EFFECTS OF MEMBRANE PENETRATION ON MODULUS AND POISSON'S RATIO FOR UNDRAINED CYCLIC TRIAXIAL CONDITIONS

SATOSHI YAMASHITAⁱ⁾, SHOSUKE TOKIⁱⁱ⁾ and TERUYUKI SUZUKIⁱⁱⁱ⁾

ABSTRACT

The effects of membrane penetration on the cyclic deformation properties for undrained cyclic triaxial tests of granular soils are discussed in this paper. The soils used are six types of sands with different grain size distributions. Test results indicated that the relationship between the shear modulus at small strain obtained from a cyclic torsional test and an undrained cyclic triaxial test, and between the Young's modulus obtained from the drained and undrained cyclic triaxial tests are dependent on the grain size distribution of the sands. It is assumed that the main reason for this is caused by a decrease of the Poisson's ratio under undrained cyclic triaxial condition due to the membrane penetration effect. The Poisson's ratio in undrained cyclic triaxial tests has been estimated from the membrane penetration value and the pore pressure fluctuation under undrained cyclic loading. Consequently, it was determined that the Poisson's ratio of the soil skeleton in undrained cyclic triaxial tests decreases from 0.5 at small strain, as the grain size increases. In addition, the shear modulus calculated from the estimated Poisson's ratio and the measured Young's modulus in undrained cyclic triaxial tests is almost equal to those in cyclic torsional tests irrespective of grain size.

Key words: cyclic triaxial test, drainage condition, membrane penetration, Poisson's ratio, sand, Young's modulus (IGC: D7/D6)

INTRODUCTION

It has generally been recognized that the undrained cyclic strength of saturated sand depends on the system compliance including degree of saturation, flexibility of the pore water pressure measuring system, membrane penetration, sample diameter (e.g., Lade and Hernandez, 1977; Martin et al., 1978; Chaney, 1978; Tokimatsu and Nakamura, 1986). The generation of pore water pressure is influenced by these factors. It is considered that these factors also influence the small strain deformation properties in undrained cyclic triaxial (CTX) test, as has been reported by Tanaka et al. (1990). The Young's modulus at small strain levels, for example, obtained from undrained CTX test decreases, as the pore pressure parameter B decreases (Yamashita et al., 1994a). Accordingly, it seems that the deformation properties in undrained CTX tests are also influenced by the effect of membrane penetration, which is a major cause of the system compliance (Martin et al., 1978; Toki et al., 1986).

In this study, the effects of membrane penetration on the cyclic undrained deformation characteristics of saturated sand were investigated by performing undrained

CTX tests for six types of sands with different grain size distributions. In addition, the shear modulus evaluated by the estimated Poisson's ratio considering the membrane penetration under undrained triaxial condition was compared with that obtained from the cyclic torsional shear (CTS) test.

TEST PROGRAM

The soils used were Toyoura, Ishikari, Soma (Yamashita and Toki, 1994b) and three kinds of silicious sands (K-3, K-5 and K-7 sands) with different grain size distributions. The mean grain sizes of these sands range from 0.12 mm to 1.25 mm. The physical properties and the grain size distribution curves of these sands are shown in Table 1 and Fig. 1.

Two samples were employed in this experiment: a solid cylindrical and a hollow cylindrical sample. Solid samples 17 cm in height and 7 cm in diameter were used for the CTX tests on K-3, K-5 and K-7 sands. Hollow samples 20 cm in height, 10 cm in outer diameter and 6 cm in inner diameter were used for the CTX tests on Ishikari, Toyoura and Soma sands and the CTS tests. The samples

ⁱ⁾ Associate Professor, Department of Civil Engineering, Kitami Institute of Technology, 165 Koen-cho, Kitami 090.

ⁱⁱ⁾ Professor, Department of Civil Engineering, Hokkaido University, Kita-13, Nishi-8, Kita-ku, Sapporo 060.

ⁱⁱⁱ⁾ Professor, Department of Civil Engineering, Kitami Institute of Technology, 165 Koen-cho, Kitami 090.

Manuscript was received for review on August 17, 1995.

