0	0	>0	<0	0	>0	>0		∞	$\frac{1}{3G}$
B	>0	>0	>0	<0	0	0	0		<0	>0
C						<0	<0	<1	<0	>0
D								1	0	∞
E								>1	>0	<0

$\dot{\epsilon}_d/\dot{q} = \infty$ when $|y| = 1$ (Point D, peak).

Remarks:

It must be noted that in the above simulation, γ_0 is assigned either a small positive or a negative value to make the initial value of β negative to reflect the fact that the relative density of the soil is very small. During a drained loading, however, the density will change. The change will be significant in the above case since the initial value of plastic modulus is relatively small, giving rise to significant (positive) plastic volumetric strain. For realistic simulation of the behavior, the value of γ_0 must be allowed to increase to reflect densification effect. While this must be done even for a soil that is initially dense, the volume change during shear will be small for such a soil, and thus the use of a constant value for γ_0 may not lead to unrealistic overall behavior.

COMPARISON WITH TRIAXIAL DATA

A set of triaxial experiments are simulated by the model, and the results compared and discussed in this section. As part of an NSF-funded project named VELACS (Arulmoli et al., 1992), a series of undrained and drained tests were performed on a sand named Nevada sand and on a silt named Bonnie silt by The Earth Technology Corporation in California. The validity of test results was verified independently by repeating some of the tests at a

number of institutions, including The Johns Hopkins University under supervision of the writer. A report containing the test data was prepared by Arulmoli et al. (1992), and submitted to the National Science Foundation. The data used in this paper are obtained from this report. In addition, some data concerning the static liquefaction behavior of very loose Nevada sand from the doctoral thesis of Yogachandran (1990) is also utilized.

Nevada sand was found to have a specific gravity of 2.67, maximum dry density of 17.33 kN/m^3 (or $e_{\min} = 0.511$) and minimum dry density of 13.87 kN/m^3 (or $e_{\max} = 0.888$). Sieve analyses indicated that the percentages of sand passing sieves of size 2, 0.850, 0.425, 0.250, 0.100 and 0.075 mm were 100.0, 100.0, 99.7, 97.3, 49.1 and 7.7% respectively. The soil, therefore, is a uniformly-graded fine sand.

Bonnie silt was found to have a specific gravity of 2.67, liquid limit of 29% and a plastic limit of 14%. The percentages of soil passing sieves of size 2, 0.250, 0.075, 0.032, 0.0125, 0.0065, 0.0032, 0.0014 mm were found to be 100.0, 99.0, 91.4, 39.9, 20.9, 13.3, 9.5, and 5.7% respectively. The soil thus contains 8.5% of sand ($0.08 \text{ mm} < \text{diameter} < 2.0 \text{ mm}$), 84.5% of silt ($0.002 \text{ mm} < \text{diameter} < 0.08 \text{ mm}$) and 7.0% of clay (diameter $< 0.002 \text{ mm}$). The behavior of Bonnie silt is more close to typical behavior of sands than clays.

Model Simulation of Undrained Behavior Assuming $p_{init}=p_0$

In this section, the behavior of Nevada sand is simulated and compared with test results. Undrained triaxial compression tests were carried out at 40, 50, 60 and 70 % relative densities and drained tests at 40 and 60% relative densities. The samples were prepared by dry pluviation, back pressure saturated, isotropically consolidated to 40, 80 and 180 kPa and sheared to failure. All samples, therefore, were normally consolidated.

First, let us consider the behavior of undrained specimens. The behavioral pattern was found to vary as the initial density of the specimens increased. The behavior of specimens at the extreme relative densities, i.e., $D_r=40$ and 70%, is first considered herein for detailed analysis. The test results, along with model simulations, are presented in Figs. 5(a) and 5(b) in p - q and ϵ_1 - q spaces, where ϵ_1 is the vertical, axial strain. The overall behavior ($\epsilon_1=0-10\%$) is shown in Fig. 5(a), and for clarity, the low strain behavior ($\epsilon_1=0-2\%$) is shown in Fig. 5(b).

The effect of overconsolidation, initial density, and initial confining pressure are all intermixed in a complex manner for granular materials. Thus, even when the sam-

ple is actually normally consolidated as in the case of Nevada sand samples considered herein, it is not clear as to whether or not the initial confining pressure p_{init} may be assumed to be the same as p_0 appearing in the constitutive model.

