MODEL TESTS AND THEORETICAL ANALYSIS OF REINFORCED SOIL SLOPES WITH FACING PANELS

TAKESHI KODAKA(i), AKIRA ASAOKA(ii) and GYANESWOR POKHAREL(iii)

ABSTRACT

A series of model tests on plain, reinforced and panel faced slopes were carried out. Models were prepared for two types of slope face gradient. Behavior of these models was observed from initial loading to ultimate failure. Special attention was given to the development of axial strains in the reinforcement, footing load, footing settlement and the shape of the failure surface. The linear elastic finite element method (LEFEM) and the rigid plastic finite element method (RPFEM) were used in the numerical simulations for the model test results corresponding to the first tangent slope in the footing pressure-settlement curve and the ultimate failure load, respectively. The pattern of the axial strain distributions recorded in the model test is supported by numerically calculated tensile forces. Similarly, the type of failure modes observed in the model tests are also clearly predicted from numerical analysis results. Both observed and computed results confirm that the reinforcement itself is more effective and that the addition of facing reduces the lateral movement thereby avoiding local failure near the slope face.

Key words: deformation, finite element method, loading test, model test, reinforced soil, sandy soil, stability analysis (IGC: E12/E14)

INTRODUCTION

The tensile (or compressive in some cases) force acting axially on the reinforcing members, e.g. soil nailing bars and geotextile, is a typical "internal force" within the reinforced soil system. Such internal force should develop, under given external forces, only when the reinforcement material and the reinforced soil restrain the deformations from each other. The internal force developed during deformation or failure can not, therefore, be estimated without existence of the deformation constraining mechanism in the soil system. The "internal force" is then naturally distinguished from the "external force" that can be controlled from outside the soil system. A typical example for the latter is the tightening force acting on an anchor whose inner end is fixed in bed rock.

The reinforced soil system at the limiting equilibrium state was recently formulated by Asaoka et al. (1994) based on the rigid plastic finite element method (RPFEM). In their formulation, a linear constraint condition which requires the length between two soil nodes along the reinforcement to remain constant during failure, sometimes referred to as "no length change" condition, is imposed upon the velocity field in the soil mass at limit state. The rate of plastic energy dissipation of the soil is minimized under this constraint condition, and mathematically it has been shown that the Lagrange multiplier for this constraint condition is interpreted as an axial force developed along the reinforcement when the equilibrium of forces of the system is considered. Based on their method of analysis a factor of safety/failure load, the axial force in the reinforcement and the velocity field of the soil mass are simultaneously determined related to the reinforced soil system at limiting equilibrium state.

Following the same methodology, the bending moment that develops along the reinforcement due to internal action can also be taken into consideration. In this study, an additional "no-bending" condition is introduced assuming that the flexural rigidity of the reinforcing material is very high compared with the stiffness of the fill soil. This is another linear constraint condition imposed upon the velocity/deformation field of the soil. A slope facing/reinforced wall, shotcrete slope and sheet pile are examples of real structures where flexural rigidity plays a major role in resisting the deformation of a soil mass (e.g. Cazzuffi et al., 1994; Christopher et al., 1994; Miki et al., 1994; Tatsuoka, 1992). Such rigidity develops

---

(i) Research Associate, Department of Civil Engineering, Nagoya University, Chikusa-ku, Nagoya 464-01.
(ii) Professor, ditto.
(iii) Doctoral Student, ditto.

Manuscript was received for review on May 18, 1994.
Written discussions on this paper should be submitted before October 1, 1995 to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4 F, kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.
the bending moment along the reinforcing members due to earth pressure not axial but lateral to the reinforcement. Furthermore, these linear constraint conditions such as “no-length change” and “no-bending” conditions are shown to be equally applicable to the deformation analysis using Linear Elastic Finite Element Method in addition to the stability analysis. Based on these two analyses, the behavior of the reinforced soil structure from initial to failure loading can be predicted approximately.

The applicability of these analysis methods was evaluated by analyzing the results of a series of medium scale 1 g model tests on the reinforced soil slopes. These slopes are about 1 m high and made of sandy soil fill, in which the reinforcing steel bars and slope facing members are installed. Deformation of the slope and the failure mode are monitored during vertical load application at the top of the slope. Advantages and limitations of the aforementioned analysis methods were assessed using these observations of the real model test.

