PROPORTIONAL LOADING BEHAVIOUR OF SAND UNDER MULTIAXIAL STRESSES

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ABSTRACT

A study of sand deformation under proportional loading under generalized stress conditions is presented. The loading includes a variety of stress paths in which the principal axes of stresses have a constant inclination to the axes of the vertically water deposited specimens. This was intended to capture the effects of inherent anisotropy in sand during proportional loading. It is shown that under such loading conditions increments of volumetric and maximum shear strains depend both on the initial stress and density state as well as on the inclination of principal stresses to the material axes.

Key words: anisotropy, generalized stresses, proportional loading, sand, strain increment direction (IGC: D6)

INTRODUCTION

Most sedimentary soils are inherently anisotropic. Their deformation response will therefore depend on the directions of principal stress increments in relation to the deposition direction. Virtually all constitutive models of soil behaviour assume material isotropy (e.g., Saada and Bianchini, 1989). Occasional attempts to model anisotropic behaviour make use of material parameters derived from triaxial tests in which principal stress axes are coincident with axes of material anisotropy and no principal stress rotation is involved (Atukorala, 1990).

Deformations in soil induce further anisotropy that gets superimposed on the inherent anisotropy (Arthur and Menzies, 1972). This induced anisotropy is much more pronounced if on loading the principal axes of stress rotate than if they do not (Rowe, 1971). Proportional loading is the term used for the latter type of loading paths. These paths are of fundamental importance in the study of deformation response of metals and other solids (Mehan, 1961; Rees, 1981). Sand is a frictional material for which the strain response and failure are strongly dependent on the level of effective stress ratio. Proportional loading paths, in which the stress components increase proportionally to each other and hence no overall change in stress obliquity occurs, are therefore of even more fundamental importance in the study of deformation response of sands. Comprehensive studies of proportional loading behaviour of sand in two dimensions, i.e. using the triaxial test, have been presented by

other researchers (Barden et al., 1969; El Sohby, 1969; Rowe, 1971; Negussey and Vaid, 1986).

This paper presents a study of the proportional loading behaviour of sand in three dimensions, by making use of the University of British Columbia, Hollow Cylinder Torsional shear apparatus (UBC-HCT). The objectives of the study are to demonstrate experimentally under such loading paths, the nature of inherent and deformation induced anisotropy, and the manner they are affected by the initial stress and density state of the sand. Studies of this nature provide data on the fundamental mechanical response of real soils against which constitutive models of behaviour must be calibrated.

EXPERIMENTATION

A detailed description of the UBC-HCT has been given by Vaid et al. (1990). The UBC-HCT device is capable of subjecting a hollow cylindrical specimen of soil (15 cm O.D., 10 cm I.D., and 30 cm high) to an axial load, torque about the vertical axis, and different internal and external pressures. These induce stresses σ_z (vertical), σ_r (radial), σ_θ (tangential) and $\tau_{z\theta}$ (circumferential shear) in the wall of the specimen. The corresponding strain components are ϵ_z , ϵ_r , ϵ_θ and $\gamma_{z\theta}$. Independent computer control of these externally applied stresses enables the soil specimen to be loaded along a prescribed stress path in the four dimensional stress space: $R = \sigma'_1 / \sigma'_3$, $\sigma'_m = 1/3(\sigma'_1 + \sigma'_2 + \sigma'_3)$, $b = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)$ and $\alpha_\sigma =$ angle of σ_1 direction with the vertical deposition direction. Using

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finite element analysis, Sayao and Vaid (1989) and Wijewickreme and Vaid (1991) have shown that the behaviour of sand over a wide domain of the stress space, R , σ'_m , b and α_σ can be explored by the UBC-HCT device without introducing significant stress non-uniformities within the wall of the specimen.

Medium Ottawa sand ASTM-C-109 was used as the testing material for the investigations presented herein. Ottawa sand is a rounded quartz sand with the following physical properties: $G_s=2.67$; $D_{50}=0.4$ mm; $C_u=1.9$; $e_{\max}=0.82$; $e_{\min}=0.50$. Saturated loose HCT samples (relative density, $D_r=30\%$) were prepared by pluviation of sand through water. This specimen reconstituting process is considered to duplicate fabric of water deposited fluvial and hydraulic fills sands (Oda, 1972) and therefore, provides convenient means of studying properties of such sands in the laboratory. Higher relative densities were obtained by densifying the loosely deposited sand by imparting low energy high frequency vibrations to the outer forming jacket. The detailed procedure adopted for the preparation of HCT test specimens and the assembly of the UBC-HCT test device have been described by Vaid et al. (1990). The initial stress state of all specimens following reconstitution and apparatus assembly was hydrostatic at $\sigma'_m=50$ kPa. The measured volumetric strains were corrected for the effects of membrane penetration in accordance with the method proposed by Vaid and Negussey (1984).

