MECHANISM OF LARGE POST-LIQUEFACTION DEFORMATION IN SATURATED SAND

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ABSTRACT

A new mechanism of large post-liquefaction shear deformation in saturated sand was established on the physical basis that the shear deformation is governed by two types of volumetric strains due to dilatancy, i.e., an irreversible dilatancy component, $\varepsilon_{v,ir}$, and a reversible dilatancy component, $\varepsilon_{v,re}$. It was found that: 1) the shear strain is composed of a shear strain component depending on change in effective stress γ_d , and a shear strain component independent of effective stress γ_o ; 2) post-liquefaction γ_o -value is triggered principally in the state of zero effective confining stress, and its current magnitude has a nearly unique relationship with the preceding maximum shear strain γ_{max} for sand of a given density; and 3) γ_d is determined by a good correlation existing between $d\varepsilon_{v,re}/d\gamma_d$ and deviator-isotropic stress ratio, q/p' . Based on the formulation for the above experimental findings and stress-dilatancy concept, a new approach is proposed to evaluate the large post-liquefaction shear strain $\gamma=(\gamma_o+\gamma_d)$ in saturated sand. The results predicted by the proposed method compared favorably with experimental observation.

Key words: cyclic loading, dilatancy, fully saturated sand, irreversible volume change, laboratory test, post-liquefaction deformation, undrained (IGC: D7/E2)

INTRODUCTION

Large lateral ground deformation, inducing severe damage to various infrastructure and lifeline systems, was observed during past earthquakes in liquefied sandy soil deposits with level or inclined surfaces. A number of laboratory experiments shows that large deformation always occurs after sand liquefaction, and it can especially be developed when the effective confining stress in sand momentarily goes through zero state during undrained loading, either cyclically or monotonically. The phenomenon that the zero effective stress state occurs for the first time was termed 'initial liquefaction' for cyclic undrained loading (Seed and Lee, 1966), which may classify the liquefaction behavior into pre-liquefaction and post-liquefaction behavior. Many studies on sand liquefaction in the past decades have focused mainly on the methods to evaluate the likelihood of liquefaction and the pre-liquefaction stress-strain response. Several studies on post-liquefaction deformation behavior have also been performed by Castro et al. (1975 and 1985), Goto et al. (1992), Katada et al. (1993), Yasuda et al. (1994), Yoshida et al. (1994) and Vaid and Thomas (1995). Our understanding of the reason why large post-liquefaction deformation can occur, however, seems quite limited.

A new mechanism on large post-liquefaction shear

deformation in saturated sand presented in this paper is established based on the experimental fact that the shear deformation is controlled by two types of volumetric strains due to stress dilatancy, i.e., an irreversible dilatancy component, and a reversible dilatancy component. In addition, the post-liquefaction shear strain is found to be constituted with two shear strain components, one that depends on change in effective stress and the other that is independent of effective stress. Based on this stress-dilatancy concept and a great deal of experimental analysis, a new approach to the evaluation of large post-liquefaction shear strain is proposed and then confirmed by experimental observations.

TWO TYPES OF SHEAR STRAIN COMPONENTS

In order to examine the post-liquefaction shear behavior of saturated sands, cyclic or monotonic undrained torsion tests after initial liquefaction were run on samples of Toyoura sand ($\rho_s=2.65 \text{ g/cm}^3$, $D_{50}=0.18 \text{ mm}$, $e_{max}=0.973$ and $e_{min}=0.635$). The samples were prepared by pluviating dry sand through air, and saturated by circulating CO_2 gas, percolating de-aired water and then by applying a back pressure of 100 kPa. B -values of more than 0.95 were obtained for all the saturated samples used in the tests.

Figure 1 shows the post-liquefaction shear stress-shear

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Fig. 1. Two post-liquefaction shear strain components observed in a typical cyclic undrained torsion test: (a) shear stress-shear strain relationship, (b) effective stress path

strain relationship and the corresponding effective stress path. It should be noted, that the term 'post-liquefaction' used in this study indicates the undrained response of sands to either cyclic or monotonic loading after initial liquefaction. As seen from Fig. 1, the phenomenon that the effective stress state point moves repeatedly through states of zero effective confining stress along the same path is accompanied with a significant development of double amplitude of shear strain.

Comparing Figs. 1(a) and (b), the double amplitude of shear strain is found to increase with increasing number of loading cycles under the same effective stress path. Such a post-liquefaction stress-strain response can be easily reproduced in cyclic undrained tests for saturated sands. It implies that there exist two shear strain components during post-liquefaction cyclic undrained loading. One is the shear strain component that depends on the change in effective stress, and the other one is independent of change in effective stress. Let us further compare one stress-strain hysteresis curve with another in Fig. 1. We note that each hysteresis curve is divided into two parts corresponding to states of non-zero and zero effective confining stress. Obviously, the phases of the stress-strain hysteresis curves associated with non-zero changes in effective stress are parallel to each other for loading or unloading, and consequently, nearly the same incremental changes in shear strain are induced. Conversely, the shear strain induced at the time intervals that the effective stress point goes through states of zero effective stress in-

creases with increasing number of load cycles, but its change does not relate to the change in effective stress. It is particularly worthy of note that such a shear strain component independent of effective stress governs the development of post-liquefaction shear strain.