FORMULATION OF REINFORCED SOIL SYSTEM

The “no-length change” and “no-bending” conditions were first formulated in terms of a velocity field assuming that the soil mass exists at the limit equilibrium state. This is only for the sake of incorporating these conditions into the rigid plastic finite element method. Later however, a similar formulation was shown to be applicable to a deformation problem during loading based on the linear elastic finite element method.

No-length Change Condition

Asaoka et al. (1994) explained that reinforcements in the soil mass were designed to fasten each soil element that touches the reinforcement during the failure process. The plastic flow of soils at the limit equilibrium state maintains a constant length between the soil elements along the reinforcement, i.e. “no-length change condition”. Referring to Asaoka et al. (1994), the “no-length change condition” is derived using the following relationship, i.e. \( |\Delta l| = |l + \Delta l| \), in Fig. 1 which illustrates the plastic flow of A and B to A’ and B’, respectively, for a small time, increment \( dt \). Finally, the condition reduces to the following linear equation.

\[
(X_1 - X_0)\hat{u} = \begin{pmatrix} \hat{u}_2 - \hat{u}_1 \end{pmatrix} = 0
\]

This equation can be derived for the global reinforced soil system from the aforementioned single reinforcing element using the following matrix \( C_I \),

\[
C_I \hat{u} = 0
\]

where \( \hat{u} \) is the vector of all nodal velocities. The matrix \( C_I \) retrieves “constrained nodes” in \( \alpha \) and gives Eq. (1) only to those respective reinforcing nodes.

No-bending Condition

In this study, the rigid material under bending, e.g. facing panel, shotcrete, sheet pile, etc., are for the first time assumed to maintain the relative positions of soil elements along these rigid materials during plastic flow of soils at limit state. Thus, three points A, B and C forming a straight line flow keeping the straight line of those points during failure as shown in Fig. 2. The no-bending condition is formulated here together with the no-length change condition corresponding to the reinforced soil nodes. Referring to Fig. 2 and the no-length change condition, the components paralleling the reinforcement are as follows:

\[
\bf{\hat{u}}_1 \cos \beta_1 = \bf{\hat{u}}_2 \cos \beta_2 = \bf{\hat{u}}_3 \cos \beta_3
\]

Using this assumption, the no-bending condition is expressed in the following form:

\[
|l_1| : (|\hat{u}_2| \sin \beta_2 - |\hat{u}_1| \sin \beta_1) = |l_2| : (|\hat{u}_3| \sin \beta_3 - |\hat{u}_1| \sin \beta_3)
\]

which yields

\[
-l_2 |\hat{u}_1| \sin \beta_1 + (|l_1| + |l_2|) |\hat{u}_1| \sin \beta_2 = -l_1 |\hat{u}_1| \sin \beta_1 = 0.
\]

Using the first term of Eq. (5), it can be rewritten as follows:

\[
-l_2 |\hat{u}_1| \sin \beta_1 = |l_2| \cos \alpha |\hat{u}_1| \sin (\alpha + \beta_3)
\]

\[
-l_1 |\hat{u}_1| \cos \alpha |\hat{u}_1| \cos (\alpha + \beta_1) = l_2 \hat{u}_{1y} - l_2 \hat{u}_{1x}
\]

in which subscript \( x \) and \( y \) express the \( x \) and \( y \) component of vectors, respectively. A geometric relationship between the second term and the third term is illustrated in Fig. 3. Using Eq. (6), the “no bending condition” can be
Incorporation of Constraint Condition

Incorporating into the RPFEM

On the basis of the upper bound theorem on plasticity, the rigid plastic finite element method (RPFEM) is obtained through minimizing the rate of internal plastic energy dissipation with respect to the kinematically admissible velocity field under several linear constraint conditions (Tamura et al., 1984). In this study, these are summarized as follows: (1) soils are assumed to exhibit no rate of volume change at limit state like the plastic flow of Mises material. (2) Loading is made through a velocity/displacement boundary similar to a rigid footing. In addition to these constraints, (3) "no length change" and (4) "no-bending" conditions are imposed upon the velocity along the reinforcement. Mathematically, these are all expressed in the following linear constraint condition on the velocity at three reinforced points as shown in Fig. 2.