Written discussions on this paper should be submitted before July 1, 1997 to the Japanese Geotechnical Society, Sugayama Bldg. 4F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

Table 1. Physical properties of sands

Sand name	Density of soil particle ρ_s (g/cm ³)	Uniformity coefficient U_c	Mean grain size D_{50} (mm)	Fines content ($\leq 75 \mu\text{m}$) F_c (%)	Maximum void ratio e_{max}	Minimum void ratio e_{min}
K-3	2.62	1.27	1.25	0	0.953	0.680
Soma	2.64	1.6	0.74	0	0.804	0.553
K-5	2.61	1.38	0.60	0	1.053	0.665
Toyoura	2.64	1.3	0.20	0	0.970	0.618
K-7	2.65	2.54	0.13	15	1.178	0.633
Ishikari	2.74	1.7	0.12	10	1.416	0.837

Fig. 1. Grain size distribution curves

were prepared with a vibration method (Yamashita and Toki, 1993). The thickness of rubber membrane to enclose the specimens was about 0.2 mm.

After each sample was set up in the cell, the cell pressure was increased to 30 kPa, and carbon dioxide was percolated through the sample. Subsequently, the deaired water was permeated through the sample. Back pressure of 196 kPa was, thereafter, applied for about one hour to ensure the saturation of the sample. By this procedure, the pore pressure parameter B of greater than 0.96 was obtained in all samples. All samples were isotropically consolidated.

Multi-stage cyclic loading tests were conducted on the samples. The test procedures have been described in detail by Toki et al. (1995). The axial (or shear) stress and the axial (or shear) strain were measured by a two-component load cell and a pair of proximity transducer placed inside the cell, respectively.

The test results were interpreted in terms of the equivalent shear modulus G_{eq} , the equivalent Young's modulus E_{eq} and the hysteretic damping ratio h , where G_{eq} , E_{eq} and h are defined from the hysteresis loop of the stress-strain relationship at tenth loading cycle in each stage. The equivalent shear modulus G_{eq} and the shear strain γ in the CTX test were estimated from the measured equivalent Young's modulus E_{eq} and the axial strain ϵ_a , as defined by

$$G_{eq} = \frac{E_{eq}}{2(1+\nu)} \quad (1)$$

$$\gamma = (1+\nu)\epsilon_a \quad (2)$$

in which ν = the Poisson's ratio.

TEST RESULTS

Comparisons of Triaxial and Torsional Tests

Figures 2(a) through (c) show comparisons for the

Fig. 2. Comparisons of undrained cyclic triaxial and cyclic torsional tests: (a) K-3 sand, (b) K-5 sand, (c) K-7 sand

drained CTS test results and the undrained CTX test results on K-3, K-5 and K-7 sand samples. In these figures, G_{eq} and γ in undrained CTX tests were estimated from the measured E_{eq} and the ε_a by assuming the ν equal to 0.5.

For the K-7 sand, $G_{eq(CTS)}$ in CTS tests were almost equal to $G_{eq(CTX)}$ in CTX tests. In the cases of K-3 and K-5 sands, $G_{eq(CTS)}$ is larger than $G_{eq(CTX)}$ at small strain levels.

Figure 3 shows the ratios of $G_{eq(CTS)}$ to $G_{eq(CTX)}$ at single amplitude shear strain of 10^{-5} versus the mean grain size D_{50} for these sand samples. In this figure, the previous test results of Ishikari, Toyoura and Soma sands are also plotted (Yamashita and Toki, 1994b). It was found that

Fig. 3. Relationship between ratios of $G_{eq(CTS)}$ to $G_{eq(CTX)}$ and mean grain size

the ratio of G_{eq} is greater for sands with a larger grain size than that with smaller one. It seems that one of the sources for this is the bedding error in the triaxial test (Tatsuoka and Shibuya, 1992). However, the effect of bedding error on Toyoura and Soma sand samples was relatively small (Yamashita and Toki, 1994b).

Accordingly, it is seen that the difference between $G_{eq(CTS)}$ and $G_{eq(CTX)}$ is attributed to not only the effect of the bedding error in the triaxial test but also the effects of other factors. For example, it is considered that the Poisson's ratio ν in undrained CTX test is lower than 0.5 for sands with a larger grain size due to the effect of membrane penetration.

Effects of Drainage Condition on Triaxial Test Results

Figures 4(a) through (d) show the relationship of E_{eq} or h versus logarithmic single amplitude axial strain $\log(\varepsilon_a)_{SA}$ obtained from drained and undrained CTX tests on Ishikari, K-7, Soma and K-3 sand samples. The CTX tests on K-3 and K-7 sands were performed on solid samples. The CTX tests on Ishikari and Soma sands were performed on hollow samples. For Soma and K-3 sands, $E_{eq(undrained)}$ in undrained tests at small strain is almost equal to $E_{eq(draind)}$ in drained tests. In the cases of Ishikari and K-7 sands, $E_{eq(undrained)}$ is larger than $E_{eq(draind)}$.