A similar phenomenon can also be observed in the monotonic undrained shear tests after complete or incomplete liquefaction, as shown in Fig. 2. In this figure Curve 3 through Curve 6 correspond to the cases after complete liquefaction, and Curves 1 and 2 to those after incomplete liquefaction. Comparing Figs. 2(a) and (b), it is found that all the shear stress-shear strain relations are nearly parallel to each other, but their shear strain values at the same shear stress level are quite different, although their corresponding effective stress paths almost coincide except for the initial phases of loading for Curves 1 and 2. There indeed exist two shear strain components stated above also for monotonic undrained loading after liquefaction, and obviously, the shear strain component independent of effective stress, induced at state of zero effective confining stress, plays a decisive role in the development of large post-liquefaction shear strain.

If the shear strain component depending on change in effective stress is denoted as γ_d , and the other one independent of effective stress as γ_o , then the post-liquefaction shear strain γ can be equated as

$$\gamma = \gamma_d + \gamma_o \quad (1)$$

Fig. 2. Two post-liquefaction shear strain components observed in monotonic undrained torsion tests after incomplete or complete liquefaction: (a) shear stress-shear strain relationship, (b) effective stress path

To reveal the mechanism resulting in this phenomenon and then to establish a methodology for evaluating γ were the objectives of this study. Attention was particularly paid to the understanding of γ_0 whose existence has been neglected. It is as an example, whether the current magnitude of γ_0 relates with the maximum shear strain induced by the preceding cyclic undrained loading, as illustrated in Fig. 2(a).

TWO VOLUMETRIC STRAIN COMPONENTS DUE TO DILATANCY

Figure 3(a) shows time histories of the changes in shear and volumetric strains with increasing number of cycles measured in a cyclic drained torsion test on a saturated, isotropically consolidated Toyoura sand. Because a constant confining stress σ'_c of 30 kPa was maintained during the entire cyclic shear testing, the resulting volumetric strain was induced by changes in shear-normal stress ratio τ/σ'_c or shear strain γ . Such a volumetric strain induced by shear application is usually called 'volumetric strain due to dilatancy' and hereafter denoted as ε_{vd} . It can be seen that during a continuing cyclic shear application, ε_{vd} is not only gradually accumulated, but also fluctuates in magnitude along with the repeated increase and decrease in γ . All the ε_{vd} -values taken at states of zero shear strain are plotted in Fig. 3(a) with a broken curve. It shows an irrecoverable, monotonic increase of the volumetric strain component which is determined mainly by the past shear history, but was independent of the current γ -value. In addition, if this volumetric strain component is subtracted from ε_{vd} and then the result is plotted in Fig. 3(b), the change of the new volumetric strain component obtained can be found to be recoverable over the entire shear process. Typical volumetric and shear strain relationships for three shearing cycles from 18th to 20th cycle as shown in Fig. 3(b) are illustrated in Fig. 4. The

Fig. 3. Changes in reversible and irreversible dilatancy components during cyclic drained shear while maintaining a constant effective confining stress

Fig. 4. Reversible volumetric strain change during cyclic drained shear (the data shown in this figure are the same as in Fig. 3)

change in such a recoverable volumetric strain component depends closely on the current shear strain, although the relationship between the volumetric and shear strains is different for loading and unloading.

Based on this observation, it can be concluded that two types of volumetric strain components exist due to dilatancy during shear. The one is characterized by its reversibility and dependency on the magnitude and direction of the current shear strain, and conversely, the other one by its irreversibility and dependency on past shear history. The former is hereafter called 'reversible dilatancy component' and denoted as $\varepsilon_{vd,rev}$, and the latter 'irreversible dilatancy component' as $\varepsilon_{vd,ir}$. Hence, the volumetric strain due to dilatancy can be expressed in general as

$$\varepsilon_{vd} = \varepsilon_{vd,rev} + \varepsilon_{vd,ir} \quad (2)$$

It should be noted the change in $\varepsilon_{vd,rev}$ is governed by the current magnitude and direction of shear strain. The change rate in $\varepsilon_{vd,ir}$ however is independent of the current magnitudes of shear strain and stress.

For a soil element subjected to a general loading, its volumetric strain may also be induced by the change in mean effective confining stress. Such a volumetric strain component is hereafter denoted as ε_{vc} . Thus, the total volumetric strain, ε_v , should be written as

$$\varepsilon_v = \varepsilon_{vd} + \varepsilon_{vc} = \varepsilon_{vd,rev} + \varepsilon_{vd,ir} + \varepsilon_{vc} \quad (3)$$

In this paper the volumetric strain takes either a positive or negative sign depending on contraction or expansion of the soil volume.