First, we attempted to do the simulations by assuming $p_{init}=p_0$ for all specimens. As $\zeta^g=1$ under this assumption, $H=0$ and the material parameters h_0 , h_1 and m_1 , therefore, are irrelevant. λ^* was determined from the results of consolidation tests. M_f was determined from the p - q relation. G_0 and n_e were determined by matching initial slopes of the experimental and model ϵ_1 - q relations. As $\zeta^g=1$, $f=g$ and thus α_g and n_g are irrelevant. R and ν were assumed to be 2.0 and 0.3 respectively. Based on the understanding of their influence on the stress-strain behavior, the deviatoric hardening parameters γ_0 , γ_1 and m were found by trial-and-error so as to obtain a good overall match. γ_0 , γ_1 and m have a significant influence on the theoretical behavior at high stress ratios, i.e., when the stress state is more close to the failure line relative to p -axis.

The parameters λ^* and M_f depend on the initial relative density. However, they are determined directly from

Fig. 5(a). Stress-strain behavior of Nevada sand under undrained compression for specimens with $D_r=40$ and 70%-overall behavior (Theory: $\zeta^g=1$, $H=0$, $K_b=K_p=K_{b1}+K_{b2}$)

the data as described above. It was found that different values had to be chosen for the parameters γ_0 and m as well. γ_1 was kept a constant for all densities. The model parameters are listed in Table 4. While the comparison is rather poor in the low strain range (Fig. 5(b)), the model simulates the overall behavior satisfactorily (Fig. 5(a)).

A number of important features may be noted from the test results, and the theory is able to capture some of them as follows. The samples' behavior is dilative from the beginning (Note from Fig. 5(b) that the effective stress path lies on or to the right of vertical.), whereas the model predicts a contractive behavior for $q/p=0$ to M_f , since $M_f=M_p$ and thus $g=f$ in this range. This will subsequently be discussed further in detail.

The slope of the ε_1 vs q changes in somewhat of a non-linear manner. After the initial elastic behavior, the curve is somewhat flatter initially, becoming steeper subsequently and flattens as the failure approaches. The model is able to capture this feature as follows. On the p -axis, $n_p=1$ and $n_q=0$ and, therefore, only the density hardening will operate, i.e., $K_{b1}>0$ and $K_{b2}=0$. However, once the stress path reaches near the failure line, K_{b1} relative to K_{b2} becomes very small since n_p is very near zero. The vari-

ation of plastic modulus with strain is presented in Fig. 6(a) (low strain range) and Fig. 6(b) (for the rest of loading) for the case with $D_r=40\%$ and $p_{init}=40$ kPa. It may be noted that K_{b1} has become negative in the range $\varepsilon_1=0.2-10\%$, indicating the state of stress is above the failure line ($n_p<0$).

It is seen from Eq. (27) that K_{b2} is proportional to p_0 . As the stress proceeds to the right, p_0 increases along with it, and the stress state continues to lie on the surface. The increase in p_0 ($I_0=3p_0$) and K_p is shown in Fig. 7 for all samples. Note that in these figures, the values of p_0 and K_p have higher values for larger p_{init} at the initial stage. The slope of ε_1 vs q changes from flat to steep due to the increase in p_0 . During this part of the loading, the effective stress path is almost coincident with the failure line, and thus n_p and n_q do not change much. It follows from Eq. (46) that an increase in K_p increases the slope of ε_d vs q relation. Note that G also increases (Eq. (11)), which further increases the slope.

However, as p_0 continues to increase in addition to the increase in ξ^p , β becomes smaller (Eq. (22)), making K_{b2} to decrease (Eq. (27)). The total plastic modulus K_p decreases, making the ε_1 vs q curve flatter at the latter

Fig. 5(b). Stress-strain behavior of Nevada sand under undrained compression for specimens with $D_r=40$ and 70%-low strain behavior (Theory: $\zeta^g=1$, $H=0$, $K_b=K_p=K_{b1}+K_{b2}$)

Table 4 Model parameters for simulations with deviatoric hardening and without shape hardening ($\zeta=1$)

Dr (%)	λ^*	M_f	γ_0	m	γ_1	R	G_0	n_e
N. Sand (40%)	.015	1.35	122	2.38	25	2	90	1
N. Sand (50%)	.013	1.40	122	2.38				
N. Sand (60%)	.010	1.50	200	5.30				
N. Sand (70%)	.008	1.57	237	6.77				
N. Sand (15%)	.020	0.76	-123	0.95				
Bonnie Silt	.040	1.20	10	1.10	5	2	20	0.5

Fig. 6. Variation of K_{b1} , K_{b2} and $K_{b1}+K_{b2}$ during theoretical simulation in Figs. 5(a) and 5(b) for sample with $D_r=40\%$ and $p_{init}=40$ kPa **Fig. 7.** Variation of p_0 and K_p during theoretical simulation in Figs. 5(a) and 5(b)

stage.

