CORRELATIONS BETWEEN SHEAR WAVE VELOCITY AND CONE TIP RESISTANCE IN NATURAL CLAYS

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ABSTRACT

Empirical correlations for estimating the shear wave velocity (V_s) from cone penetration tip resistance (q_c) are derived from measured field data taken in 31 different natural clays. Despite the fact that V_s and q_c are operational at the opposite extremes of a highly nonlinear stress-strain-strength relationship, the two parameters can be interrelated because they are both influenced by effective confining stress level, anisotropic K_0 -stress state, mineralogy, aging, bonding, and other factors. Regression analyses indicate that the estimate is significantly improved if measurements of void ratio (e_0) with depth are also known, resulting in an expression that relates V_s to both q_c and e_0 . Results are applied to an independent recent case study involving a sensitive marine clay where the relevant parameters are well documented.

Key words: clay, cohesive soil, cone penetration, field test, in-situ test, nondestructive test, penetration test, shear wave, wave propagation (IGC: C2/ C3/ D6/ D7)

INTRODUCTION

The shear wave velocity (V_s) is a fundamental property of soil, rock, concrete, steel, and other civil engineering materials. In geotechnical problems, the evaluation of V_s is especially important in defining the maximum value of soil stiffness at small strains, particularly since the stress-strain-strength response of soils is extremely nonlinear (Jardine et al., 1986; Burland, 1989). This stiffness is commonly expressed in terms of the low-amplitude shear modulus, $G_{\max} = \rho V_s^2$, where ρ = mass density of the material. At very small shear strains ($\gamma_s < 10^{-3}$ percent), $G_{\max}$ is an important parameter relevant to both static and cyclic loading behavior of soils (Tatsuoka and Shibuya, 1992; Lo Presti et al., 1993).

In ground vibration problems, V_s is required for determining the actual level of shear strain (γ_s) during transient and steady-state events in the field. The magnitude of shear strains for plane waves is given by the expression $\gamma_s = \dot{u} / V_s$, where $\dot{u}$ = peak particle velocity (Stokoe et al., 1978). Applicability of field strain measurements includes situations involving damage control of construction vibrations and nondestructive pavement evaluation by either heavy mass vibrator or falling weight deflectometer. In foundation vibration problems, a value of V_s is needed for the determination of the dimensionless frequency $a_0 = \omega r_0 / V_s$, where ω = circular frequency and

r_0 = foundation radius, as well as for the evaluation of soil foundation stiffness (Dobry et al., 1986).

Measurements of V_s for soil materials can be obtained on laboratory-trimmed specimens using either resonant column tests or bender elements, but are best accomplished in the field utilizing geophysical techniques. The primary in-situ test methods include the crosshole test (CHT), downhole test (DHT), and spectral analysis of surface waves (SASW). These field techniques are advantageous in that the soil is tested in its natural environment, under anisotropic stress states, and with minimal disturbance effects, as compared with laboratory methods.

In many practical instances, it is not economically feasible to conduct CHT or DHT surveys as part of routine geotechnical exploration programs. Moreover, although the SASW procedure needs no borehole and can be performed quickly and economically, data reduction is complex because of the dispersive nature of surface waves. Also, although the seismic cone penetration test (SCPT) offers an expedient means of profiling of V_s in a downhole configuration (Campanella et al., 1985), specialized equipment and technical expertise are necessary to ensure that the data are properly obtained and evaluated. Consequently, if possible, the development of empirical relationships between V_s and penetration tests has merit.

It is important to note that V_s and q_c reflect soil behav-

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Written discussions on this paper should be submitted before January 1, 1996 to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4 F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

ioral responses at vastly different and opposite ends of the nonlinear stress-strain-strength relationship. While V_s reflects a true elastic response corresponding to small strains on the order of $\gamma_s < 10^{-5}$ (decimal), the penetration resistance (q_c) relates to the undrained shear strength (s_u) or peak shear stress at strains associated with failure! Despite this paradox, both parameters share a functional dependence on similar quantities, such as effective confining stress level, K_0 stress state, mineralogy, and aging. It was therefore the intent of this study to evaluate possible statistical trends between V_s and q_c in natural clay deposits.

Early research using resonant column testing (Hardin, 1978) suggested that V_s depended on such variables as mean effective stress (σ'_0), void ratio (e_0), and overconsolidation ratio (OCR). More recently (Jamiolkowski et al., 1994), however, the dependency on OCR has been shown to be illusory and artificially caused by the independent influences of the vertical and horizontal stresses (σ'_{v0} , and σ'_{h0}). Since the cone tip resistance is also apparently controlled by the effective geostatic stress state given by the magnitudes of σ'_{v0} , and σ'_{h0} (Crooks et al., 1988), it is possible to interrelate V_s and q_c empirically.