MECHANISM OF LARGE POST-LIQUEFACTION SHEAR DEFORMATION

No volume change occurs for a saturated sand subjected to a shear application in an undrained condition, or, $\varepsilon_v = 0$. Accordingly, Eq. (3) becomes:

$$\varepsilon_{vd,rev} + \varepsilon_{vd,ir} + \varepsilon_{vc} = 0 \quad (4)$$

As a result, excess water pore pressure Δu builds up in the sand. In the entire loading course, ε_{vc} decreases in a

swelling manner as Δu increases, but it cannot become less than its value at state of zero effective confining stress (or its maximum volumetric strain of expansion) denoted as $\varepsilon_{vc,o}$. On the other hand, $\varepsilon_{vd,ir}$ always displays an irreversible increase in contraction throughout a continuing cyclic undrained shear process whether before or after initial liquefaction. In general, the $\varepsilon_{vd,ir}$ -value at state of zero effective stress is much more than $-\varepsilon_{vc,o}$, namely, $\varepsilon_{vd,ir} > -\varepsilon_{vc,o}$. This means that the following conditions are maintained, namely:

$$\varepsilon_{vc} \geq \varepsilon_{vc,o} \quad (5)$$

$$\varepsilon_{vd,ir} + \varepsilon_{vc,o} > 0 \quad (6)$$

Obviously, $\varepsilon_{vd,ir}$ is required to have sufficient reduction to satisfy Eq. (4) representing the condition of zero volume change, i.e., the volumetric strain component of expansion due to dilatancy should become large enough to maintain $\varepsilon_{vd,ir} = -(\varepsilon_{vd,ir} + \varepsilon_{vc,o})$. In order to induce such a large $\varepsilon_{vd,ir}$ -value, therefore, a large shear strain γ needs to be triggered because an increase in $\varepsilon_{vd,ir}$ can be obtained only by an increase in γ .

As illustrated in Figs. 1 and 2, the change in post-liquefaction shear strain is induced principally at states of zero effective confining stress. At these time intervals, no change of the shear strain component depending on the change in effective stress γ_d defined before occurs, and in addition, no change in $\varepsilon_{vc,o}$, because of no change in effective stress. At this time, an increase in $\varepsilon_{vd,ir}$ is balanced with a reduction in $\varepsilon_{vd,ir}$ induced by a shear strain whose occurrence is in a state of zero effective confining stress. Such a shear strain is the shear strain component independent of effective stress γ_o defined in Eq. (1). With further increasing cyclic loading after initial liquefaction, $\varepsilon_{vd,ir}$ continues to increase more and more. Thus, the γ_o -value accordingly must increase thereby causing a sufficient decrease in $\varepsilon_{vd,ir}$. As a consequence, γ_o exhibits a monotonic increase with increasing number of cycles.

This analysis provides the answers why γ_o can occur, and why the double amplitude of cyclic shear strain after initial liquefaction always displays a significant increase although the exactly same change in effective stress path occurs as shown in Fig. 1. Similarly, different $\varepsilon_{vd,ir}$ -changes resulting from different preceding undrained cyclic shear applications are responsible for conspicuously different changes in induced post-liquefaction shear strain as seen in Fig. 2.

Based on the same analysis, the condition triggering the flow deformation in a saturated sand is:

$$-\varepsilon_{vd,ir} < (\varepsilon_{vd,ir} + \varepsilon_{vc,o}) \quad (7)$$

It is clear that the smaller the initial sand density, the smaller $-\varepsilon_{vd,ir}$ and the larger $\varepsilon_{vd,ir}$. Catastrophic flow deformation therefore can occur in a saturated, contractive loose sand subjected to cyclic or monotonic undrained shear, because the value of $\varepsilon_{vd,ir}$ induced may become much larger than that of $-\varepsilon_{vd,ir}$.

CONSTITUTIVE RELATIONS FOR POST-LIQUEFACTION SHEAR STRAIN COMPONENT INDEPENDENT OF EFFECTIVE STRESS

Relation between γ_o and $\varepsilon_{vd,ir}$ Based on Plasticity Theory

If a saturated sand is assumed to be purely plastic subject to undrained shear after initial liquefaction, the plastic volumetric and deviator strains, ε_v^p and γ^p , may be expressed as follows based on the theory of plasticity.

$$d\varepsilon_v^p = d\lambda \frac{\partial Q}{\partial p'} \quad (8)$$

$$d\gamma^p = d\lambda \frac{\partial Q}{\partial q}$$

in which Q is a plastic potential function, $d\lambda$ is the coefficient of proportionality, the superscript 'p' means plasticity, and the stress parameter (q, p') with respect to the effective principal stress state (σ'_i , $i=1, 2$, and 3) is defined as

$$p' = \frac{1}{3} (\sigma'_1 + \sigma'_2 + \sigma'_3) \quad (9)$$

$$q = \frac{1}{\sqrt{2}} \sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2}$$