Figure 5 shows the ratios of $E_{eq(undrained)}$ to $E_{eq(draind)}$ at single amplitude axial strain of 10^{-5} with respect to logarithmic mean grain size $\log D_{50}$. It was found that the ratio of E_{eq} decreases, as grain size of sand increases. In addition, the ratio of E_{eq} for hollow samples is smaller than that for solid samples.

Fig. 4. Effects of drainage condition on cyclic triaxial test results: (a) Ishikari sand, (b) K-7 sand, (c) Soma sand, (d) K-3 sand

Fig. 5. Relationship between ratios of $E_{eq(undrained)}$ to $E_{eq(draind)}$, and mean grain size

Fig. 6. Effects of grain size of sand on excess pore water pressure under undrained cyclic triaxial condition: (a) K-3 sand, (b) K-5 sand, (c) K-7 sand

Since the shear modulus obtained from the CTS test was not affected by the drainage condition (e.g., Yamashita and Toki, 1994b), the shear modulus obtained from the undrained CTX test should coincide with that from the drained CTX test. Thus, the relationship between $E_{eq(draind)}$ and $E_{eq(undrained)}$ may be defined by

$$G_{eq} = \frac{E_{eq(draind)}}{2(1+\nu_{drained})} = \frac{E_{eq(undrained)}}{2(1+\nu_{undrained})} \quad (3)$$

in which $\nu_{drained}$ and $\nu_{undrained}$ = the Poisson's ratio under drained and undrained loading.

Accordingly, in Soma and K-3 sand samples, it is assumed that either the Poisson ratio in the drained CTX test is almost equal to 0.5, or the Poisson's ratio in the undrained CTX test is lower than 0.5.

Figures 6(a) through (c) show the relationship of the measured pore water pressure change divided by initial effective confining pressure $\Delta u_{measured}/\Delta \sigma'_c$ at tenth loading cycle for each stage (see Fig. 7) versus the single amplitude axial strain $(\epsilon_a)_{SA}$ on K-3, K-5 and K-7 solid samples. In these figures, the pore pressure change for an ideal case in which an isotropic and elastic soil specimen is saturated with an incompressible pore fluid (i.e., $\Delta u = \Delta \sigma_a/3$) is also plotted. In the case of K-7 sand, $\Delta \sigma_a/3$ is almost equal to $\Delta u_{measured}$. For K-3 and K-5 sands with greater grain size, $\Delta u_{measured}$ is lower than $\Delta \sigma_a/3$.

Figure 8 shows the ratios of $\Delta u_{measured}$ to $\Delta \sigma_a$ against mean grain size. The ratio of Δu to $\Delta \sigma_a$ of isotropic and elastic sample without system compliance is one third. It is found that the ratio of $\Delta u_{measured}$ to $\Delta \sigma_a$ decreases, as grain size of sand increases. In addition, the ratios of $\Delta u_{measured}$ to $\Delta \sigma_a$ of hollow samples are smaller than those of solid samples. From these figures, it can also be seen that in the case of undrained CTX test, the effect of membrane penetration is significant at small strain levels.

Fig. 7. Typical time histories of deviator stress, axial strain and pore pressure during undrained cyclic triaxial test

Fig. 8. Relationship between ratios of $\Delta u_{\text{measured}}$ to $\Delta \sigma_a$ and mean grain size

EVALUATION OF MEMBRANE PENETRATION EFFECT

Estimate of Membrane Penetration Value

Volume change of sample caused by membrane penetration on granular soils, wherein the effective confining pressure varies, was first recognized by Newland and Allely (1959). Many attempts subsequently have been made to evaluate the value of membrane penetration (e.g., Roscoe et al., 1963; Vaid and Negussey, 1984, Frydman et al., 1973).

Figure 9 shows the value of membrane penetration of K-3, K-5 and K-7 sands against logarithmic effective confining pressure, where values of membrane penetration are estimated by assuming isotropic soil behavior during isotropic unloading as suggested by Vaid and Negussey (1984).

There is approximately a linear relationship between logarithmic effective confining pressure and membrane penetration on each sand. Baldi and Nova (1984) showed that this relation is expressed as

$$\Delta v_m = S \Delta \log \sigma'_c \quad (4)$$

Fig. 9. Relationship between unit membrane penetration and effective confining pressure

in which S = the normalized membrane penetration and Δv_m = unit membrane penetration and is defined by

$$v_m = \varepsilon_{vm} V / A_m \quad (5)$$

in which ε_{vm} = volumetric strain by membrane penetration, V = volume of specimen and A_m = surface area covered by membrane.

Figure 10 shows the relationship of S and D_{50} for K-3, K-5 and K-7 sands with reported data by Tokimatsu and Nakamura (1986), where S of all sands are estimated by method of Vaid and Negussey.