A comprehensive review of the deformational characteristics of soils and rocks from field and laboratory testing (Tatsuoka and Shibuya, 1992) has found somewhat global relationships existing between the maximum Young's modulus (E_{max}) defined at strains $\gamma_s < 10^{-5}$ (decimal) and maximum deviator stress, $q_{max} = (\sigma_1 - \sigma_3)_{max}$. Since V_s in fact represents a measure of E_{max} and q_c reflects a measure of the undrained strength of the clay ($s_u \approx \frac{1}{2} q_{max}$), the proposed $V_s - q_c$ interrelationships herein are consistent with previously published $E_{max} - q_{max}$ correlations.

COMPILED CPT DATABASE

Data from 31 different clay sites which were subjected to both cone penetration testing (CPT) and shear wave velocity measurements (CHT, DHT, SASW) have been compiled and reported by Mayne and Rix (1993). The clays range from intact to fissured materials and have wide ranges in plasticity characteristics ($8 < PI < 300$), sensitivities ($2 < S_t < 200+$), and overconsolidation stress states ($1 < OCR < 100+$). In the initial study, relationships were explored between the small strain shear modulus (G_{max}) in terms of cone tip resistance (q_c) and/or void ratio (e_0). Herein, direct correlations for V_s in terms of q_c and e_0 have been sought. One justification for this additional study is that V_s is a fundamental measurement, whereas G_{max} is a calculated value obtained from V_s and either an assumed or measured value of mass density (ρ). Another purpose for obtaining a direct $V_s - q_c$ trend is for cases when an estimate of V_s is needed for evaluating shear strain levels during ground vibration monitoring or nondestructive pavement analyses.

Empirical correlations between V_s and penetration test results have been investigated previously. In particular, Ohta and Goto (1978) and Imai and Tonouchi (1982)

related V_s to the N -value from standard penetration test, with separate trends noted for clays, sands, and gravels. Alternative $V_s - N$ relationships have been evaluated, primarily for sands (Jamiolkowski and Robertson, 1988). Prior attempts to correlate V_s directly with q_c include site-specific studies for (1) very highly plastic Mexico City clay with $200 \leq PI \leq 450$ (Jaime and Romo, 1988); and (2) two Italian sands (Baldi et al., 1989). The intent herein is to investigate general $V_s - q_c$ trends applicable to a variety of natural clays.

The observed trend between the measured V_s and q_c for both intact and fissured clay deposits is presented in Fig. 1. Despite the obvious scatter, it is evident that shear wave velocities and cone resistances increase with consistency from soft to stiff to hard clay materials. Log regression analyses were conducted on the data and the more significant relationships are reported in Table 1. It is important to note that 22 of the sites were tested using piezocones so that the cone tip resistances are corrected ($q_c \rightarrow q_T$) for pore water pressure effects acting on unequal areas behind the cone (Campanella et al., 1985). Furthermore, 5 additional sites included very stiff fissured materials in which these corrections are likely negligible (Mayne et al., 1990). As a consequence, 87 percent of the sites reflect corrected tip resistances (q_T).

Considering all of the available data for intact and fissured clays, the general regression trend is given by ($n=481$, $r^2=0.736$):

$$V_s = 1.75(q_c)^{0.627} \quad (1)$$

where V_s is in units of meters/sec and q_c in kPa.

From considerations of cone penetration as a bearing capacity problem, it is more appropriate to utilize a net cone tip resistance, such as $(q_c - \sigma_{v0})$, or more properly, $(q_T - \sigma_{v0})$, where σ_{v0} is the total overburden stress. This "more correct" term was also examined in light of the

Fig. 1. Direct correlation between shear wave velocity and cone tip resistance in intact and fissured clays. Database from Mayne and Rix (1993)

Table 1. Results of statistical regression analyses from shear wave velocity data