\[(l_{2y} - l_{2x}, -(l_{1y} + l_{1z}), l_{1y} - l_{1x})
\]

\[
(\hat{u}_{1x}, \hat{u}_{1y}, \hat{u}_{1z}, \hat{u}_{2x}, \hat{u}_{2y}, \hat{u}_{2z})^T = 0
\] (7)

This equation can also be expanded corresponding to the all nodal velocity vectors using the matrix \(C_b\) similarly as in the "no length change condition", see Eq. (2), i.e. the matrix, \(C_c\):

\[C_b \hat{u} = 0\] (8)

where \(C_b\) in the case shown in Fig. 4 can be expressed as follows:

\[
(C_b) = \begin{pmatrix}
0 & l_{y(M-N)} & -l_{x(M-N)} & -l_{y(L-M)} & l_{x(L-M)} & -l_{y(L-M)} & o \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\end{pmatrix}
\] (9)

Points \(L, M\) and \(N\) represent a unit reinforcing system in all reinforcement.

Linear constraint conditions.

The formulation is employed by introducing the Lagrange multipliers \(\lambda, \mu, \nu\) and \(\xi\), and finally, the following function is minimized.

\[
\mathcal{F}(\hat{u}, \lambda, \mu, \nu, \xi)
\]

\[= \int_V D(\hat{u})dV + \lambda^T(L\hat{u} - o) + \mu^T(C\hat{u} - o)
\]

\[+ \nu^T(C\hat{u} - o) + \xi^T(C_b\hat{u} - o)
\] (10)

in which \(D\) is the rate of internal plastic energy dissipation. \(L\) is the matrix defined such as \(\hat{v} = L\hat{u}\) where \(\hat{v}\) is the rate of volume changes in all elements. Therefore the first constraint condition, \(L\hat{u} = o\) indicates that no rates of volume change occur at all elements in the limit state. The vector \(o\) is a prescribed vector only at the displacement boundary and the matrix \(C\) retrieves the velocity vector at the displacement boundary when all nodal velocity vectors, \(\hat{u}\), are multiplied by \(C\). Therefore, the second
constraint condition, \( C\dot{u} = a \) defines the provisional norms of velocity vector beneath the rigid loading plate. The third constraint condition, \( C\dot{u} = o \) indicates the "no length change condition", see Eq. (2), while the fourth one, \( C\dot{u} = o \), indicates the "no-bending condition", see Eq. (8). As the rate of internal plastic energy dissipation, \( D(\dot{u}) \), is the convex function of \( u \), a local stationary condition of \( \Psi \) gives the global minimum of \( \Psi \). Then taking the derivative of \( \Psi \) with respect to \( u, \dot{u}, \mu, v, \zeta \), one has the following equilibrium equation of forces at limit state and accompanied constraint conditions.

\[
\begin{align*}
\int_V B^T s dV + L^T\lambda + C^T\mu + C^T v + C^T \xi = 0 \\
L\dot{u} = 0 \\
C\dot{u} = a \\
C\dot{u} = o \\
C\dot{u} = o
\end{align*}
\]  \( \text{(11)} \)

in which \( s \) denotes the deviator stress vector while Lagrange multipliers \( \lambda, \mu, v, \zeta \) are interpreted as the indeterminate isotropic stress and as a contact pressure at the prescribed displacement boundary, respectively (Tamura et al., 1984).

The Lagrange multiplier \( v \) is interpreted as the unit nodal force acting on constrained nodes along the reinforcement direction (Asaoka et al., 1994). \( \zeta \) is also interpreted as the unit nodal force acting on constrained nodes but along the direction perpendicular to the reinforcement. The forces calculated as \( \zeta \) appear at the reinforced system which remains straight, e.g. points A, B and C in Fig. 2, to resist the bending of the reinforced system. The forces calculated as \( v \) and \( \xi \) in the horizontal reinforcement as shown in Fig. 5 can be explained. The fourth term in Eq. (11), \( C^T v \), which can be rewritten by the \( x, y \) component of the points A, B and C as follows:

\[
C^T v = (l_{0x}v_1, 0, -l_{1x}v_1 + l_{2x}v_2, 0, -l_{2x}v_2, 0)^T
\]  \( \text{(16)} \)