Estimate of Poisson's Ratio

For a given change in effective confining pressure, where the sample is fully saturated and the volume change of the pore pressure measurement system is very small, the total volume change ΔV is the sum of volume changes caused by membrane penetration ΔV_m and soil deformation ΔV_s and may be expressed as

$$\Delta V = \Delta V_s + \Delta V_m \quad (6)$$

Equation (6) can be rewritten as

$$\begin{aligned} \Delta \varepsilon_v &= \Delta \varepsilon_{vs} + \Delta \varepsilon_{vm} \\ &= \Delta \varepsilon_a + 2\Delta \varepsilon_r + \Delta \varepsilon_{vm} \end{aligned} \quad (7)$$

in which ε_r = lateral strain caused by soil deformation.

In an undrained triaxial test, the total volume change of the closed system is zero. Accordingly, from Eq. (7) and $\Delta \varepsilon_v = 0$, rearranging the terms results in

$$\Delta \varepsilon_r = -(\Delta \varepsilon_a + \Delta \varepsilon_{vm}) / 2 \quad (8)$$

The Poisson's ratio of soil skeleton is defined by

$$\nu = -\Delta \varepsilon_r / \Delta \varepsilon_a \quad (9)$$

Substitution of Eq. (8) into Eq. (9) leads to

$$\nu = 0.5 + \Delta \varepsilon_{vm} / 2\Delta \varepsilon_a \quad (10)$$

From Eqs. (4) and (5), the volumetric strain by membrane penetration is given by

$$\Delta \varepsilon_{vm} = S (A_m / V) \Delta \log \sigma'_c \quad (11)$$

Fig. 10. Relationship between normalized membrane penetration and mean grain size

in which $\Delta \log \sigma'_c$ = change of effective confining pressure under undrained cyclic loading and is expressed as

$$\Delta \log \sigma'_c = \log (\sigma'_0 - u_{\max}) - \log (\sigma'_0 - u_{\min}) \quad (12)$$

in which $\Delta \log \sigma'_c$ is negative value, σ'_0 = initial effective confining pressure, $u_{\max}$ and $u_{\min}$ = maximum and minimum excess pore pressure at each cycle (see Fig. 7).

From Eq. (12), $\Delta \varepsilon_{vm}$ in Eq. (11) becomes negative value and thus ν in Eq. (10) becomes smaller than 0.5.

Figure 11 shows the Poisson's ratios estimated by Eq. (10) and Fig. 10 for six types of sands. The estimated

Poisson's ratios $\nu_{\text{estimated}}$ for four sands except for K-7 and Ishikari sands are lower than 0.5 at small strain level.

Figures 12(a) through (d) show the relationship between the G_{eq} or h and $\log (\gamma)_{SA}$ in the undrained CTX tests, where G_{eq} and γ are calculated by the measured E_{eq} and $\nu_{\text{estimated}}$ or $\nu=0.5$. In these figures, the results of CTS tests are also plotted. It was found that the shear modulus obtained from undrained CTX tests assuming the Poisson's ratio equal to 0.5 is under-estimated. And also, the CTS test results are almost equal to the CTX test results estimated by $\nu_{\text{estimated}}$ irrespective of the grain size of sands.

In order to precisely evaluate the small strain stiffness of granular soils in the CTX test, either the effect of membrane penetration on the Poisson's ratio in undrained condition should be exactly evaluated, or the Poisson's ratio in drained condition should be evaluated by measured volumetric and axial strains (e.g., Yasuda and Matsumoto, 1993) or by measured axial and lateral strains (e.g., Shibuya et al., 1994).

In this discussion, the sample is assumed to be isotropic and an elastic body at small strain. The effect of fabric anisotropy, the difference of intermediate stress between the CTX and CTS tests were not considered in this paper. Accordingly, further research will be necessary to clarify these factors.

Fig. 11. Effect of membrane penetration on the Poisson's ratio under undrained triaxial condition

CONCLUSIONS

The effects of membrane penetration on the small

Fig. 12. Comparisons of cyclic torsional test and undrained cyclic triaxial test results, where G_{eq} and γ in triaxial test are calculated from measured E_{eq} and $\nu_{\text{estimated}}$: (a) Toyoura sand, (b) K-5 sand, (c) Soma sand, (d) K-3 sand

strain stiffness in undrained cyclic triaxial tests are discussed in this paper. The results can be summarized as follows:

- 1) The Poisson's ratio at small strain under undrained cyclic triaxial condition decreases from 0.5, as the grain size increases due to the effect of membrane penetration.
- 2) The shear modulus calculated from the estimated Poisson's ratio and the measured Young's modulus in undrained cyclic triaxial tests is almost equal to that in cyclic torsional tests irrespective of grain size.