Since the values used for γ_0 increases with increasing D_r , K_{b2} is larger for specimens with $D_r=70\%$ than that for specimens with $D_r=40\%$, giving rise to a stiffer initial behavior for specimens with $D_r=70\%$ relative to that for specimens with $D_r=40\%$. p_0 , therefore, increases more rapidly with ϵ_1 for denser specimens, however, making K_{b2} to decrease earlier as ϵ_1 increases (Fig. 7). Thus, ϵ_1 vs q relation becomes flatter at a smaller strain for these sam-

ples than that for specimens with $D_r=40\%$. This also reduces the effect of the initial confining pressure on the behavior during the latter stage of the shear loading. In fact, it may be observed that the larger the p_{init} , the earlier the K_p becomes smaller, the earlier the ϵ_1 - q becomes flatter. Observe from Fig. 5 that there is a significant difference between the stress-strain curves associated with $p_{init}=40, 80$ and 160 kPa for samples with $D_r=40\%$.

The comparisons for specimens with $D_r=50$ and 60%

Fig. 8(a). Stress-strain behavior of Nevada sand under undrained compression for specimens with $D_r=50$ and 60%-overall behavior (Theory: $\zeta=1$, $H=0$, $K_b=K_{b1}+K_{b2}$)

are shown in Fig. 8(a) (overall behavior) and in Fig. 8(b) (low strain behavior), which indicate that the match is as good as that for specimens with $D_r=40$ and 70%. Note that data for the case with $p_{init}=40$ kPa and $D_r=40\%$ is not available. The model parameters used are listed in Table 4.

The available test data base includes a set of data on undrained behavior of normally consolidated Bonnie silt. Three specimens were tested in triaxial compression, each with $p_{init}=40$, 80 and 160 kPa. The behavior was simulated as was done for Nevada sand, i.e., with the assumption that $p_{init}=p_0$. The model simulations are compared with test results in Fig. 9. The model parameters employed are listed in Table 4. The model simulates the behavior of this silt satisfactorily. The soil shows a contractive initial behavior with a subsequent dilative behavior, the degree of the latter is much smaller than that of the Nevada sand. Consequently, a value of γ_0 (10.0), much smaller than that used for Nevada sand (>122) is found to be sufficient.

In summary, the model with the assumption that $p_{init}=p_0$, thus ignoring the shape hardening effect, yields satisfactory results for the overall stress-strain behavior

for Nevada sand and Bonnie silt.

Model Simulation of Undrained Behavior Assuming $p_{init} \neq p_0$

As pointed out earlier, the model at the beginning of loading predicts a contractive behavior with the assumption $p_{init}=p_0$, whereas the experimental data show that the material dilates from the beginning (Figs. 5(b) and 8(b)).

This suggests that, even though p_{init} was the largest confining pressure ever applied to the samples, they may not be treated as normally consolidated as it would normally be done for cohesive soils. A combined hardening using a p_0 that is different from p_{init} is necessary for this sand. In addition to the deviatoric hardening parameters γ_0 , γ_1 and m , the shape hardening parameters h_0 , h_1 and m_1 are also needed in this case, which were again determined by a trial-and-error procedure. These parameters were chosen primarily by matching the initial pore pressure responses. Since the state of stress is initially inside the surfaces, parameters α_g and n_g are also now required. These were also obtained by trial-and-error by matching the overall responses. The model parameters employed

Fig. 8(b). Stress-strain behavior of Nevada Sand under undrained compression for specimens with $D_r=50$ and 60%-low strain behavior (Theory: $\zeta=1$, $H=0$, $K_b=K_{b1}+K_{b2}$)

Fig. 9. Stress-strain behavior of bonnie slit under undrained compression (Theory: $\zeta^g=1$, $H=0$, $K_b=K_{b1}+K_{b2}$)

Table 5. Model parameters with combined hardening for Nevada sand of $D_r=40\%$

λ^*	R	M_f	G_0	n_e	γ_0	γ_1	m	h_0	h_1	m_1	α_g	n_g	p_0
.015	2	1.35	22	1.2	122	25	2	10	8	.2	.4	10	480 (kPa)

are listed in Table 5.