These forces act to resist the extension of the distance between the constrained nodes along the reinforcement during failure. The fifth term in Eq. (11), \( C^T \xi \) can be rewritten similarly to Eq. (16) as follows:

\[
C^T \xi = (0, -l_{2x}\xi_1, 0, (l_{2x}+l_{1x})\xi_2, 0, -l_{1x}\xi_2)^T
\]  \( \text{(17)} \)

These forces, which are interpreted as shear forces, resist bending of the straight line due to earth pressure during failure. Furthermore the resistance to bending moment developed in the reinforced system is calculated as follows:

\[
l_{1x}l_{2x}\xi_2
\]  \( \text{(18)} \)

A unit reinforced system resisting bending is determined by the neighboring three nodes and each system can be overlapped with each other. The resistance to bending moment can be obtained by superimposing each moment.

**Stress-Strain Rate Relationships of Soil at Limit State**

Eqs. (11) to (15) define a statically indeterminate limit-ing equilibrium problem and these equations are solved with the aid of a constitutive relationship of soils at the limit state. Since the no rate of volume change is already assumed, the following Mises type plastic flow is employed (Asaoka and Kodaka, 1992):

\[
s_{ij} = \sigma_0 \cdot \hat{e}_{ij}^p, \quad \vec{e} = \sqrt{\hat{e}_{ij}^p \cdot \hat{e}_{ij}^p}
\]

where \( \hat{e}_{ij}^p \) denote a plastic strain rate. In order to simulate the model tests, the Mises constant \( \sigma_0 \) is assumed in the present study to follow "\( c \) and \( \phi \)" characteristics, that is,

\[
\sigma_0 = \sqrt{2} c \cos \phi + \sqrt{2} p \sin \phi
\]

in which \( p \) denotes confining mean pressure and \( c \) and \( \phi \) are the cohesion and frictional angle in a triaxial compression test, respectively. Note that the relationship in Eq. (20) is obtained under plane strain condition. Although this problem seems to follow the non-associated flow rule (Tamura et al., 1987), the solutions obtained by iterative calculations satisfy the associate flow rule at the time of convergence, (see the flow chart in Fig. 6). Eqs. (19) and (20) define the \( c-\phi \) soil as an assembly of the inhomogeneous Mises materials where each soil portion consists of different \( \sigma_0 \) with respect to corresponding confining pressure, \( p \), (Asaoka et al., 1994).

**Incorporating into the LEFEM**

The initial tangential load-deformation curve of the reinforced soil structure was estimated for this study (approximately) by the use of the linear elastic finite element method (LEFEM).
The potential energy is minimized by introducing constraint conditions on the deformation field, which is almost the same as minimizing the rate of internal plastic energy dissipation corresponding to the aforementioned RPFEM. The minimization of the following energy function

$$
\Phi(u, v, \xi) = \int_C \frac{1}{2} \sigma : e D V + v^T(C_u - o) + \xi^T(C_S u - o) - F^T u
$$

(21)
yields the equilibrium equations with constraint conditions as follows:

$$
\int_C B^T D B d V + C_f^T v + C_S^T \xi - F = 0 \quad (22)
$$

$$
C_S u = o \quad (23)
$$

$$
C_S^T u = o \quad (24)
$$
in which the matrix $D$ defines the constitutive relationship of the linear elastic material based on the Hooke's law, i.e. $\sigma = D e$. The Lagrange multipliers can be interpreted similarly as in the RPFEM.

OUTLINE OF THE MODEL TESTS

Testing Materials and Apparatus

The grain size distribution curve for the sand fill used is shown in Fig. 7. The fill material is classified as a silty sand (SM). Some physical and mechanical test results are presented in Table 1. The test model which consists of four rigid concrete walls of 200 mm thickness is shown in Fig. 8. In order to reduce the friction between the soil mass and the wall surface, the following treatments were made. (1) Teflon sheets (1 mm thick) were fastened to each wall and then grease was applied on the surface of those sheets. (2) The greased surface was covered by another soft silicon mixed paper sheet which can tear under very small stress. Since the side walls were tied to each other by fourteen steel bolts and the side wall friction was also maintained very small, all model tests can be considered as "Plane strain condition" tests. The loading system used was a rigid rough footing, which is made of steel plate (500 (length) \times 900 (width) mm) and its undersurface was covered by rough sandpaper. This loading system is manually operated with a hydraulic jack.