REFERENCES

- 1) Baldi, G. and Nova, R. (1984): "Membrane penetration effects in triaxial testing," *Jour. of Geotechnical Engineering*, ASCE, Vol. 110, No. 3, pp. 403-420.
- 2) Chaney, R. C. (1978): "Saturation effects on the cyclic strength of sands," *Earthquake Engineering and Soil Dynamics*, GED, ASCE, Vol. 1, pp. 343-358.
- 3) Frydman, S., Zeitlen, J. G. and Alpan, I. (1973): "The membrane effect in triaxial testing of granular soils," *Jour. of Testing and Evaluation*, Vol. 1, No. 1, pp. 37-41.
- 4) Lade, P. V. and Hernandez, S. B. (1977): "Membrane penetration effects in undrained tests," *Jour. of Geotechnical Engineering Division*, ASCE, Vol. 103, No. GT2, pp. 109-125.
- 5) Martin, G. R., Finn, W. D. L. and Seed, H. B. (1978): "Effects of system compliance on liquefaction tests," *Jour. of Geotechnical Engineering Division*, ASCE, Vol. 104, No. GT4, pp. 463-479.
- 6) Newland, P. L. and Allely, B. H. (1959): "Volume changes during undrained triaxial tests on saturated dilatant granular materials," *Géotechnique*, Vol. 9, No. 4, pp. 174-182.
- 7) Roscoe, K. H., Schofield, A. H. and Thurairajah, A. (1963): "An evaluation of test data for selecting a yield criterion for soils," *ASTM, STP361*, pp. 111-128.
- 8) Shibuya, S., Park, C., Tatsuoka, F., Abe, F., Teachavorasinskun, S., Kohata, Y. and Sato, T. (1994): "The significance of local lateral-strain measurement of soil specimens for a wide range of strain," *Soils and Foundations*, Vol. 34, No. 2, pp. 95-105.
- 9) Tanaka, Y., Kokusho, T., Yoshida, Y. and Kudo, K. (1990): "Consideration of effects of system compliance on the shear modulus of gravelly soils," *Proc. of 25th Japan National Conf. on SMFE*, pp. 757-758 (in Japanese).
- 10) Tatsuoka, F. and Shibuya, S. (1992): "Deformation characteristics of soils and rocks from field and laboratory tests," *Proc. of 9th Asian Regional Conf. on SMFE*, Vol. 2, pp. 101-170.
- 11) Toki, S., Miura, S. and Yamashita, S. (1986): "Effect of pore pressure coefficient-B on undrained cyclic triaxial strength of saturated sand," *Proc. of 21th Japan National Conf. on SMFE*, pp. 565-568 (in Japanese).
- 12) Toki, S., Shibuya, S. and Yamashita, S. (1995): "Standardization of laboratory test method to determine the cyclic deformation properties of geomaterials in Japan," *Pre-failure Deformation of Geomaterials*, Balkema, Vol. 2, pp. 741-784.
- 13) Tokimatsu, K. and Nakamura, K. (1986): "A liquefaction test without membrane penetration effects," *Soils and Foundations*, Vol. 26, No. 4, pp. 127-138.
- 14) Vaid, Y. P. and Negussey, D. (1984): "A critical assessment of membrane penetration in the triaxial test," *Geotechnical Testing Journal*, Vol. 7, No. 2, pp. 70-76.
- 15) Yamashita, S. and Toki, S. (1993): "Effects of fabric anisotropy of sand on cyclic undrained triaxial and torsional strengths," *Soils and Foundations*, Vol. 33, No. 3, pp. 92-104.
- 16) Yamashita, S., Shibuya, S., Inahara, H. and Furukawa, T. (1994a): "Effects of B-value on cyclic deformation characteristics of sand and clay," *Proc. of Symposium on Deformation Characteristics of Geomaterials Associated with Dynamic Problems of Ground and Soil-Structures*, JSSMEF, pp. 137-142 (in Japanese).
- 17) Yamashita, S. and Toki, S. (1994b): "Cyclic deformation characteristics of sands in triaxial and torsional tests," *Pre-failure Deformation of Geomaterials*, Balkema, Vol. 1, pp. 31-36.
- 18) Yasuda, N. and Matsumoto, N. (1993): "Dynamic deformation characteristics of sands and rockfill materials," *Canadian Geotechnical Journal*, Vol. 30, No. 4, pp. 747-757.