INHERENT ANISOTROPY

The degree of inherent or initial anisotropy in sand is reflected by the relative values of different strain components under hydrostatic loading. Identical principal strains $\varepsilon_1=\varepsilon_2=\varepsilon_3$ (or $\varepsilon_{\text{vol}}=3\varepsilon_1$) would be observed if sand

Fig. 1. Strain response under hydrostatic loading

was isotropic. Results from hydrostatic compression of loose ($D_r=34\%$) sand, starting from an initial hydrostatic stress state of $\sigma'_m=50$ kPa and proceeding up to $\sigma'_m=300$ kPa are shown in Fig. 1. A linear strain path $\varepsilon_{\text{vol}}=4.5\varepsilon_1$ may be noted. This implies higher deformability in the horizontal than in the vertical deposition direction (i.e. $\varepsilon_r=\varepsilon_\theta=1.75\varepsilon_z$), which is typical of cross-anisotropic granular materials (Oda, 1972; Arthur and Menzies, 1972). Due to the linearity of the strain path, it may be pointed out that the degree of initial anisotropy is not altered by hydrostatic compression at moderate stress levels ($\sigma'_m < 300$ kPa). The behaviour in Fig. 1 is consistent with the observations in the triaxial device reported by Negussey and Vaid (1986) on the same material. In the case of pluviated sands, the degree of inherent anisotropy has been shown to decrease with increase in relative density (Negussey and Vaid, 1986; Sayao and Vaid, 1989).

PROPORTIONAL LOADING BEHAVIOUR

Typical examples of stress changes during proportional loading at $R=2$ are illustrated in Fig. 2. The initial stress state ($R=2$, $b=0.5$, $\sigma'_m=50$ kPa, $\alpha=0$) prior to initiating proportional loading in Fig. 2(a) was reached by imposing sequential changes in b and R at constant $\sigma'_m=50$, $\alpha=0$; and in Fig. 2(b) by increasing R at constant $\sigma'_m=50$ kPa, $b=0.0$ and $\alpha=45^\circ$. It may be noted that, as expected in such a path, all principal stress ratios (σ'_1/σ'_2 ,

Fig. 2. Stress components during proportional loading in general stress space

σ'_2/σ'_3 and $R=\sigma'_1/\sigma'_3$ stay constant, together with the direction α_σ of σ'_1 . This implies that the overall obliquity of the applied stress vector remains fixed.

A systematic study of sand behaviour under proportional loading paths in general stress space ($0 \leq \alpha \leq 90^\circ$; $0 < b \leq 1$) is now presented. The strain response of loose Ottawa sand ($D_r=34\%$) is examined first under selected inclinations α_σ . The observations are based on initial stress conditions represented by $\sigma'_m=50$ kPa, $b=0$ and $R=2$. The investigations are then extended to other initial density and stress conditions.

Proportional Loading at Various α_σ

Stress-strain behaviour

Initial conditions for this test series were $D_r=34\%$, $R=2$, $b=0$ and $\sigma'_m=50$ kPa. Three different loading directions α_σ were selected: $\alpha_\sigma=0, 25$, and 45° . During each test, σ'_m was monotonically increased from 50 to 300 kPa, under constant, R , α_σ and b .

Figure 3 shows the stress-strain behaviour of Ottawa sand under proportional loading. For a given increment in σ'_m , both volumetric contractions and shear distortions may be noted to increase with increase in α_σ . In particular, the response for $\alpha_\sigma=0$ is much stiffer than for other α_σ directions. These observations are consistent with shear and hydrostatic compression results reported in Fig. 1, which indicated a marked initial (or inherent) anisotropy.

Fig. 3. Effect of loading direction α_σ on strain development due to proportional loading

It is interesting to note that, on drained loading at higher values of α_σ , large shear strains are induced under changes in σ'_m only. Undrained loading under similar conditions, would therefore result in positive excess pore pressures much larger than the increment in σ_m .

Direction of strain increment

The observed directions (α_{de}) of the major principal strain increments are compared to the corresponding constant principal stress directions α_σ in Fig. 4. Behaviour under proportional loading is such that α_{de} stays nearly constant but slightly greater than α_σ , for α_σ -values greater than zero.* This may again be explained by the initial cross-anisotropy of the sand, which causes a small deviation of ϵ_1 towards the horizontal direction. Constancy of α_{de} under constant inclination of the applied stress vector suggests that effects of strain induced anisotropy may be considered insignificant in proportional loading paths, at the initial stress and density conditions considered in this investigation.