Construction of the Model Slopes

Two types of model slopes were constructed. The first on was 1:0.5 (vertical: horizontal) face slope (mild slope) and the other was 1:0.2 face slope (steep slope). The dimensions of the mild slope is shown in Fig. 8. Although the slope height in the case of steep slope models was changed to 941 mm, the length of the top horizontal surface (platform) in all models and the loading positions were unaltered, i.e. kept the same as in the previous case (mild slope).

The construction sequence adopted for each model slope was similar to the following: (1) Soil was mixed by a large mixer in order to homogenize the water content of 10%. (2) The model consists 3 layers in the base and 9 layers in the slope part in the case of mild slope (8 layers in the case of steep slope). Each layer was compacted with a wooden tamper weighing 10 kgf until the same fixed height and the average unit wet weight of 18.13 kN/m$^3$ was obtained. At the time of compaction, the face of the slope was rigidly supported by a wedge shaped form which was made of EPS (expanded polystyrene) and detachable into a separate layer.

Test Conditions

Table 1 shows the overall model test schemes. The reinforcement was 3 mm diameter and 500 mm long steel bar whose surface was coated with the fill sand by gluing. Five or six reinforcements were placed laterally at mid-

![Fig. 7. Grain size distribution of sand fill](image_url)

![Fig. 8. Model test configurations and reinforcement details](image_url)

<table>
<thead>
<tr>
<th>Table 1. Physical and mechanical properties of fill material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material parameters</td>
</tr>
<tr>
<td>Specific gravity, $G_s$</td>
</tr>
<tr>
<td>Maximum dry density, $\gamma_{max}$ kN/m$^3$</td>
</tr>
<tr>
<td>Opt. moisture content, $w_{opt}$ (%)</td>
</tr>
<tr>
<td>Cohesion, c, kN/m$^2$ (CD Triaxial Test)</td>
</tr>
<tr>
<td>Angle of internal friction, $\phi$, (deg.)</td>
</tr>
<tr>
<td>Particles passed the 2 mm sieve (grain size ana.)</td>
</tr>
<tr>
<td>Particles passed the 0.074 mm sieve</td>
</tr>
<tr>
<td>Uniformity coefficient</td>
</tr>
<tr>
<td>Coefficient of curvature</td>
</tr>
</tbody>
</table>
Table 2. Model test scheme

<table>
<thead>
<tr>
<th>Model</th>
<th>Face slope</th>
<th>Reinforcements</th>
<th>Facing panels</th>
<th>Length of reinf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Mild slope 1V:0.5H</td>
<td>Non-reinforced</td>
<td>No-facing</td>
<td>—</td>
</tr>
<tr>
<td>B</td>
<td>Mild slope 1V:0.5H</td>
<td>Reinforced</td>
<td>5 mm thick panel</td>
<td>500 mm</td>
</tr>
<tr>
<td>C</td>
<td>Steep slope 1V:0.2H</td>
<td>Non-reinforced</td>
<td>No-facing</td>
<td>—</td>
</tr>
<tr>
<td>D</td>
<td>Steep slope 1V:0.2H</td>
<td>Reinforced</td>
<td>5 mm thick panel</td>
<td>500 mm</td>
</tr>
</tbody>
</table>

Figure 8. The panel facing was an acrylic resin board (5 mm thick) which had sufficient rigidity against the lateral earth pressure developed during loading. Figure 9 shows the arrangement of the facing panel for the mild slope. Each reinforcement bar was fixed at the center position of each facing panel. Therefore, the angle between the reinforcement bar and facing panel does not change during the entire loading period. The facing panels overlap with each other in the lateral direction as shown in Fig. 9, but, just touch each other in the vertical direction.

Figure 10 shows the loading and measurement system. The parameters measured during loading were as follows: (1) Load (loading stress), (2) vertical settlement of the loading plate, (3) axial strain in the reinforcement and (4) horizontal movement of the facing panels or slope face. The load was measured with a load cell installed at the center of the loading plate, and the loading pressure was an average of the load per unit area. Slip surface and deformation of the model slopes at failure were observed by removing the side wall after the complete loading operation.