The results obtained using combined hardening and assuming $p_{init} \neq p_0 = 480$ kPa are shown in Fig. 10. It may be observed that the overall behavior, as well as the initial behavior, can now be simulated satisfactorily by the model.

Model Simulation of Triaxial Drained Behavior Assuming $p_{init}=p_0$

The available test data base includes a series of triaxial drained tests performed on Nevada sand specimens with initial relative density of 40 and 60%, and on Bonnie silt specimens. The tests were performed under a constant mean normal pressure condition.

Using the same model parameters (Table 4) as those

employed in the undrained simulation described earlier with the assumption that $p_{init}=p_0$, we simulated the drained behavior and compared the results with the test data. The results are shown for the sands in Fig. 11, and the silts in Fig. 12. It may be observed that the overall model behavior compares well with the experimental data. The maximum value of q in the ϵ_1 vs q relationships are, however, over predicted by the model for the sands. The dilation of Bonnie silt at $p_{init}=40$ kPa is under predicted by the model. The apparent discrepancy has to do primarily with the shape of the distorted ellipse and the nature of the deviatoric hardening function. The model predictions may be improved by choosing a distorted ellipse (i.e., by choosing different values for n_1 and n_2) that bends more inward, becoming closer to the failure

Fig. 10. Model Simulation of the behavior of Nevada sand with combined hardening for specimens with $D_r=40\%$

Fig. 11. Drained behavior of Nevada sand in triaxial compression (Theory: $\zeta^g=1$, $H=0$, $K_b=K_{b1}+K_{b2}$)

line. The hardening function may also be further improved. Further comparison studies, utilizing data on other sand types, seem desirable before making such modifications to the model.

Model Simulation of Static Liquefaction of Nevada Sand Assuming $p_{init}=p_0$ and $\beta < 0$

The Nevada sand tested in undrained triaxial compression was found to show static liquefaction by Yogachandran (1990) when the initial relative density was 15%. The behavior was simulated by the model and compared to the test data in Figs. 13(a) and 13(b). The model parameters used in the simulation are listed in Table 4. Note that a negative value for γ_0 and a very low value for M_f were employed.

The model simulates the observed behavior very well.

The variation of $y=K_p^*/Kn_p^2$ (Eq. (47)) is plotted against p in Fig. 13(c). As the analysis presented earlier indicated, y changes from positive to negative, with the peak on the p - q relation occurring at $y=-1$. The variations of K_{b1} , K_{b2} and $K_p=K_{b1}+K_{b2}$ with the deviatoric strain is presented in Fig. 13(d). It may be observed that as a result of the use of negative value for β , K_{b2} is negative, making $K_p=K_{b1}+K_{b2}$ to become negative quickly as the strain increases.

SUMMARY AND CONCLUSIONS

An elasto-plastic model employing a density and a deviatoric hardening rule is presented for simulating the stress-strain behavior of granular materials under monotonic loading conditions. A shape hardening based on

 Fig. 12. Drained behavior of Bonnie slit in triaxial compression (Theory: $\zeta^g=1$, $H=0$, $K_b=K_{b1}+K_{b2}$)

 Fig. 13. Simulation of static liquefaction behavior of Nevada sand in undrained compression at $D_r=15$ per and at $p_0=210$ kPa (Theory: $\zeta^g=1$, $H=0$, $K_b=K_{b1}+K_{b2}$)

the concept of bounding surface is employed to account for the hardening effect of the material for stress states which are within the bounding surface. A second surface, referred to as the loading surface, which is embedded within the bounding surface, is used to model the phenomenon of phase transformation. The bounding and the loading surfaces are represented by a composite surface consisting of an ellipse and a distorted ellipse. The model is based on the associated flow rule.

The model's behavior is first discussed qualitatively and it is shown that the model has the theoretical potential for capturing most of the salient features of the stress-strain behavior of granular materials. A set of undrained and drained triaxial data are then used to verify the potential of the model to simulate the experimentally observed behavior.

The model is shown to simulate satisfactorily the observed triaxial behavior of the soils considered. Also, it is shown that, while a combined hardening that includes density, deviatoric and shape hardening is necessary to capture the entire behavior including the fine details, the shape hardening may in some cases be excluded to capture approximately the overall behavior, thus eliminating the need for some of the required parameters.

Also, the model is shown to capture the static liquefaction behavior observed for very loose sands in a realistic manner.

As a result of employing an associated flow rule, the model produces a symmetric stiffness matrix, which should allow finite element analyses employing this model to be carried out in an efficient manner.

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