Furthermore, as can be seen from Figs. 1(b) and 2(b), all the effective stress states for post-liquefaction loading always lie on a Critical Stress state Line (CSL). If Mohr-Coulomb criterion is used to describe those stress states, they may be determined as

$$\frac{q}{p'} = M_{CS,o} \quad (10)$$

in which $M_{CS,o}$ is the post-liquefaction critical deviator-isotropic stress ratio, obtained from post-liquefaction cyclic undrained tests.

From Eq. (10), a field function may be written as

$$F = q - M_{CS,o} \cdot p' \quad (11)$$

Plastic flow of a saturated sand under a post-liquefaction undrained loading is assumed to be associated, and therefore,

$$\begin{aligned} Q &= F = q - M_{CS,o} \cdot p' \\ \frac{\partial Q}{\partial p'} &= \frac{\partial F}{\partial p'} = -M_{CS,o} \\ \frac{\partial Q}{\partial q} &= \frac{\partial F}{\partial q} = 1 \end{aligned} \quad (12)$$

Thus, the relationship between ε_v^p and γ^p can be derived from Eqs. (8) and (12) and given by

$$\frac{d\varepsilon_v^p}{d\gamma^p} = -M_{CS,o} \quad (13)$$

or

$$d\varepsilon_v^p = -M_{CS,o} \cdot d\gamma^p \quad (14)$$

This equation shows a linear change in ε_v^p with γ^p which does not relate with change in effective stress. For a post-liquefaction sand subjected to shear in a state of zero effective confining stress, we may write $d\varepsilon_v^p = d\varepsilon_{vd, re}$ and $d\gamma^p = d\gamma_o$, because 1) elastic strains can be omitted, 2) $\gamma_d = d\varepsilon_{vc} = 0$ for zero change in effective stress, and 3) only change in $\varepsilon_{vd, re}$ depends on current change in shear strain. Therefore, $d\varepsilon_v^p$ may be regarded as being the increment of $\varepsilon_{vd, re}$ induced by the increment of γ_o independent of effective stress, or,

$$d\varepsilon_{vd, re} = -M_{CS, o} \cdot d\gamma_o \quad (15)$$

Relation between γ_o and $\varepsilon_{vd, ir}$ Based on Concept of Relative Compression

A series of cyclic undrained triaxial tests followed by drained consolidation for saturated sands was conducted by Shamoto et al. (1995) in order to predict the earthquake-induced settlements in sand deposits. This study shows that the volume change of various sands over a wide density range can be uniquely related to relative compression as defined by Eq. (16).

$$R_c = \frac{\Delta e}{e_o - e_{\min}} \quad (16)$$

in which e_o = initial void ratio, $e_{\min}$ = minimum void ratio, and Δe = change in void ratio. The study has also shown that the experimental relationship between the logarithm of the relative compression R_c and the logarithm of the maximum shear strain $\gamma_{\max}$ induced during undrained cyclic loading is approximately linear, namely

$$R_c = R_o \cdot \gamma_{\max}^m \quad (17)$$

in which R_o and m are constant coefficients determined by tests.

The effective confining stress state in a saturated, isotropically consolidated sample undergoes a change when subjected to cyclic undrained loading. The effective confining stress state reached after such a cyclic loading can return to its initial state by means of drained testing in which the induced excess pore water pressure dissipates to zero while maintaining the initial confining stress constant. Because both of $\varepsilon_{vd, re}$ and ε_{vc} as shown in Eq. (3) return to zero at the state corresponding to zero excess pore pressure after the dissipation, the final volumetric strain measured in the drained testing can only be the irreversible volumetric strain component $\varepsilon_{vd, ir}$. In this way, the $\varepsilon_{vd, ir}$ -value induced at any one moment of a cyclic undrained loading process can be determined.

Based on the above consideration and Eqs. (16) and (17), we can write $\varepsilon_{vd, ir}$ as

$$\varepsilon_{vd, ir} = \frac{\Delta e}{1 + e_o} = \frac{e_o - e_{\min}}{1 + e_o} R_c = \frac{e_o - e_{\min}}{1 + e_o} R_o \cdot \gamma_{\max}^m \quad (18)$$

This equation indicates that the current $\varepsilon_{vd, ir}$ -value depends on the maximum volume decrease potential ($e_o - e_{\min}$) and the preceding $\gamma_{\max}$ -value.