TEST RESULTS

Observation of Model Tests

Figure 11 shows the failure slip surface corresponding to a mild slope which was observed by removing the side wall at the end of each test. Details of the observations for each test during initial loading to ultimate failure are as follows:

Type A (plain slope): As soon as the loading began, some comparatively dry sand rolled down on the slope surface. As the loading level was gradually increased, the loading plate was inclined towards the slope face and a horizontal crack passing through the face was observed on the upper part of the slope surface. When the footing load reached the upper limit, another large crack appeared on the lower part of the slope surface. Figure 11(a) indicates that the failure surface is shallow and is confined near the slope face covering the full height of the slope.

Type B (reinforcement without facing): When the load reached 170 kPa, a horizontal crack appeared on the second layer from the top of the slope and subsequently another crack appeared around the center of the slope height. At that moment, the loading plate inclined towards the front of the slope, as in case Type A. The upper part of the slope surface from the second layer of the slope collapsed at 270 kPa loading level. A new horizontal crack appeared on the toe of the slope at 290 kPa loading level. Figure 11(b) shows that there are three main slip surfaces and many local failure surfaces are confined within the shallowest failure surface. During the first half of the loading level, only the shallow failure zone seemed to exist, which is similar to Type A. Due to the effect of the reinforcement however, the shallow failure didn’t cause the overall failure of the slope. The final failure was due to the deep failure initiated from the inner end.
of the footing.
Type C (reinforcement with facing): Since the rigidity of the facing material was considerably high and the overlap joint between the facing panels was so strong that nothing could be observed outside the slope. Figure 11(c) shows that there was no local failure near the facing, which is quite different compared to the Type B test. The failure surface passes through the inner end of each reinforcing bar, i.e. also parallel to the slope face. Such a failure mode may be classified as “block failure”.

Figure 12 shows the failure slip surface for the case of the steep slope. The tendency of each test, i.e. plain slope, reinforcement without facing and reinforcement with facing, is similar to the mild slope case. In the case of Type D, see Fig. 12(a), the shallow failure which occurred in the lower face of the slope was so large that the loading procedure could not be continued at the early loading level. Figure 12(b) shows that the failure surface initiated from the inner end of the footing does not pass through the bottom or below the slope. Figure 12(c) shows that Type F exhibits a block type failure similar to Type C.

Based on these observations of the failure surfaces, it is clear that the reinforcement bar is effective in preventing the collapse by a local failure from occurring close to the surface and furthermore the facing panel is effective for prevention local failure.

Failure Load and Vertical Displacement
Figures 13 and 14 show the footing load versus the footing settlement recorded for the tests corresponding to mild slopes and steep slopes, respectively. In the case of mild slopes, see Fig. 13, the effect of reinforcement and panel facing can be observed from the start of loading (i.e. first tangent) to the failure stage. The first tangent line is comparatively steep in the case of a reinforced slope with facing panels (Type C) while reinforced without facing panels (Type B) lies in between the panel faced slope and the plain slope (Type A). This result shows that the reinforcement and the panel facing are quite effective to restrain the displacement. Failure loads
for both Type B and Type C cases are very high compared with Type A, which means that the reinforcement and the panel facing are also effective for the stability of slopes. Furthermore, Types B and C show that there is considerable footing settlement before the final failure state is reached.

In the case of steep slopes, see Fig. 14, the first tangent lines are the same as explained for the previous case, i.e. mild slope case. Although, the facing case (Type F) shows a higher failure load compared with other cases as the same footing settlement, the final failure load of type F does not reach the failure load of the reinforced slope without facing (Type E). Type E shows that there is considerable footing settlement before the final state is reached as in the case of the similar model for the mild slope case, i.e. Type B. However, overall the reinforcements, whether facing panel exists or not, are very effective to stabilize slopes.