Clay types	Regression equations	S.E.	r^2	n
Intact	$V_s = 1.85(q_c)^{0.618}$	0.154	0.495	387
	$V_s = 8.60(q_c - \sigma_{v0})^{0.410}$	0.157	0.471	387
	$V_s = 9.44(q_c)^{0.435}(e_0)^{-0.532}$	0.088	0.832	339
	$V_s = 40.3(q_c - \sigma_{v0})^{0.219}(e_0)^{-0.508}$	0.116	0.708	339
Intact & fissured	$V_s = 1.75(q_c)^{0.627}$	0.146	0.736	481
	$V_s = 6.53(q_c - \sigma_{v0})^{0.461}$	0.152	0.714	481
	$V_s = 14.02(q_c)^{0.364}(e_0)^{-0.490}$	0.093	0.846	364
	$V_s = 41.16(q_c - \sigma_{v0})^{0.212}(e_0)^{-0.490}$	0.114	0.768	364

NOTES:

1. V_s =shear wave velocity in meter/sec.
2. q_c =cone tip resistance in kPa.
3. σ_{v0} =total vertical overburden stress in kPa.
4. e_0 =in-situ void ratio.
5. S.E.=standard error of the dependent variable from log regression.
6. r^2 =coefficient of determination.
7. n =number of data sets in regression analysis.

aforementioned trends with the results also summarized in Table 1. However, no significant statistical improvement occurs, probably because of the added uncertainties associated with the calculation of σ_{v0} with depth at each of the particular sites.

Since the importance of void ratio and degree of fissuring were notable in the prior study, an evaluation of V_s in terms of q_c and e_0 was conducted for data on intact clays only. The various clays were sorted according to plasticity characteristics as well and multiple regression analyses provided the summary relationship presented in Fig. 2. A

Fig. 2. Measured versus predicted shear wave velocities for intact clays

secondary trend of increasing V_s with decreasing plasticity index (PI) is also evident in this figure. For intact clays, the multiple regression correlation is ($n=339$, $r^2=0.832$):

$$V_s = 9.44(q_c)^{0.435}(e_0)^{-0.532} \quad (2)$$

with V_s given in meters/sec and q_c in kPa.

Generally, the profile of e_0 will likely not be known beforehand while conducting in-situ testing with an electric cone or piezocone at a particular clay site. However, some innovative tools such as the neutron-moisture cone or RI-penetrometer (Shibata et al., 1992, 1994) permit a continuous mapping of e_0 in saturated clay deposits. In this instance, the mass density is not estimated but actually known, therefore permitting an accurate conversion from V_s to G_{max} .

CASE STUDY APPLICATION

A recently published and well-documented case study affords an opportunity to evaluate the predictive relationship between V_s , q_c , and e_0 . Profiles of in-situ V_s resulting from SASW surveys at the St. Alban site, Quebec are reported by Lefebvre et al. (1994). The St. Alban clay is an intact sensitive marine clay of low plasticity ($5 \leq PI \leq 30$) with a relatively constant OCR ≈ 2.2 , and variable profile of void ratio ($1.1 \leq e_0 \leq 2.9$). The results of piezocone tests at the site have been previously reported (Roy et al., 1982) and details on the Laval penetrometer (LaRochelle et al., 1988) permit a correction of the measured resistance (q_c) to the proper corrected tip resistance (q_T).

Comparisons of measured and estimated profiles of V_s using both (1) and (2) are presented in Fig. 3. The importance of including both parameters q_T and e_0 in the prediction is evident. Considering the fact that none of the sensitive Champlain Sea clays were available for inclusion in the original database used for the correlation, the agree-

Fig. 3. Comparison of measured and predicted V_s for sensitive Saint Alban clay

ment for Eq. (2) appears quite reasonable and better than other empirical procedures that rely on separate evaluations or estimates of additional soil parameters, including overconsolidation ratio (OCR), mean effective stress (σ'_{oct}) or lateral stress coefficient (K_0), and rebound exponent (k), as discussed by Lefebvre et al. (1994).

CONCLUSIONS

Relationships between the in-situ shear wave velocity (V_s) and cone penetration tip resistance (q_c) have been investigated. Although V_s corresponds to the small-strain region ($\gamma_s < 10^{-3}\%$) of the stress-strain-strength curve and q_c relates to failure strain levels, statistical regression analyses on data from 31 natural clays indicate that trends exist between V_s and q_c , particularly if the variation of void ratio (e_0) with depth is also known. A validation check on the approach was afforded by independent data reported from Quebec.

ACKNOWLEDGMENTS

Gratitude is extended to Dr. Mehmet T. Tumay the former, and Dr. Priscilla Nelson, the current director of the NSF Geomechanics Program and Dr. Clifford J. Astill of the NSF Siting and Geotechnical Systems Program for financial support of the first author on piezocone testing of clays and the second author on field wave propagation measurements, respectively.

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