A Constitutive Relation for Evaluating γ_o

Considering that γ_o is triggered almost at state of zero effective confining stress, and substituting the complete undrained condition of Eq. (4) and then Eq. (18) into Eq. (15), we can obtain:

$$\gamma_o = -\frac{\varepsilon_{vd, re}}{M_{CS, o}} = -\frac{\varepsilon_{vd, ir} + \varepsilon_{vc}}{M_{CS, o}} = \frac{1}{M_{CS, o}} \left(\frac{e_o - e_{\min}}{1 + e_o} R_o \cdot \gamma_{\max}^m + \varepsilon_{vc, o} \right) \quad (19)$$

in which $\varepsilon_{vc, o} = \varepsilon_{vc}$ at state of zero effective confining stress as defined before, and $\varepsilon_{vc, o} < 0$. Since the $\gamma_{\max}$ -value at $\gamma_o = 0$ marks the start of γ_o and also is the minimum shear strain triggering initial liquefaction, we call it 'entrance shear strain' denoted as γ_{entry} . Substituting $\gamma_o = 0$ and $\varepsilon_{vc} = \varepsilon_{vc, o}$ into Eq. (19) gives:

$$\gamma_{entry} = \left[-\frac{1 + e_o}{R_o(e_o - e_{\min})} \cdot \varepsilon_{vc, o} \right]^{1/m} \quad (20)$$

This equation shows that γ_{entry} depends only on the initial void ratio and $\varepsilon_{vc, o}$ -values of which both are constant for sand of a given density subjected to a constant confining stress. It appears that γ_{entry} may be regarded as an index to judge whether initial liquefaction and γ_o occur.

Since γ_{entry} and $\gamma_{\max}$ after initial liquefaction are more than 1%, we take $\gamma_{\max}^m - \gamma_{entry}^m \approx (\gamma_{\max} - \gamma_{entry})^m$ approximately for convenience of parameter determination. Thus, Eq. (19) becomes

$$\gamma_o = \frac{R'_o}{M_{CS, o}} \frac{e_o - e_{\min}}{1 + e_o} (\gamma_{\max} - \gamma_{entry})^m \quad (21)$$

It should be noted that constant R_o in Eq. (19) is replaced with another constant R'_o in consideration of the probable difference of both them in magnitude due to the above stated approximation. Equation (21) implies that the current γ_o -value may be evaluated by the preceding $\gamma_{\max}$ -value for saturated sand with a given density.

CONSTITUTIVE RELATIONS FOR POST-LIQUEFACTION SHEAR STRAIN COMPONENT DEPENDING ON CHANGE IN EFFECTIVE STRESS

Based on the previous studies by Pradhan et al. (1989), an experimental stress-dilatancy equation for saturated sand subjected to cyclic drained shear may be expressed as

$$\frac{q}{p'} = M_o - \alpha \frac{d\varepsilon_{vd}}{d\gamma^p} \quad (22)$$

in which ε_{vd} = volumetric strain due to dilatancy, γ^p = plastic shear strain, M_o = deviator-isotropic stress ratio at zero dilatancy, and α = a constant coefficient. Equation (22) is identical to the original Cam clay flow rule at $\alpha = 1$.

For describing large post-liquefaction deformation of saturated sands, elastic deformation may be neglected, and therefore, $d\gamma^p = d\gamma_d$. In addition, it is seen from Fig. 3 that the accumulative value of $\varepsilon_{vd, ir}$ increases, but its

rate of change obviously tends to decrease with increasing number of cycles in the past. Conversely, the rate of change in $\varepsilon_{vd, re}$ exhibits an increase with growing rate in γ_d . It can thereby be reasoned that $d\varepsilon_{vd, ir} \ll d\varepsilon_{vd, re}$ or $d\varepsilon_{vd} = d\varepsilon_{vd, re} + d\varepsilon_{vd, ir} \approx d\varepsilon_{vd, re}$ during the further undrained loading on a sand having experienced a larger shear strain γ_o at state of zero effective stress. Thus, Eqs. (4) and (22) may be simplified:

$$\varepsilon_{vd, re} + \varepsilon_{vc} = 0 \quad (23)$$

$$\frac{q}{p'} = M_o - \alpha \frac{d\varepsilon_{vd, re}}{d\gamma_d} \quad (24)$$

Furthermore, the volumetric strain component of expansion ε_{vc} due to change in p' may be described by the equation similar to the Martin et al.'s empirical formula (1975):

$$\varepsilon_{vc} = K \left(\frac{p'_i}{p_a} \right)^A \left(\frac{p'}{p'_i} \right)^B \quad (25)$$

in which p_a =barometric pressure, 100 kPa, p'_i =initial effective confining pressure, p' =current effective confining pressure, K , A and B =constant coefficients determined by drained unloading tests.