**Axial Strains in the Reinforcing Bars and Lateral Displacement of Slope Surface**

Figures 15 and 16 show lateral displacements of the slope surface and axial strains in the reinforcing bars recorded during loading. From these figures, at the same loading level, the reinforcement and the facing panel is considerably effective not only in controlling the vertical settlement but also in reducing the lateral displacement. In the case of reinforcement with facing especially the effect is much higher. A significant outward movement at the middle of the slope height due to the local failure can
compared to the axial strains in the case of Types B and E respectively, and furthermore significant axial strains developed at the face side of the reinforcing bars in these former cases can be clearly observed. In all the cases, a peak in the axial strain distribution curve can be seen moving towards the inner direction as the reinforcement position becomes lower and lower. Furthermore, twin peaks in the axial strain distribution curve may occur at the lower positions of reinforcement.

**NUMERICAL SIMULATION AND DISCUSSIONS**

**Conditions of the Simulation**

The numerical simulation work is performed using a linear elastic finite element method (LEFEM) and the rigid plastic finite element method (RPFEM). In both cases constraint conditions for displacement and velocity are introduced, respectively as mentioned in earlier chapters. In this study, the LEFEM is used to analyze the first tangent line of the vertical settlement versus footing load curve, while the RPFEM is used to compute the ultimate failure footing load. The finite element array used in these analyses is shown in Fig. 17. A rigid rough loading condition is assumed, which restrains the displacement or velocity under the footing to move only in a vertical direction. Another assumption to be mentioned here is the friction between floor and soil mass. At first the friction seemed considerably high, so that the boundary conditions corresponding to floor nodes were assumed to be

---

**Fig. 16.** Observed axial strain distribution along the reinforcing bars for the steep slope case

be observed in the case of a reinforced slope without facing, i.e. Types B and E, while a progressive rotation which pushed out from the footing to the slope toe is significant in the case of reinforcement with facing, i.e. Types C and F.

Axial strains in the case of Types C and F are larger

---

**Fig. 17.** Finite element array used in the numerical simulations
fixed in the mild slope cases. Later, however, particularly for the steep slope tests, such assumption of frictional boundary indicated a conservative tendency of the results, and therefore, free boundary condition (rollers) was assumed.

In order to simulate the reinforced slope, the nodes connected by the thick lines in Fig. 17 are confined by "no-length change conditions", while the nodes positioned along the facing are confined by the "no-length change condition" along with "no-bending condition" for the facing direction only in the case of reinforced slopes with facing, i.e. Types C and F. In this analysis, since the bar spacing was close enough, the reinforcement shown as a thick line in Fig. 17 was assumed to be a continuous sheet for simplicity. This assumption seems more appropriate for the case of reinforced slope with facing, because the slope with facing cannot deform except in the lateral direction due to the panels restraining each other.

The soil properties required in the LEFEM as well as the RPFEM analysis were obtained by back analysis of the first model test on an unreinforced plain slope i.e.,

---

Fig. 18. Comparisons of computed first tangent line on the footing pressure – settlement curve and failure load with model test observations
Table 3. Material parameters obtained by performing back analysis for the unreinforced case

<table>
<thead>
<tr>
<th>Linear elastic analysis</th>
<th>Rigid plastic analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu = 0.3$</td>
<td>$\phi = 25^\circ$</td>
</tr>
<tr>
<td>$E = 7000; \text{kPa}$</td>
<td>$c = 20; \text{kPa}$</td>
</tr>
</tbody>
</table>

Type A, the results of which are given in Table 3, because of the difficulty in using the properties of unsaturated soil obtained directly from the laboratory tests. In the case of the LEFEM, when Poisson’s ratio is assumed equal to 0.3, the Young’s modulus in the table resulted in good matching with the first tangent line for the Type A model test. Similarly, in the case of the RPFEF, when the angle of internal friction was assumed to be equal to $25^\circ$, the failure load of the type A model matched well to the computed failure load using the cohesion shown in the table. The soil properties determined in the Type A model test were used consistently to analyze all the remaining cases including the steep slope case.