Strain paths

Any induced anisotropy in proportional loading may also be assessed by examining the strain paths followed in principal strain spaces (ϵ_1, ϵ_3) and (ϵ_2, ϵ_3). It is apparent, from the results shown in Fig. 5, that proportional loading paths result in linear strain paths, regardless of inclination α_σ . These observations give support to the above suggestion that no change in inherent anisotropy occurs on straining under proportional loading at $R=2$ and $b=0$. This could be attributed to rather small levels of strains (less than 1%) that accrue on proportional loading. Similar conclusions, based on proportional loading in the triaxial device ($\alpha_\sigma=0$), have been presented by Rowe (1971) and Vaid and Negussey (1986).

Fig. 4. Strain increment directions during proportional loading

* There is a coincidence of axes of loading and anisotropy when pluviated sands are loaded vertically. Thus, $\alpha_{de}=\alpha_\sigma=0$ under this condition.

Fig. 5. Effect of loading direction α_σ on strain paths during proportional loading

Marked differences in strain response as a function of α_σ are also apparent in Fig. 5. Under vertical loading conditions ($\alpha_\sigma=0$), all principal strains are compressive and are of relatively small magnitudes. Under inclined loading conditions ($\alpha_\sigma=25^\circ$ and 45°), a substantial increase in major principal strains (ϵ_1) may be noted (Fig. 5(a)). As already pointed out, this is due to the inherently anisotropic characteristics of the sand. Associated with these larger compressive strains ϵ_1 , extensional strains ϵ_3 are observed in the plane of the specimen wall, at $\alpha_\sigma=25^\circ$ and 45° . These significant differences in principal strains ϵ_1 and ϵ_3 explain the occurrence of much larger shear strains ($\gamma_{\max}$) at $\alpha_\sigma>0$, shown in Fig. 3.

Proportional Loading at Other Initial Conditions

The results of proportional loading tests presented in Figs. 3 to 5 correspond to loose sand at initial stress conditions $R=2$ and $b=0$. Additional series of proportional loading tests were carried out at other selected values of D_r , R and b , in an attempt to verify the extension of the experimental observations pointed out in the previous section.

Interpretation of results is made simpler by keeping σ_1 in the vertical direction in all remaining tests. As a consequence, the directions of principal stress and strain increments were coincident with the axis of initial anisotropy ($\alpha_\sigma=\alpha_{de}=0$).

Effect of relative density

Figure 6 shows volumetric and shear strain responses of very loose ($D_r=20\%$), loose ($D_r=33\%$) and dense ($D_r=60\%$) Ottawa sand. Monotonic increase in σ'_m from 50 to 300 kPa was imposed at constant stress parameters $R=2$, $b=0.5$ and $\alpha_\sigma=0$. As expected, induced volume contractions and distortions are noted to increase with

Fig. 6. Effect of relative density D_r on strain development during proportional loading

Fig. 7. Effect of relative density D_r on strain paths during proportional loading

decrease in relative density.

The observed strain paths in principal strain space are plotted in Fig. 7. It is important to point out that behaviour in proportional loading, at the selected stress conditions described above, is such that the major principal strain ϵ_1 which is compressive, occurs in the radial direction, regardless of the value of D_r . Accordingly, ϵ_2 is verti-

cal and ε_3 occurs in the circumferential direction. This is directly related to the inherent cross-anisotropy of the sand tested. If the sand was isotropic, strain response under $\alpha_\sigma=0$ condition would be necessarily such that $\varepsilon_1=\varepsilon_z$. In addition, the stress system ($R=2$, $b=0.5$) imposed on the specimen also contributes to the extensional strain response in the circumferential direction. This is discussed in the next section on the effect of b .

A detailed examination of Figs. 7(a) and (b) reveals that, at a given level of vertical strain ($\varepsilon_z=\varepsilon_2$), larger compressive radial strains ($\varepsilon_r=\varepsilon_1$), and slightly less extensional circumferential strain ($\varepsilon_\theta=\varepsilon_3$) are associated with lower relative densities. As a result, overall deformations in the horizontal plane are greatest for the loose ($D_r=20\%$) sand. This behaviour suggests that a higher degree of initial anisotropy is present in the loose sand. Such conclusion is consistent with the greater effect of principal stress rotation observed on loose sand and with hydrostatic compression data on this sand reported by Negussey and Vaid (1986).

Another important aspect to be pointed out in Fig. 7 is related to the linearity of the strain paths. The dense sand exhibits essentially linear strain paths, which are again an indication that the initial anisotropy is maintained during proportional loading over the range of stresses imposed. On the other hand, a small curvature may be noted in the strain paths followed by the loose ($D_r=20\%$) sand. This suggests that the larger strains experienced by the loose sand tend to induce a small but gradual change in the initial anisotropy. Consistent with the above observations, a very small curvature may be noted in the strain paths corresponding to an intermediate relative density ($D_r=33\%$), between loose and dense. But, the strain paths followed by the loose sand may be considered as nearly linear, for all practical purposes. Small changes in inherent anisotropy under proportional loading may therefore be confined to very loose sands only.