In addition, the effective stress path for the post-liquefaction loading phase in an undrained condition always lies on the Critical Stress state Line (CSL) as shown in Figs. 1(b) and 2(b). Therefore, $q = M_{CS} p'$, where M_{CS} =slope of CSL. It should be noted that M_{CS} is different from $M_{CS, o}$ defined in Eq. (10), since the latter indicates the critical deviator-isotropic stress ratio near the state of zero effective confining stress. Based on Eqs. (23), (24) and (25) as well as $q = M_{CS} p'$, the shear strain component γ_d can be evaluated by

$$\gamma_d = \frac{\alpha K}{M_{CS} - M_o} \left(\frac{p'_i}{p_a} \right)^A \left(\frac{q}{M_{CS} \cdot p'_i} \right)^B \quad (26)$$

EXPERIMENTAL BASIS OF PROPOSED POST-LIQUEFACTION CONSTITUTIVE RELATIONS

Constitutive Relations for Post-Liquefaction Deformation

Based on those studies, large post-liquefaction shear strain may be evaluated by Eqs. (1), (21), and (26), namely

$$\gamma = \gamma_o + \gamma_d \quad (1)$$

$$\gamma_o = \frac{R'_o}{M_{CS, o}} \frac{e_o - e_{min}}{1 + e_o} (\gamma_{max} - \gamma_{entry})^m \quad (21)$$

$$\gamma_d = \frac{\alpha K}{M_{CS} - M_o} \left(\frac{p'_i}{p_a} \right)^A \left(\frac{q}{M_{CS} \cdot p'_i} \right)^B \quad (26)$$

and moreover, post-liquefaction volumetric strain may be determined by Eqs. (4), (18), (27), and (25), or

$$\varepsilon_v = \varepsilon_{vd, re} + \varepsilon_{vd, ir} + \varepsilon_{vc} = 0 \quad (4)$$

$$\varepsilon_{vd, ir} = \frac{\Delta e}{1 + e_o} = \frac{e_o - e_{min}}{1 + e_o} R_c = \frac{e_o - e_{min}}{1 + e_o} R_o \cdot \gamma_{max}^m \quad (18)$$

$$\begin{aligned} \varepsilon_{vd, re} &= -M_{CS, o} \int_0^{\gamma_o} d\gamma_o - \frac{M_{CS} - M_o}{\alpha} \int_0^{\gamma_d} d\gamma_d \\ &= -M_{CS, o} \gamma_o - \frac{M_{CS} - M_o}{\alpha} \gamma_d \end{aligned} \quad (27)$$

$$\varepsilon_{vc} = K \left(\frac{p'_i}{p_a} \right)^A \left(\frac{p'}{p'_i} \right)^B \quad (25)$$

in which there are eleven constant coefficients: $M_{CS, o}$, M_{CS} , M_o , γ_{entry} , m , R_o , R'_o , α , A , B , and K that need to be determined by tests. Note that the reversible volumetric strain $\varepsilon_{vd, re}$ can be induced at states of both zero and non-zero effective confining stress, and correspondingly, its magnitude may be evaluated respectively by Eqs. (15) and (24). In general, the $\varepsilon_{vd, re}$ -value induced during post-liquefaction undrained monotonic shear is accumulated by the reversible volumetric strains produced at states of zero and non-zero effective stress. Based on Eqs. (15) and (24), therefore, Eq. (27) provides a formula for calculating $\varepsilon_{vd, re}$ as the sum of the two integrals with respect to shear strain paths at states of zero and non-zero effective stress.

Experimental Relationship between Current γ_o and Preceding γ_{max}

In order to examine the correlation between the current γ_o and the preceding γ_{max} shown in Eq. (21), the results of cyclic undrained torsion tests on the saturated Toyoura sand were analyzed. Figure 5 shows the time history of a typical test result in which the induced shear strain component γ_o and the preceding maximum double amplitude of cyclic shear strain γ_{max} are plotted. In order to more clearly indicate the method for determining γ_o -value, the portion in Fig. 5 within the dotted box is enlarged and shown in Fig. 6. This figure provides a criterion with which the γ_o -value is determined as the shear strain induced at state of nearly zero effective

Fig. 5. Time history of a typical cyclic undrained torsion test on a hollow cylindric sample: (a) shear stress ratio, (b) excess pore pressure, (c) shear strain

Fig. 6. Determination of shear strain component γ_o induced at zero effective stress state (the data shown in this figure are the same as in Fig. 5)

Fig. 7. Relationship between current γ_o and preceding γ_{max} for samples of different densities

confining stress at which the excess pore pressure Δu ranges from 99.5 kPa to 100 kPa for a constant initial confining pressure σ'_c of 100 kPa.

Considering that the relative density D_r of sands in an actual subsoil profile related with engineering problems usually ranges from about 40% to 80%, the samples used in this study were prepared within such a density range. Figure 7 shows a good correlation existing between γ_{max} and γ_o for samples with different D_r -values which suggests that γ_o increases either with an increase in γ_{max} for the same D_r -value or with a decrease in D_r for the same γ_{max} -value, just as implied in Eq. (21).