**Results and Discussions**

The calculated vertical displacement versus the footing pressure curves based on the proposed method are shown in Figs. 18(a) to (c) corresponding to the mild slope. Both the first tangent line and failure load based on LEFEM and RPFEF analyses respectively, can explain the model test results. This figure also shows that the effects of both the reinforcement and the facing panel can be approximately modeled without changing the soil parameters but incorporating the proposed constraint conditions of no-length change and no-bending into the finite element method. The results of the steep slope case are shown in Figs. 18(d) to (f). Although the soil parameters remain the same as in the mild slope case, the effect of restraining the soil deformation, i.e. the first tangent line, is nicely explained regarding the test results for all the remaining cases. The effect of the reinforcement on the stability is also explained corresponding to Type E, i.e. reinforced slope without facing. The failure load recorded in the case of Type F (with facing) was smaller than that of Type E (without facing). This result cannot be explained by the proposed method where the plastic energy is to be minimized subject to the more constraining conditions in a panel facing case, i.e., “no-length change” plus “no bending” conditions.

Figure 19 shows the displacement fields obtained by LEFEM in the case of the mild slope, where magnitudes are relatively exaggerated. The difference between Types A and B is not so clear whereas the difference between Types B and C is comparatively clear. Due to the constraint conditions for the facing panel, the displacement along the reinforcement from face of the slope to the end of reinforcement depends on the displacement of the slope facing. The direction of displacements around the reinforced region is aligned parallel to the slope face direction corresponding to the Type C case. This may be the reason why the failure surfaces observed in Types C and F are smaller than that for Types B and E respectively, see Figs. 13 and 14. Such behavior could not be explained based on the RPFEF result because it assumes the soil mass to be an incompressible material at the failure stage. Figure 20 shows the velocity fields obtained based on RPFEF in the case of mild slope. Figure 20(a) illustrates that the plain slope case can explain the failure mode of the model test where only the upper part of the slope failed. From Fig. 20(b), the plastic region expands to the toe of the slope which is similar to the failure surface of the model test. Furthermore, due to the incorporated constrain conditions on the slope face, the plastic region expands more deeply as shown in Fig. 20(c).

Figure 21 shows the computed mean stress distributions at failure based on RPFEF. In Type B, a considerable high stress is concentrated towards the face side of the reinforcement. This tendency is more clearly seen in Type C where high stress concentration appears just behind the panel facing as shown in Fig. 21(c). In the plain slope case, a comparatively high stress concentration is observed around the inner edge of the footing, whereas in the reinforced slope case, a high stress concentration is observed around the outer edge of the footing. This may
be the reason why the footing plate was inclined toward the face side in the plain slope model test.

The computed tensile force distribution at failure based on RPFEM and the axial strain distribution recorded in the model test are shown in Fig. 22. It should be noted that the axial force is calculated per unit width, 1 m. Although the calculated distributions are quite approximate due to the coarse finite element array, the pattern of the axial strain recorded in the model test is explained by this calculated tensile force, e.g. a position of peak, moving the peak towards the inner side with a lower bar position and appearance of twin peaks in the low positioned bars, etc.

CONCLUSIONS

A series of model tests on plain, reinforced and panel faced slopes were carried out. Models were prepared for two types of slope face. Behavior of those models were observed from initial loading to ultimate failure. In this study, the mechanism proposed by Asaoka et al. (1994) was employed and an additional mechanism was proposed to compute the resistance to bending of reinforcing bars and facing panels developed due to the earth pressure. Numerical simulations were carried out based on the linear elastic finite element method and rigid plastic finite element method. The following conclusions were drawn based on this study.

1) Model test results clearly indicate that there is substantial change in the response of soil to applied stresses when the soil mass is reinforced. The mode of lateral and vertical displacement of the soil mass from initial loading to failure shows excellent improvement in response of the soil mass with respect to applied external stresses. The model test results are well supported by the numerical analysis procedures adopted in the numerical simulations.

2) The mechanism of the reinforcement and facing is modeled by introducing the two linear constraint conditions “no-length change condition” and “no bending condition” in energy functions.

3) The first tangent line for footing pressure-settle-
REINFORCED SOIL SLOPES

Fig. 22. Comparison of observed axial strains with computed axial force distribution obtained with the RPFEM

4. Horizontal strain distributions in the reinforcements show that the efficiency of reinforcing bar increases when facing panels are provided. This observed pattern is also supported by numerical analysis.

ACKNOWLEDGMENT

The authors wish to express their thanks to Messrs. Ochiai, Kato and Kodama of Yahagi Construction Co., Ltd. for their cooperation in performing the model test.

REFERENCES