This important characteristic response of loose sand has apparently not been detected in previous investigations on proportional loading, and is probably due to the fact that those studies were carried out in the triaxial device, where uniform sand specimens looser than about 30% are difficult to prepare. Moreover, the stress system in triaxial compression is limited to $b=0$ conditions only. The effect of b on the strain response to proportional loading is examined in the following section.

Effects of Intermediate Stress Parameter

In this test series, three loose specimens ($D_r=33\%$) were subjected to initial stress state $R=2.0$, $\alpha_\sigma=0$ and different values of $b=0.0$, 0.3 and 0.5, prior to increasing σ'_m from 50 to 300 kPa. The range of b selected represents the majority of practical loading situations ranging from axisymmetric to plane strain.

Volumetric and maximum shear strains on proportional loading are shown in Figs. 8(a) and (b), respectively. It is apparent that, even though volumetric contractions are nearly similar, shear distortions are highly dependent on b -value.

Fig. 8. Effect of intermediate stress parameter b on strain development due to proportional loading

Fig. 9. Effect of intermediate stress parameter b on strain paths during proportional loading

The observed differences in shear strain response to proportional loading may be explained by examining the strain paths followed in principal strain space (Fig. 9). Under axisymmetric conditions ($b=0$, $\alpha_\sigma=0$), all three

principal strains are compressive and the major strain ε_1 occurs in the vertical direction. On the other hand, under more general stress conditions ($b=0.3$ and 0.5), extensional minor principal strains ε_3 are noted. Moreover, major compressive principal strains ε_1 are developed in the radial direction. This observed switch in ε_1 from vertical to radial direction, when b changes from 0.0 to either 0.3 or 0.5 , may be attributed to two main factors: the stress system imposed on the specimen and the inherent cross-anisotropy of the sand. As already illustrated in Fig. 2, increase in $\sigma'_r (= \sigma'_2)$ is greater when proportional loading is imposed at $b=0.5$ than at $b=0$. In addition, increase in σ'_3 is also smaller at $b=0.5$. Consequently, if the stress ratio is relatively low, ε_r becomes the major principal strain in an inherently anisotropic sand.

It may be noted in Fig. 9 that ε_1 and ε_3 increase in magnitude (with opposite signs) with an increase in b -value. This explains the differences in shear strain response under different b -values already pointed out (Fig. 8).

The strain paths shown in Figs. 9(a) and (b) are essentially linear. Thus, the initial cross-anisotropy of the medium-loose sand remains unaltered during proportional loading at the selected stress conditions, $R=2.0$ and $\alpha_\sigma=0$.

Effect of Stress Ratio Level

All proportional loading tests reported in the previous sections correspond to only one R -level ($R=2.0$). In this test series, three medium-loose ($D_r=33\%$) sand specimens were initially subjected to different values of $R=1.3, 2.0$ and 3.0 , and to identical $b=0.5$ and $\alpha_\sigma=0$, prior to imposing a monotonic increase in σ'_m from 50 to 300 kPa.

Figure 10(a) shows the stress-strain behaviour of medi-

Fig. 10. Effect of stress ratio level R on strain development due to proportional loading

Fig. 11. Effect of stress ratio level R on strain paths during proportional loading

um-loose sand in terms of volumetric deformations. Larger contraction is induced in sand at $R=3.0$, for a given level of σ'_m . At $R \leq 2.0$, volumetric contractions are seen to be nearly similar. In contrast, much larger differences in shear distortion may be noted (Fig. 10(b)) as R increases from 1.3 to 3.0 .

The strain paths observed in this test series are shown in Fig. 11. In all tests, $\varepsilon_1 = \varepsilon_r$ and $\varepsilon_3 = \varepsilon_\theta$. At $R=1.3$ and 2.0 , principal strains induced in the medium-loose sand follow nearly linear paths. At higher stress ratio ($R=3.0$), however, principal strain paths show visible nonlinearity. This suggests that the large strains may have induced gradual changes in the initial anisotropy of the sand tested at $R=3.0$.

CONCLUSIONS

Experimental results on proportional loading in general stress space indicate that volumetric contractions and shear distortions induced in Ottawa sand are highly dependent on the initial stress and density conditions. At a given σ'_m level, deformations are observed to increase with decrease in D_r and increase in α , b and R .

Examination of principal strain paths suggest that the sand's inherent anisotropy gets gradually affected by induced strains at low relative density ($D_r=20\%$) or at high stress ratio ($R=3.0$). Under all other initial conditions investigated, strain paths were shown to be essentially linear during proportional loading, implying preservation of the nature of inherent cross-anisotropy.

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