Vaid and Thomas (1995) were the first to pay attention to the axial strain ϵ_o induced by monotonic undrained loading at nearly zero effective stress state for saturated, consolidated, and then cyclically liquefied triaxial samples of the looser Fraser sand, and also to the relation of ϵ_o with the preceding maximum double amplitude of cyclic axial strain $\epsilon_{a,max}$. Their data as transformed into a $\gamma_{max}-\gamma_o$ relationship are shown in Fig. 8. It is seen from this figure that γ_o increases with increasing γ_{max} , which is consistent with the results shown in Fig. 7. However, their γ_o -values were determined within the region from the zero effective stress state to a larger 5-kPa axial stress developed, resulting in an overestimation of γ_o particularly for denser sand.

Fig. 8. Shear strain induced near zero effective stress state as a function of maximum double amplitude of cyclic shear strain (modified after Vaid and Thomas, 1995)

Fig. 9. Two patterns of irregular cyclic loadings

Undrained torsion tests of two types of irregular cyclic loadings shown in Fig. 9 were further performed in order to examine the influence of irregular cyclic loading on the $\gamma_{max}-\gamma_o$ relationship. By using the method determining γ_{max} and γ_o as illustrated in Figs. 5 and 6, the $\gamma_{max}-\gamma_o$ relationship was obtained as shown in Fig. 10 for the $D_r=70\%$ samples. Also plotted in the same figure for comparison are the $D_r=70\%$ data from the cyclic undrained loading tests of constant amplitude shown in Fig. 7. The data obtained from the irregular cyclic tests shows good agreement with those from the constant amplitude cyclic tests.

The influence of pre-shear history on the $\gamma_{max}-\gamma_o$ relationship is also investigated on the basis of cyclic undrained torsion tests after smaller amplitude cyclic drained torsion shear application up to 1000 cycles on the saturated, isotropically consolidated Toyoura sand. Comparison between the results for the $D_r=60\%$ samples with and without pre-shear history is indicated in Fig. 11, which shows that the change in γ_o with γ_{max} is not related to the past pre-shear history before cyclic undrained loading.

Based on Eq. (21) and the above experimental results shown in Figs. 7, 10 and 11, the relationships between γ_{max} and γ_o for the different conditions stated before could

Fig. 10. Relationship between current γ_0 and preceding $\gamma_{\max}$ for samples subjected to cyclic loading with constant or irregular amplitude

Fig. 11. Relationship between current γ_0 and preceding $\gamma_{\max}$ for samples with and without pre-shearing history

Fig. 12. Effectiveness of Eq. (28) and determination of coefficients R_0 and m

be universally expressed in terms of the relationship between $\gamma_{\max}$ and $\gamma_0 M_{CS,0}(1+e_0)/(e_0-e_{\min})$, as shown in Fig. 12. This figure shows good correlation, suggesting the effectiveness of the proposed $\gamma_{\max}-\gamma_0$ relationship shown in Eq. (21).

Experimental Analysis Related with Determinations of Coefficients

1) Coefficients $M_{CS,0}$, M_{CS} and M_0 :

As seen from Figs. 1(b) and 2(b), the point of the effective stress state during post-liquefaction undrained loading always moves along CSL, and in addition, the post-liquefaction CSL is unique for a given density. Zhang, Shamoto and Tokimatsu (1995) show that the slope of CSL, M_{CS} , for undrained loading depends principally on the initial relative density over the stress change range associated with actual liquefaction problems, and the q/p' -value at zero dilatancy, which corresponds to that at phase-transformation state, M_0 , remains almost constant for a given sand, as demonstrated in Fig. 13. For example, $M_{CS}=1.20$ and $M_0=0.93$ when $D_r=70\%$. It is assumed that $M_{CS,0}=M_{CS}$ in this study.

2) Coefficients γ_{entry} , m and R'_0 :

As can be seen from the relationship between $\gamma_{\max}$ and $\gamma_0 M_{CS,0}(1+e_0)/(e_0-e_{\min})$ shown in Fig. 12, all the test data lie scattered within a narrow zone. The γ_{entry} -value is found to average 2.65% in the D_r range from 40% to 80%, although it varies depending on $\varepsilon_{vc,0}$ and $\varepsilon_{\min}$ as defined rigorously in Eq. (20). According to curve fitting by the formula (21) at $\gamma_{\text{entry}}=2.65\%$ for the data in Fig. 12, m and R'_0 were determined as 0.76 and 28.1.

3) Coefficient α Related with Dilatancy Rate:

Post-liquefaction drained monotonic torsion tests were run on hollow cylindric torsion shear specimens of the Toyoura sand with different densities. Based on the test results, the relationship between $d\varepsilon_{v,rc}/d\gamma_d$ and deviator-isotropic stress ratio, q/p' ($=\sqrt{3}\tau/\sigma'$ in which τ and σ' are, respectively, shear and normal stresses on horizontal shear plane of a specimen) was obtained as shown in Fig. 14. Good correlation exists between the above two variables. In this case, $\alpha=3\sqrt{3}/2$.

4) Coefficients A , B , and K Related with ε_{vc} :

After M_{CS} , M_0 , and α are determined using the methods illustrated before, A , B , and K in Eq. (26) can be determined directly by fitting curve of the experimental relations between p'/p'_i ($=q/M_{CS}p'_i$) and γ_d ($=\gamma-\gamma_0$) obtained from post-liquefaction monotonic undrained loading tests at different (p'_i/p_a) -values for sand

Fig. 13. Deviator-isotropic stress ratios at critical stress and at zero dilatancy for different density samples subjected to cyclic undrained torsion (modified after Zhang, Shamoto and Tokimatsu)

Fig. 14. Relationship between post-liquefaction reversible dilatancy and deviator-isotropic stress ratio during monotonic undrained torsion shear application

Fig. 15. Determination of coefficients K and B in Eq. (28) based on experimental relationship between γ_d and p'/p'_i

Fig. 16. Comparison between post-liquefaction shear stress-shear strain relationships measured by tests and calculated by proposed constitutive relationships

samples of different D_r . Figure 15 shows a result of the fitting curve for the case of $D_r=70\%$ and $p'_i=100$ kPa in which $B=0.44$ and $K=0.554$. Because the post-liquefaction monotonic undrained loading tests in this study were performed only under $p'_i=100$ kPa or $(p'_i/p_o)=1$, the coefficient A could not be obtained. In principle, however, there is no difficulty to determine it. In addition,

the methods used to determine R_o in Eq. (18) been proposed by the authors (1996). It has been shown that R_o averages 3.69 respectively, irrespective of type and initial density of sand.

PRELIMINARY VERIFICATION OF PROPOSED CONSTITUTIVE RELATIONS

In order to confirm the effectiveness of the proposed constitutive relations, the relationships between post-liquefaction shear stress $\tau (=q/\sqrt{3})$ and shear strain γ calculated by Eqs. (1), (21) and (26) are compared favorably with the results actually measured by tests, as shown in Fig. 16. This good agreement shows that the proposed constitutive relations can be used for evaluating large post-liquefaction shear deformation in saturated sands.

In principle, the proposed constitutive relations may be adopted for an actual evaluation of the post-liquefaction undrained stress-strain behavior of saturated sands. It means that they cannot be directly used for the conditions where partial drainage needs to be considered or larger initial driving stresses are present.

CONCLUSIONS

The following conclusions may be drawn on the basis of various experimental facts observed in a series of post-liquefaction undrained and drained torsional shear tests of both monotonic and cyclic loading.

1) Large post-liquefaction shear deformation in saturated sands is governed by two types of volumetric strains due to dilatancy, i.e., irreversible dilatancy component, $\epsilon_{v,ir}$, and reversible dilatancy component, $\epsilon_{v,re}$.

2) Shear strain is composed of a shear strain component depending on change in effective stress γ_d , and a shear strain component independent of effective stress γ_o . Post-liquefaction γ_o -value is triggered almost at state of zero effective confining stress, and post-liquefaction γ_d is controlled by deviator-isotropic stress ratio (q/p').

3) The constitutive relationships for evaluating post-liquefaction shear strain in saturated sands have been established on the analysis of mechanism, relative compression concept, theory of plasticity, and other previous available studies. As a consequence, the current γ_o -value is determined only by the preceding maximum shear strain γ_{max} for sand of a given density, and γ_d is determined by good correlation existing between $d\epsilon_{v,re}/d\gamma_d$ and q/p' .

4) The proposed constitutive relations have been preliminarily confirmed to be effective for the evaluation of large post-liquefaction shear deformation.

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NOTATION

- γ = shear strain ($\gamma_d + \gamma_o$)
 γ_d = shear strain component depending on change in effective stress
 γ_o = shear strain component independent of effective stress
 $\gamma_{\max}$ = maximum shear strain, taken as double amplitude of cyclic shear strain in this study
 ε_v = volumetric strain ($\varepsilon_{vd, re} + \varepsilon_{vd, ir} + \varepsilon_{vc}$)
 ε_{vd} = volumetric strain due to dilatancy ($\varepsilon_{vd, re} + \varepsilon_{vd, ir}$)
 $\varepsilon_{vd, re}$ = reversible dilatancy component
 $\varepsilon_{vd, ir}$ = irreversible dilatancy component
 ε_{vc} = volumetric strain component due to change in p'
 $\varepsilon_{vc, o}$ = ε_{vc} -value at state of zero effective confining stress
 M_{CS} = Post-liquefaction undrained critical deviator-isotropic stress ratio
 $M_{CS, o}$ = Post-liquefaction undrained critical deviator-isotropic stress ratio at a very small effective confining stress state

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